Snakes: Active Contour Models

CS 6640
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(some material from Zoltan Kato http://www.cab.u-szeged.hu/~kato/variational/)
Materials

- Trucco and Verri, Introductory Techniques for 3-D Computer Vision, Chapter 5
- Bryan Morse, Lecture 21 Image Understanding
Websites

• Andy Witkin’s homepage:  
  http://www.cs.cmu.edu/~aw/

• Terzopoulos: http://www.cs.ucla.edu/~dt/vision.html  
  → Snakes: Active Contour Models

• Original snake demo: Kass, Witkin, Terzopoulos  

• Other Demos:  
  – Xu/Prince: http://www.iacl.ece.jhu.edu/static/gvf/  
  – http://users.ecs.soton.ac.uk/msn/book/new_demo/  
  – http://www.markschulze.net/snakes/index.html
Snake’s Energy Function

• Position of the snake $v(s) = (x(s), y(s))$
• $E_{\text{snake}} = \int \left[ E_{\text{int}} v(s) + E_{\text{image}} v(s) + E_{\text{con}} v(s) \right] ds$

  – Internal: Internal energy due to bending. Serves to impose piecewise smoothness constraint
  – Image: Image forces pushing the snake toward image features (edges, etc…)
  – Constraints: External constraints are responsible for putting the snake near the desired local minimum
Internal Energy

\[ E_{\text{int}} = \alpha(s) |v_s(s)|^2 + \beta(s) |v_{ss}(s)|^2 \]

- First order term: membrane, \( \alpha(s) \): "elasticity"
- Second order term: thin plate, \( \beta(s) \): "rigidity, stiffness"
- If \( \alpha(s) = \beta(s) = 0 \), we allow breaks in the contour
Image Forces

• Edge Functional: negative gradient magnitude: \( E_{\text{edge}} = -|\nabla I(x,y)|^2 \)

• Better: negative gradient magnitude of Gaussian-smoothed image:

\[
E_{\text{edge}} = -|\nabla G(\sigma) \otimes I(x,y)|^2
\]

→ Attracts the snake to locations of large gradients = strong edges
External Constraint Forces

- Springs: add $-k(x_1-x_2)^2$ to $E_{con}$
- Volcano: $1/r^2$ repulsion force, combine with image potential
External Constraint Forces: Spring
Original Demo Kass et al., 1988

http://www.cs.ucla.edu/~dt/videos/deformable-models/snakes.avi
Numerical Solutions

\[-\frac{d^2}{ds^2} \left( \frac{\partial E}{\partial \left( \frac{d^2x}{ds^2} \right)} + \frac{\partial E}{\partial \left( \frac{d^2y}{ds^2} \right)} \right) + \frac{d}{ds} E_{v_s} - E_v = 0\]
Numerical Solutions

• The spline \( v(s) \) which minimizes \( E^*_{\text{snake}} \) must satisfy

\[
- \frac{d^2}{ds^2} \left( \frac{\partial E}{\partial \left( \frac{d^2 x}{ds^2} \right)} + \frac{\partial E}{\partial \left( \frac{d^2 y}{ds^2} \right)} \right) + \frac{d}{ds} E_{v_s} - E_v = 0
\]

• Solutions: Greedy local updates, Euler Lagrange etc. (see handouts)
Using Snakes for Dynamic Scenes

- Once a snake finds a feature, it "locks on"
- If feature begins to move, the snake will track the same local minimum
- Fast motion could cause the snake to flip into a different minimum
Example Movie Sequences