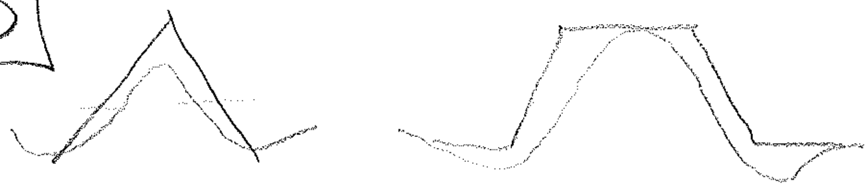
GG | 11/3/2010

Canny Operators for ridge/line profiles

$\boxed{\text{1-D}}$



$\Rightarrow$ resembles 2nd derivative of Gaussian

Filtering: $\left(\frac{d^2}{dx^2} G(x,\sigma)\right) \otimes I(x)$

linear operation:

$$\underbrace{\frac{d^2}{dx^2}}_{\text{2nd derivative}} \Big(\underbrace{G(x,\sigma) \otimes I(x)}_{\text{Gaussian smoothed image}} \Big)$$

discrete $\boxed{1 \,|\, -2 \,|\, 1}^{*} \otimes \left(G(x,\sigma) \otimes I(x) \right)$

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convolution: $\frac{d}{dx} \cdot \frac{d}{dx}$: $\boxed{-1 \,|\, 1} \otimes \boxed{-1 \,|\, 1} \rightarrow \boxed{-1 \,|\, 2 \,|\, -1}$

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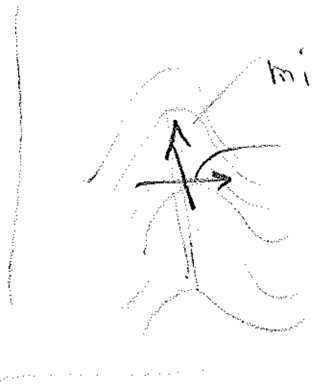
②

Canny for ridges/lines cted.

$\boxed{2\text{-D}}$ 2nd derivative in direction orthogonal to ridge = direction of maximum 2nd derivative



how to calculate?

$$\begin{bmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial x \partial y} \\ \frac{\partial^2}{\partial x \partial y} & \frac{\partial^2}{\partial y^2} \end{bmatrix} \circ I(x) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{yx} & I_{yy} \end{bmatrix}$$

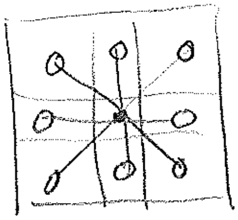
"Hessian" H

Directions of min and max 2nd derivatives?

Diagonalization: $|H - \lambda I| = 0$
Characteristic system
principal 2nd derivatives: eigenvalues $[\lambda_1 \ \lambda_2]$

principal directions: eigenvectors $[\bar{ev}_1, \bar{ev}_2]$

Simplification for discrete implementation



- calculate 2nd derivatives in 4 raster directions
- choose direction where 2nd derivative is extremal (max or min)
- discrete masks for x, y: $\boxed{1 \mid -2 \mid 1}$
- discrete masks for diagonals: $\frac{1}{2}\boxed{1, -2, 1}$ (factor due to $\sqrt{2}$ spacing of pixels)

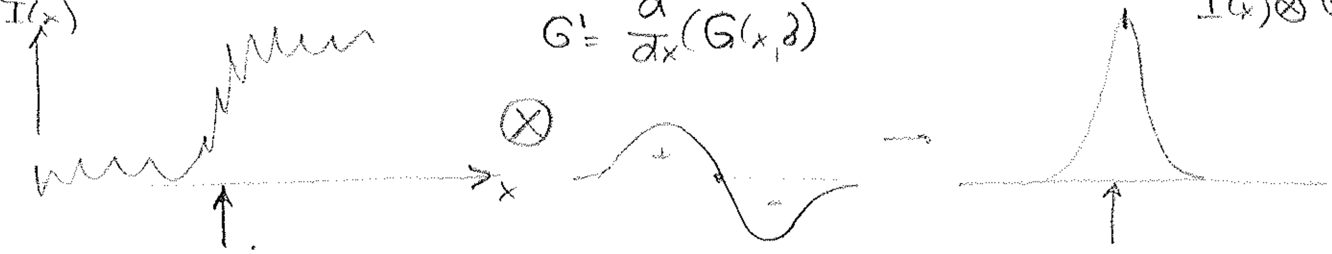
Summary 2nd derivative operator

- Gaussian blurring of image (scale); $G(\bar{x},\sigma) \otimes I(\bar{x})$
- build 2nd derivatives (discrete or via Hessian)
- choose extremal 2nd derivative perpendicular to ridge $\longrightarrow$ pixel output
- please note that you get positive and negative output for dark and bright lines

Nonmaximum suppression

1-D: (i.e., detection of maxima)

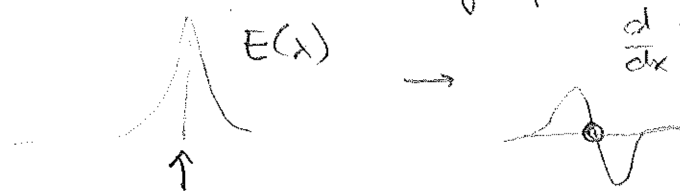
$G' = \frac{d}{dx}(G(x,\sigma))$ $\qquad$ $I(x) \otimes G' = E(x)$



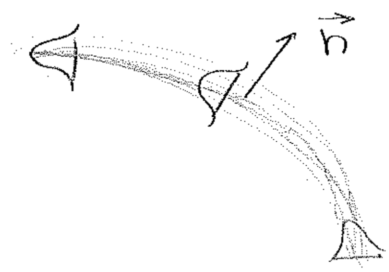
noisy step edge $\qquad$ edge filter

detection of peak: - neighbors smaller than center

- analytical: zerocrossing of derivative



2-D: Peak detection along normal direction = gradient direction

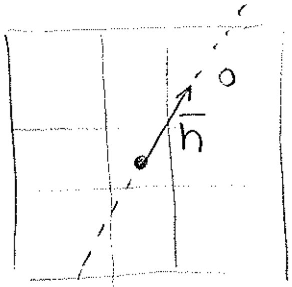


notation: $\nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix}$

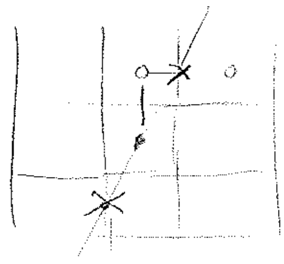
$$\bar{n} = \frac{\nabla(G \otimes I)}{|\nabla(G \otimes I)|}$$

gradient vector

normalization to length 1



- since $\bar{n}$ is continuous,
  exact values have to be interpolated:

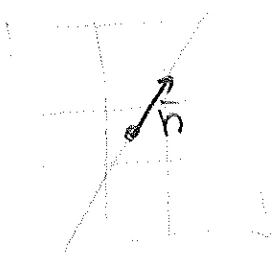


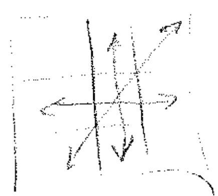
$$G_A = \frac{u_x}{u_y} G(x+1, y+1) + \frac{u_y - u_x}{u_y} G(x, y+1)$$

$$G_B = \ldots$$

Rule: keep center pixel if neighbors have smaller values

- simplification:
  - take $\bar{n}$
  - find neighbors closest to direction of $\bar{n}$
    - calculate angle α, choose discrete direction of neighbors
  - take neighbor pixels and determine peak



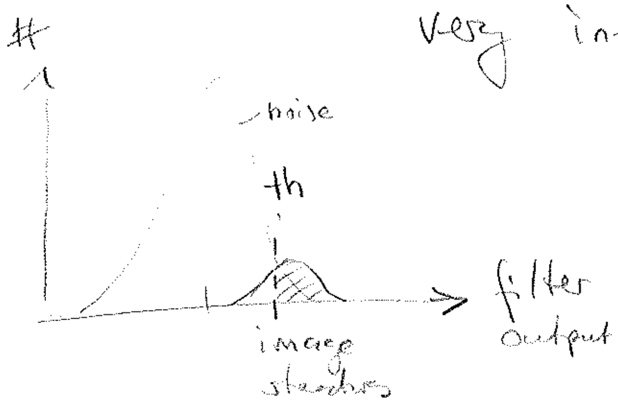


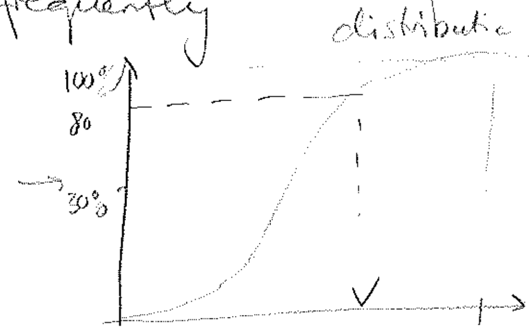
Slide

Canny: Hysteresis thresholding

goal: separation between signal and noise:

- noise: Gaussian distributed, amplitude in general lower than image structures
- edges/lines: large values occurring very infrequently





e.g. threshold at 80% quantile

① Automatic choice of threshold:

→ create histogram of filter output (after non-maximum suppression)

- calculate distribution:

$$d(x) = \int_0^x h(x)\,dx$$

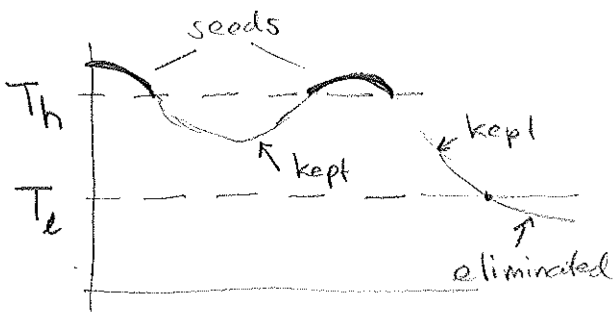
- choose 80% or other high percentile for thresholding the filter output image

Hysteresis thresholding, ctd

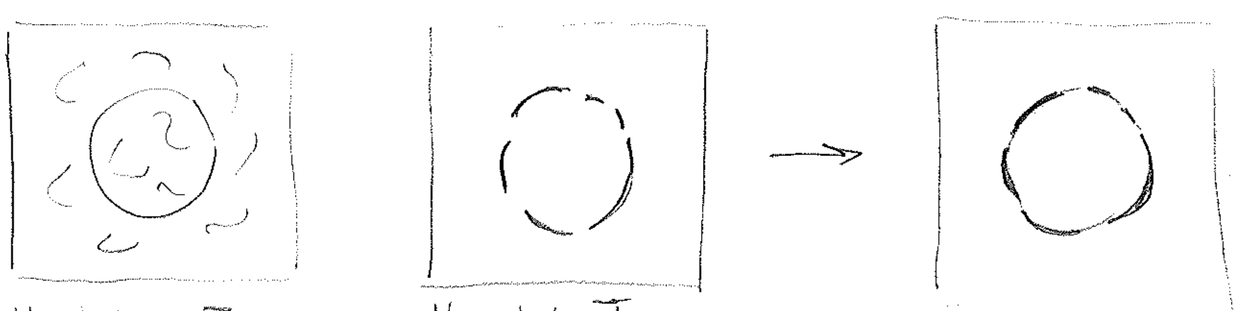
2) Hysteresis thresholding

- observation: after single threshold, edge contours still broken up since values fluctuate above and below the length of a contour

- idea: - low and high thresholds T_ℓ, T_h



- contour parts above T_h: taken as output
- segments connected to those "seed" contours and above lower threshold T_ℓ: taken as output
- connectivity for segments: $\boxed{d_m} = 4/8$
- segments with values above T_ℓ and below T_h, but not connected to "seed $> T_h$" are eliminated



threshold T_ℓ — threshold T_h "seed" contours — hysteresis