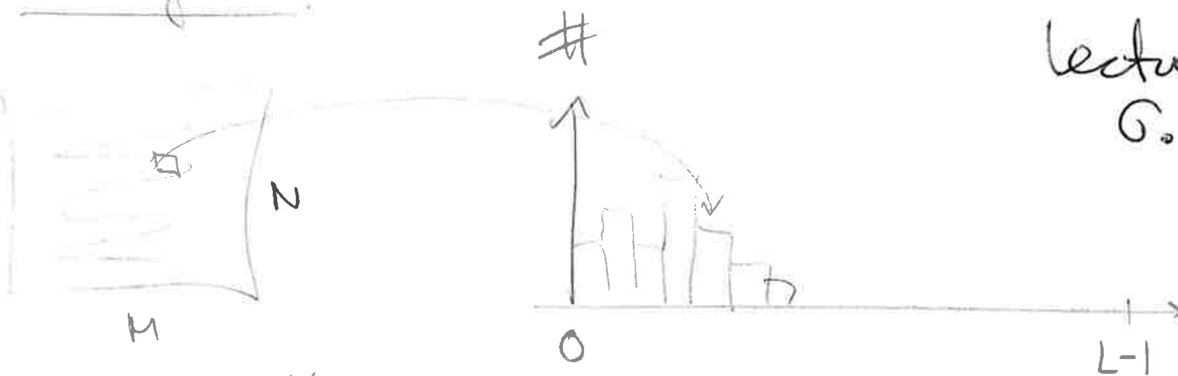
CS / BIOEN 6640 | Book DIP 3 | 9/18/2014 (1)

Histogram: repetition

Lecture Notes
G. Gerig



$\forall$ all $I(x,y)$
do x
do y
hist (I(x,y)) ++

[0 .. L-1] : 0 .. 255
8 bit

? $\int$ histogram $= \sum_{i=0}^{L-1} \text{hist}(i) = ?$ M x N pixels

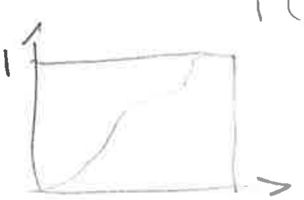
normalize: $p(i) = \frac{\text{hist}(i)}{M \cdot N}$ p(i)

$\sum_{i=0}^{l} p(i) = \frac{M \cdot N}{M \cdot N} = 1$ $\Sigma = 1$ → i

what is probability to find intensity i in image

histogram → probability density function

Cumulative Distribution Function (cdf)



$F(x) = \sum_{i=-\infty}^{x} p(i)$ F(x) monotonic, non decreasing differentiable

Slide 18/19 $F(x) = \int_{-\infty}^{x} f(q)\,dq$

↓

$f(q) \stackrel{?}{=} \left.\frac{d\,F(q)}{dq}\right|_{x} = F'(x)$

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See notes 9/3/2014

Show slides classification:

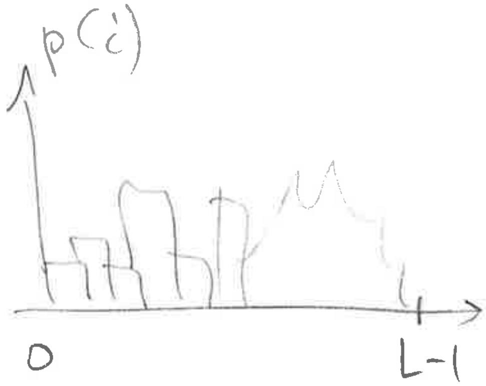
unsupervised   histogram → clustering / thresholds

supervised : select subregions →
calculate estimates of pdf
→ calculate thresholds

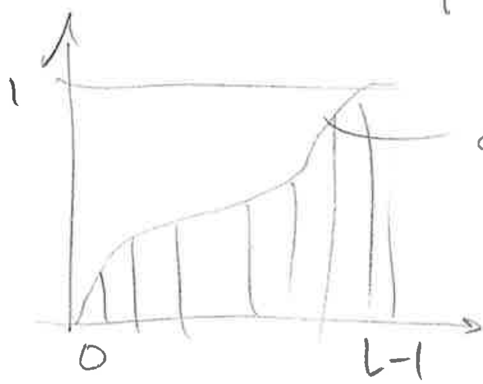
⟹ ⟹ H - equalization

Histogram Equalization   DIP book 3.3.1

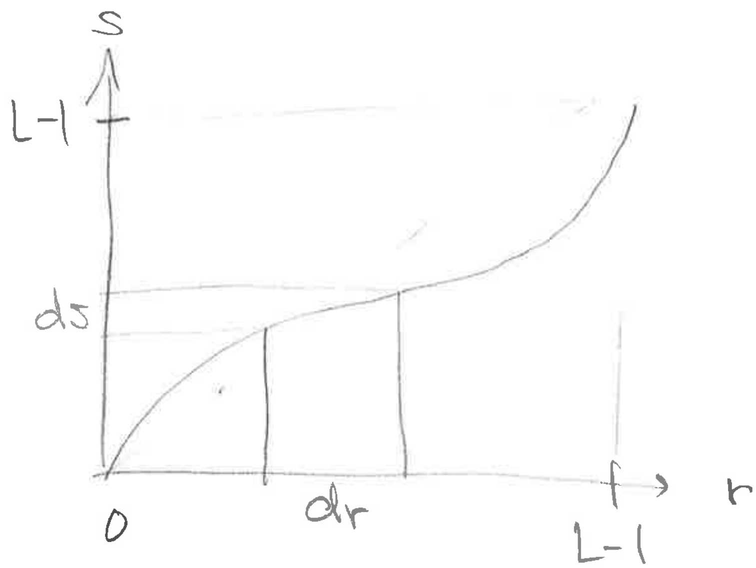
notation: L: luminance, brightness
range: [0 ... L-1]   [0 ... 255]
1 byte images



→



histogram manipulation: Mapping



$s = T(r)$ (out: s, in: r)

assumption:
- $T(r)$ monotonically incr.
- $T(r)$ has inverse

histogram converted to pdf

$p_r(s) = pdf(r)$

$p_s(s) = pdf(s)$

$$p_s(s) = p_r(s) \cdot \left| \frac{dr(r)}{ds} \right|$$

local stretching, contraction determined by $T(r)$

case of identity: $\frac{dr}{ds} = 1 \rightarrow p_s(s) = p_r(s)$

what if $T(r)$ selected as $cdf(r)$?

more precise $(L-1)\, cdf(r)$

$$\frac{ds}{dr} = (L-1)\frac{d\,(cdf(r))}{dr} = ? = (L-1)\,p_r(r)$$

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plug in:

$$p_s(s) = p_r(r) \cdot \left|\frac{dr}{ds}\right| = \cancel{p_r(r)} \cdot \frac{1}{(L-1)\,\cancel{p_r(r)}}$$

pdf of output)

input

$$= \frac{1}{L-1}$$

? what does that mean

all intensities of output same likelihood!

=> histogram shows uniform distribution across whole range!

solution histogram equalization; recipe:

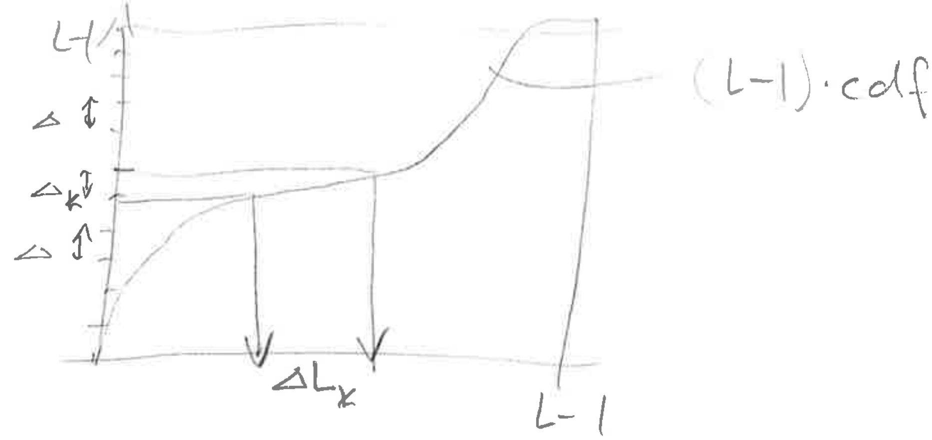
- Range [0 ... L-1]
- calculate histogram

normalize $\frac{hist(\,)}{M \cdot N} \to pdf_r(r)$

- Calculate $cdf(r) = \sum_{r=0}^{L-1} pdf(r)$

- $new\_intensity(x,y) = (L-1)\, cdf\,(intensity(x,y))$

why?



equal sampling of frequency axis

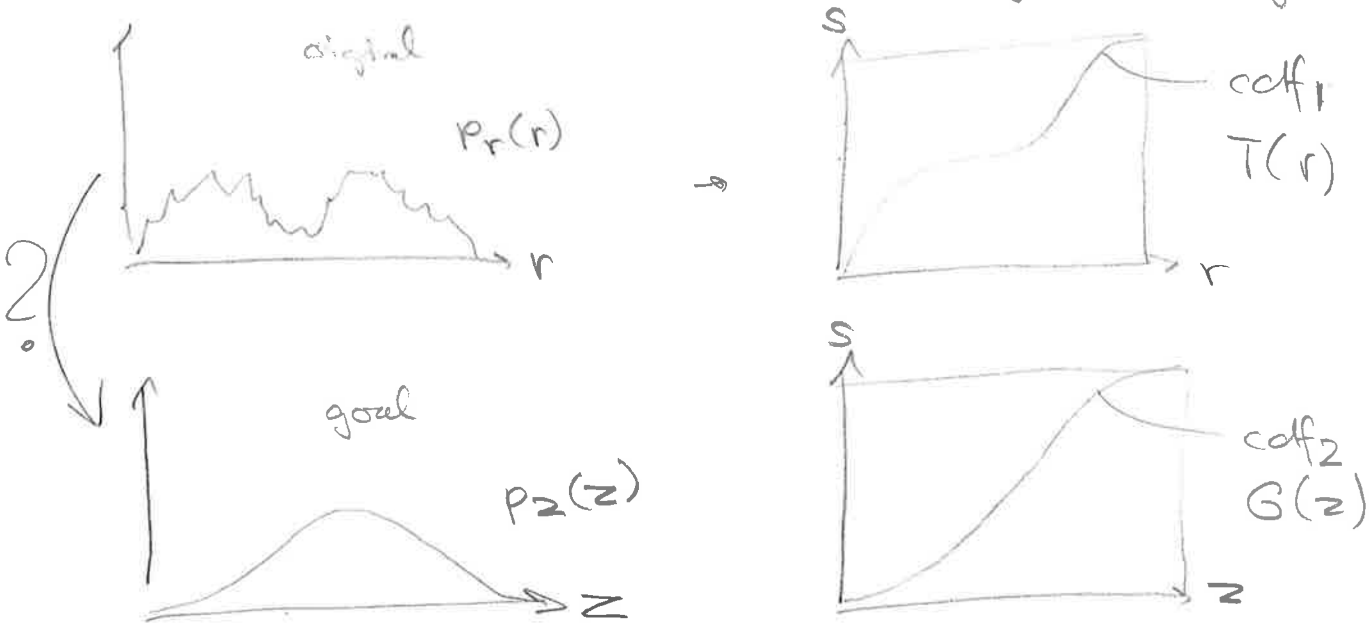
$\Rightarrow$ same "amount of pixels" in Δ_k

in interval ΔL_k

Histogram Matching [DIP book 3.3.2]

uniform is often not the best solution, "artificial"

=> what if we have a specific goal histogram?



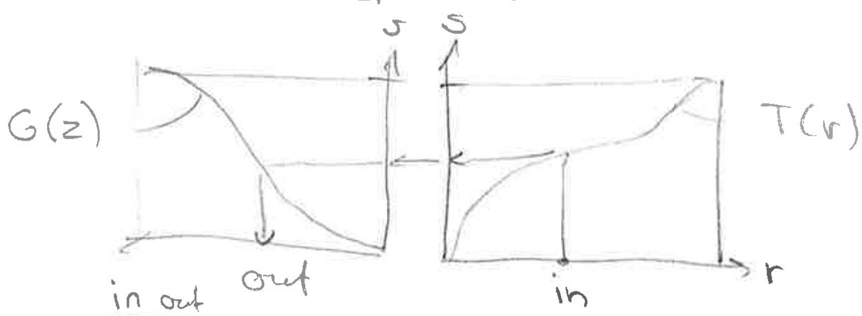
(1) $s = T(r) = cdf1(r)$ s uniform distribution

(2) also: $s = G(z) = cdf2(z)$

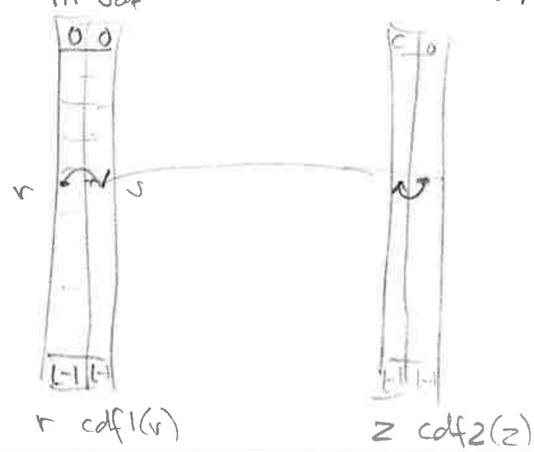
cascading: $r \rightarrow z$?   (1)∘(2)

$$z = G^{-1}(s) = G^{-1}(T(r))$$

| dual mapping?



Discrete:
Look up table



inverse: go to "out" and read "in"