Geometric Transformations and Image Warping: Mosaicing

CS 6640
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faculty.cs.tamu.edu/jchai/cpsc641_spring10/lectures/lecture8.ppt
Applications
Microscopy (Morane Eye Inst, UofU, T. Tasdizen et al.)
Mosaic Procedure

Basic Procedure

– Take a sequence of images from the same position
  • Rotate the camera about its optical center
– Compute transformation between second image and first
– Transform the second image to overlap with the first
– Blend the two together to create a mosaic
– If there are more images, repeat
Image Mosaic

Is a pencil of rays contains all views

Can generate any synthetic camera view as long as it has the same center of projection!
Image Re-projection

The mosaic has a natural interpretation in 3D

- The images are reprojected onto a common plane
- The mosaic is formed on this plane
- Mosaic is a *synthetic wide-angle camera*
Issues in Image Mosaic

How to relate two images from the same camera center?
  - image registration

How to re-project images to a common plane?
  - image warping
3D Perspective and Projection

• Camera model

\[ \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = f \begin{pmatrix} x/z \\ y/z \\ 1 \end{pmatrix} \]

Image coordinates
Image Homologies

• Images taken under cases 1, 2 are perspectively equivalent to within a linear transformation

  – Projective relationships – equivalence is

\[
\begin{pmatrix}
  a \\
  b \\
  c \\
\end{pmatrix} \equiv \begin{pmatrix}
  d \\
  e \\
  f \\
\end{pmatrix} \iff \begin{pmatrix}
  a/c \\
  b/c \\
  1 \\
\end{pmatrix} = \begin{pmatrix}
  d/f \\
  e/f \\
  1 \\
\end{pmatrix}
\]
Transformations

Translation  
Scaled Rigid  
Horizontal Shear  
Perspective  
Terrain Relief

\[
\begin{pmatrix}
X' \\
Y' \\
1
\end{pmatrix} = \begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
x \\
y \\
1
\end{pmatrix}
\]

\[
\begin{pmatrix}
X' \\
Y' \\
W
\end{pmatrix} = \begin{pmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & 1
\end{pmatrix} \begin{pmatrix}
x' \\
y' \\
1
\end{pmatrix}
\]

New image coordinates can be found as \( x' = X'/W, y' = Y'/W \)

affine

\( x', y' \) : homographies
Materials

- Excellent material to derive homography matrix:
  - www.cs.toronto.edu/~jepson/csc2503/tutorial2.pdf
Perspective Projection Properties

- Lines to lines (linear)
- Conic sections to conic sections
- Convex shapes to convex shapes
- Foreshortening
Transforming Images To Make Mosaics

Linear transformation with matrix P

\[
\bar{x}^* = P\bar{x}
\]

\[
P = \begin{pmatrix}
p_{11} & p_{12} & p_{13} \\
p_{21} & p_{22} & p_{23} \\
p_{31} & p_{32} & 1
\end{pmatrix}
\]

\[
x^* = p_{11}x + p_{12}y + p_{13}
\]

\[
y^* = p_{21}x + p_{22}y + p_{23}
\]

\[
z^* = p_{31}x + p_{32}y + 1
\]

Perspective equivalence

\[
x' = \frac{p_{11}x + p_{12}y + p_{13}}{p_{31}x + p_{32}y + 1}
\]

\[
y' = \frac{p_{21}x + p_{22}y + p_{23}}{p_{31}x + p_{32}y + 1}
\]

Multiply by denominator and reorganize terms

\[
p_{31}xx' + p_{32}yx' - p_{11}x - p_{12}y - p_{13} = -x'
\]

\[
p_{31}xy' + p_{32}yy' - p_{21}x - p_{22}y - p_{23} = -y'
\]

Linear system, solve for P

\[
\begin{pmatrix}
-x_1 & -y_1 & -1 & 0 & 0 & 0 & x_1 & y_1 \\
-x_2 & -y_2 & -1 & 0 & 0 & 0 & x_2 & y_2 \\
\vdots & & & & & & \vdots & \vdots \\
-x_N & -y_N & -1 & 0 & 0 & 0 & x_N & y_N \\
0 & 0 & 0 & -x_1 & -y_1 & -1 & x_1 & y_1 \\
0 & 0 & 0 & -x_2 & -y_2 & -1 & x_2 & y_2 \\
\vdots & & & & & & \vdots & \vdots \\
0 & 0 & 0 & -x_N & -y_N & -1 & x_N & y_N
\end{pmatrix}
\begin{pmatrix}
p_{11} \\
p_{12} \\
p_{13} \\
p_{21} \\
p_{23} \\
p_{31} \\
p_{32}
\end{pmatrix}
= \begin{pmatrix}
-x_1' \\
x_2' \\
\vdots \\
x_N'
\end{pmatrix}
\]
Image Stitching

Stitch pairs together, blend, then crop
Image Stitching

A big image stitched from 5 small images
Image Mosaicing
4 Correspondences
5 Correspondences
6 Correspondences
Mosaicing Issues

- Need a canvas (adjust coordinates/origin)
- Blending at edges of images (avoid sharp transitions)
- Adjusting brightnesses
- Cascading transformations
Recognizing panoramas

• A fully automatic 2D image stitcher system
Recognizing panoramas

- A fully automatic 2D image stitcher system

- Image matching with SIFT features
- For every image, find the M best images with RANSAC
- Form a graph and find connected component in the graph
- Stitching and blending.
Automotive Solutions

Intensity Based Image Mosaicing

- **Transformation:**

\[
x_i' = \frac{m_0x_i + m_1y_i + m_2}{m_6x_i + m_7y_i + 1}
\]

\[
y_i' = \frac{m_3x_i + m_4y_i + m_5}{m_6x_i + m_7y_i + 1}
\]

- **Problem:** Determining the transformation parameters \(m_i\) between every two adjacent images, in order to merge the set of images into a single complete image.

- **Idea:** to choose the parameters \(m_i\) such that the sum of squared difference between all pixels between the two images is minimized

\[
E = \sum_i \left[ I'(x_i', y_i') - I(x_i, y_i) \right]^2 = \sum_i e_i^2
\]

- Non-linear minimization, e.g. by Levenberg Marquardt algorithm

- See: [http://www.umiacs.umd.edu/~hismail/Mosaic/node2.html#SECTION00020000000000000000](http://www.umiacs.umd.edu/~hismail/Mosaic/node2.html#SECTION00020000000000000000)