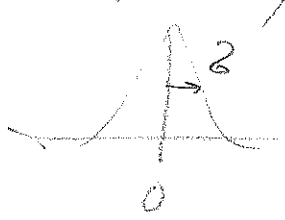
Noise reduction through {image / pixel} averaging (Add on to Slide 15)

N pixels or images : how far is estimate from true mean

zero mean / normal

model: $S_i = J + n_i$ $\quad n_i \in N(0, \sigma^2)$



$\langle n_i \rangle = 0$

$$\bar{S} = \frac{1}{N}\sum_{i=1}^{N} S_i = \frac{1}{N} Js + \underbrace{\frac{1}{N}\sum_i n_i}_{0 \text{ (zero mean)}} = \bar{S}$$

$$E[(\bar{S}-S)^2] = E\left[\left(\frac{1}{N}\sum n_i\right)^2\right] = \frac{1}{N^2} E\left[\left(\sum n_i\right)^2\right]$$

reformulate * ; linear

$$= \frac{1}{N^2} E\left[\sum_i \sum_j n_i n_j\right] = \frac{1}{N^2} \sum_i \sum_j E[n_i n_j]$$

independent, zero mean!

$$= \frac{1}{N^2}\sum_i n_i^2 + \frac{1}{N^2}\sum_{i \neq j} E[n_i]E[n_j]$$

$i = j$: sum of N comp. ; $i \neq j$: $E[n_i] = 0$

$$= \frac{1}{N^2} \cdot N \cdot E[n^2] = \frac{1}{N} E[n_i^2] = \boxed{\frac{1}{N}\sigma^2}$$

→ averaging N images: variance reduced by $\frac{1}{N}$, std reduced by $\frac{1}{\sqrt{N}}$

* $E\left[\left(\sum_{i=1}^{2} n_i\right)^2\right] = E\left((n_1+n_2)^2\right) = E\left[(n_1^2 + 2n_1n_2 + n_2^2)\right]$

$E\left[\left(\sum_{i=1}^{n} n_i\right)^2\right] = E(n_1 + n_2 + \dots n_n)^2 = E(n_1^2 + n_1n_2 + n_1n_3 + \dots)$