Canny Operators for ridge/line profiles

1-D

= resembles 2nd derivative of Gaussian

\[ G(x) \quad G'(x) \quad G''(x) \]

Filter:

\[ (\frac{d^2}{dx^2} G(x,\theta)) \otimes I(x) \]

Linear operation:

\[ \frac{d^2}{dx^2} (G(x,\theta) \otimes I(x)) \]

2nd derivative Gaussian smoothed image

discrete

\[ \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \otimes (G(x,\theta) \otimes I(x)) \]

convolve: \[ \frac{d}{dx} \frac{d}{dx} : \begin{bmatrix} 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 2 & -1 \end{bmatrix} \]
2nd derivatives

\[ \nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right) : \text{ nabla operator} \]

\[ \nabla L(\mathbf{x}) = \text{ gradient of } L : \left( \frac{L_x}{L_y} \right) \]

\[ \Rightarrow \quad \nabla \left( \nabla L \right) = \left( \frac{\partial^2 L}{\partial x^2} + \frac{\partial^2 L}{\partial y^2} \right) \quad \text{Laplacian} \]

\[ \Rightarrow \quad \nabla \left( \nabla L \right) = \begin{bmatrix} \frac{\partial^2 L}{\partial x^2} & \frac{\partial^2 L}{\partial x \partial y} \\ \frac{\partial^2 L}{\partial y \partial x} & \frac{\partial^2 L}{\partial y^2} \end{bmatrix} \quad \text{matrix of second derivatives, Hessian matrix} \]

1. Typical kernel for Laplacian:

\[ \nabla \left( \nabla L \right) = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix} \quad \Rightarrow \text{ Slicks} \]

2. Example:

\[ \nabla \left( \nabla L \right) = \left( \frac{\partial^2 L}{\partial x^2}, \frac{\partial^2 L}{\partial x \partial y} \right) = \left( \frac{\partial^2 L}{\partial x^2} + \frac{\partial^2 L}{\partial y^2} \right) \]

\[ \left( \begin{array}{cc} \frac{\partial}{\partial x} & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \end{array} \right) \quad \text{bad: zero crossing of 2nd derivative, min & max!} \]
2-D 2nd derivative in direction orthogonal to ridge = direction of maximum 2nd derivative

\[ \begin{bmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial x \partial y} \\ \frac{\partial^2}{\partial y \partial x} & \frac{\partial^2}{\partial y^2} \end{bmatrix} I(x) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{yx} & I_{yy} \end{bmatrix} \]

Hessian \( H \)

how to calculate?

Direction of min and max 2nd derivatives?

Diagonalization: \( |H - \lambda I| = 0 \)

Characteristic system: principal 2nd derivatives: eigenvalues \( \lambda_1, \lambda_2 \)

principal directions: eigenvectors \( \{\overline{v}_1, \overline{v}_2\} \)
Simplification for discrete implementation

- Calculate 2nd derivatives in 4 raster directions
- Choose direction where 2nd derivative is extremal (max or min)

- Discrete mask for x,y: \[
\begin{bmatrix}
1 & -2 & 1
\end{bmatrix}
\]
- Discrete mask for diagonal: \[
\frac{1}{2} \begin{bmatrix}
1 & -2 & 1
\end{bmatrix}
\]
  (taper due to \(\sqrt{2}\) spacing of pixels)

Summary 2nd derivative operator

- Gaussian blurring of image (scale): \(G(\sigma_0) \ast I(x)\)
- Build 2nd derivatives (discrete or via Hessian)
- Choose extremal 2nd derivative perpendicular to ridge \(\Rightarrow\) pixel output
- Please note that you get positive and negative output for dark and bright lines