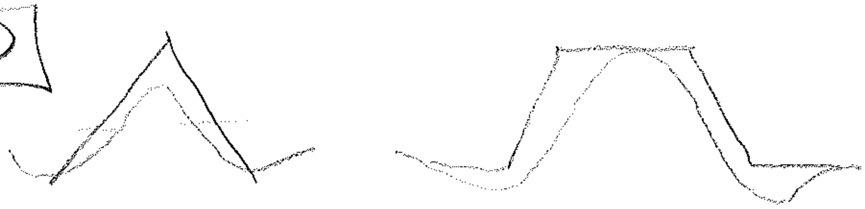
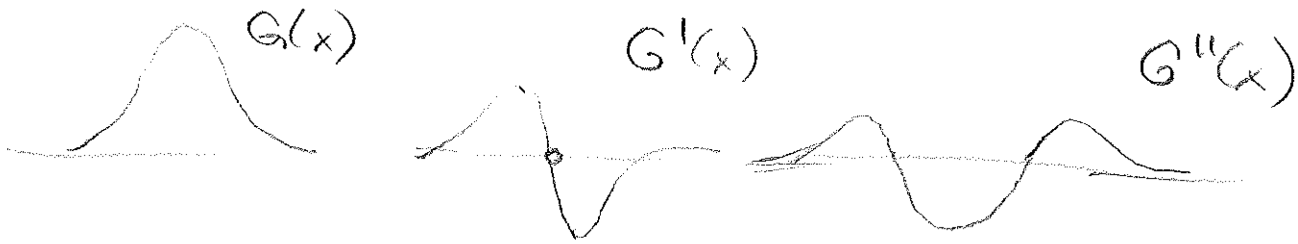
Canny Operators for ridge/line profiles

1-D

slide 28



⇒ resembles 2nd derivative of Gaussian



Filtering: $\left(\frac{d^2}{dx^2} G(x,\sigma)\right) \otimes I(x)$

linear operation:

$$\underbrace{\frac{d^2}{dx^2}}_{\text{2nd derivative}} \underbrace{\left(G(x,\sigma) \otimes I(x) \right)}_{\text{Gaussian smoothed image}}$$

discrete $\boxed{1 \mid -2 \mid 1}^{*} \otimes \left(G(x,\sigma) \otimes I(x) \right)$

convolution: $\frac{d}{dx} \cdot \frac{d}{dx}$: $\boxed{-1 \mid 1} \otimes \boxed{-1 \mid 1} \rightarrow \boxed{-1 \mid 2 \mid -1}$

2nd derivatives

$$\bar{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) \quad : \quad \text{nabla operator}$$

$$\bar{\nabla} L(\bar{x}) = \text{gradient of } L : \begin{pmatrix} L_x \\ L_y \end{pmatrix}$$

$$(1) \rightarrow \quad \bar{\nabla} \cdot (\bar{\nabla} L) = \left(\frac{\partial^2 L}{\partial x^2} + \frac{\partial^2 L}{\partial y^2}\right) \quad \text{Laplacian}$$

$$(2) \rightarrow \quad \underbrace{\bar{\nabla}(\bar{\nabla} L)}_{\text{gradient of gradient of } L} = \begin{bmatrix} \frac{\partial^2 L}{\partial x^2} & \frac{\partial^2 L}{\partial x \partial y} \\ \frac{\partial^2 L}{\partial x \partial y} & \frac{\partial^2 L}{\partial y^2} \end{bmatrix}$$

matrix of second derivatives, Hessian matrix

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(1) Typical kernels for Laplacian:

	1	
1	-4	1
	1	

1	1	1
1	-8	1
1	1	1

$\Rightarrow$ Slides

$$\bar{\nabla} \cdot \bar{\nabla} L = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) \begin{pmatrix} \frac{\partial L}{\partial x} \\ \frac{\partial L}{\partial y} \end{pmatrix} = \frac{\partial^2 L}{\partial x^2} + \frac{\partial^2 L}{\partial y^2}$$

$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} \begin{pmatrix} \frac{\partial L}{\partial x} \\ \frac{\partial L}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 L}{\partial x^2} & \frac{\partial^2 L}{\partial x \partial y} \\ & \end{pmatrix}$$

but: zerocrossings of 2nd derivative: min & max!

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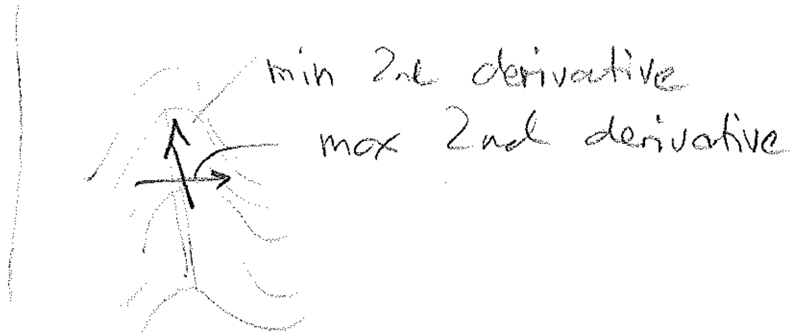
Canny for ridges/lines etc.

2-D

2nd derivative in direction orthogonal to ridge = direction of maximum 2nd derivative



how to calculate?

$$\begin{bmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial x \partial y} \\ \frac{\partial^2}{\partial x \partial y} & \frac{\partial^2}{\partial y^2} \end{bmatrix} \circ I(x) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{yx} & I_{yy} \end{bmatrix}$$

"Hessian" H

Directions of min and max 2nd derivatives?

Diagonalization: $|H - \lambda I| = 0$

Characteristic system

principal 2nd derivatives: eigenvalues $[\lambda_1 \; \lambda_2]$

principal directions: eigenvectors $[\bar{ev}_1, \bar{ev}_2]$

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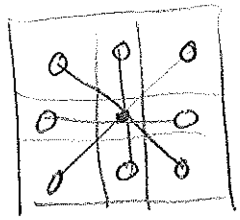
Simplification for discrete implementation



- calculate 2nd derivatives in 4 raster directions
- choose direction where 2nd derivative is extremal (max or min)
- discrete masks for x, y: $\boxed{1 \mid -2 \mid 1}$
- discrete masks for diagonals: $\frac{1}{2}\boxed{1, -2, 1}$ (factor due to $\sqrt{2}$ spacing of pixels)

Summary 2nd derivative operator

- Gaussian blurring of image (scale): $G(\vec{x},\sigma) \otimes I(\vec{x})$
- build 2nd derivatives (discrete or via Hessian)
- choose extremal 2nd derivative perpendicular to ridge $\rightarrow$ pixel output
- please note that you get positive and negative output for dark and bright lines