Mathematical Morphology: Binary Morphology

CS 650: Computer Vision

Mathematical Morphology

- The analysis of signals and images based on shape (Morphology = "study of shape")
- Uses a set-theoretic approach to modify shapes based on local operators
- Many operations are similar to convolution but use set operations
- Useful for
  - enhancing structural properties
  - segmentation
  - quantitative description

Set Operations on Binary Images

A	the image ("on" pixels)
A^c	the compliment of the image (inverse)
$A \cup B$	the union of images A and B
$A \cap B$	the intersection of images A and B
$A - B = A \cap B^c$	the difference between A and B (the pixels in A that aren't in B)
$\#A$	the cardinality of A (area of the object(s))

Translation

- The image A translated by movement vector t is

$$A_t = \{c \mid c = a + t \text{ for some } a \in A\}$$

- Literally, pick up each pixel in A and move it by the movement vector t

Dilation

- Everywhere the *structuring element* B overlaps the shape:

$$A \oplus B = \{c \mid c = a + b \text{ for some } a \in A \text{ and } b \in B\}$$

- Unioned copies of the shifted image:

$$A \oplus B = \bigcup_{t \in B} A_t$$

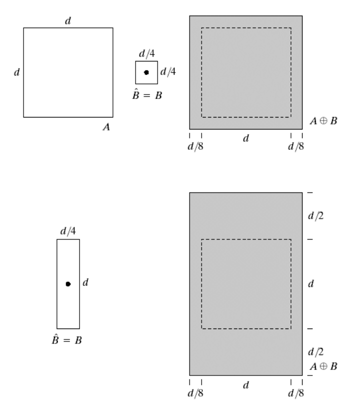
- Unioned copies of the shifted B:

$$A \oplus B = \bigcup_{t \in A} B_t$$

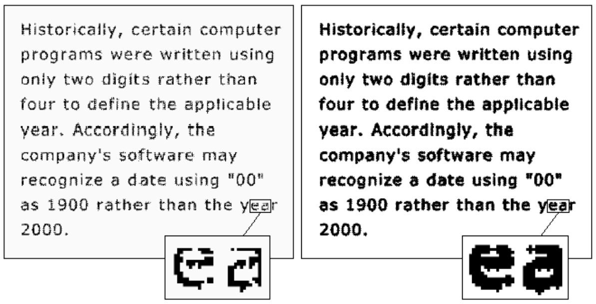
Dilation

a b c
d e

FIGURE 9.4 (a) Set A. (b) Square structuring element (dot is the center). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element.



Dilation



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Properties of Dilation

- Commutative:

$$A \oplus B = B \oplus A$$

- Associative:

$$(A \oplus B) \oplus C = A \oplus (B \oplus C))$$

Erosion

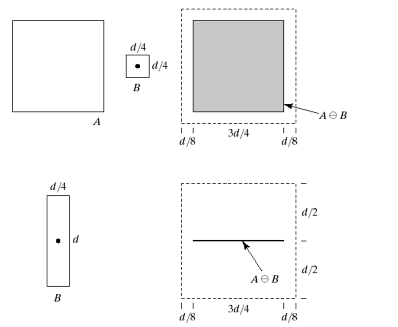
- All the positions where the structuring element B fits *entirely* inside the shape:

$$A \ominus B = \{x \mid x + b \in A \text{ for every } b \in B\}$$

- Intersected copies of the shifted image

$$A \ominus B = \bigcap_{t \in B} A_{-t}$$

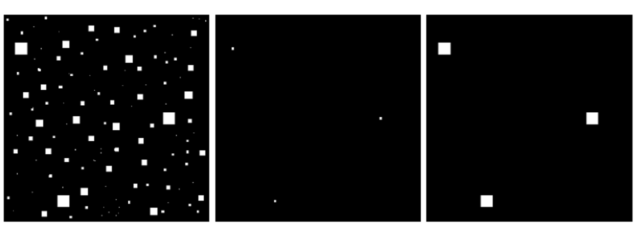
Erosion



a b c
d e

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

Erosion



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Properties of Erosion

- *Not* commutative:

$$A \ominus B \stackrel{?}{=} B \ominus A$$

- *Not* associative:

$$(A \ominus B) \ominus C \stackrel{?}{=} A \ominus (B \ominus C)$$

Duality of Erosion and Dilation

- *Dual* operations are ones that can be defined in terms of each other
- Duals are *not* inverse operations
- Dilation and erosion are dual operations:

$$\begin{aligned}(A \oplus B)^c &= A^c \ominus \breve{B} \\ (A \ominus B)^c &= A^c \oplus \breve{B}\end{aligned}$$

  where $\breve{B}$ denotes the reflection of B
- Intuition: dilating the foreground is the same as eroding the background—the structuring just element flips

Example Application: Finding Boundary Pixels

- Can find all of the boundary pixels by dilating the object and subtracting the original:

$$\text{Bound}_{\text{ext}}(A) = (A \oplus B) - A$$

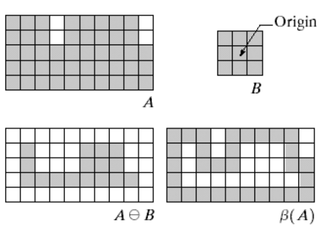
  where B is a 3×3 structuring element containing all 1s
- Or by eroding the original and subtracting that from the original:

$$\text{Bound}_{\text{int}}(A) = A - (A \ominus B)$$

Example Application: Finding Boundary Pixels

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



Hit-and-Miss

- Hit-and-miss operators find target pixel configurations:

$$A \otimes (J, K) = (A \ominus J) \cap (A^c \ominus K)$$

  - J is the target that must be "hit" (foreground)
  - K is the target that must be "missed" (background)
- Example:

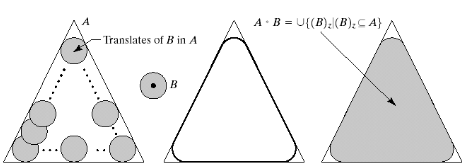
J			K				Effective Target		
0	0	0	0	1	1		x	0	0
1	1	0	0	0	1	→	1	1	0
0	1	0	0	0	0		x	1	x

Opening

- An *opening* operation is an erosion followed by a dilation:

$$A \circ B = (A \ominus B) \oplus B$$

- Used to remove small objects, protrusions, connections



a b c d

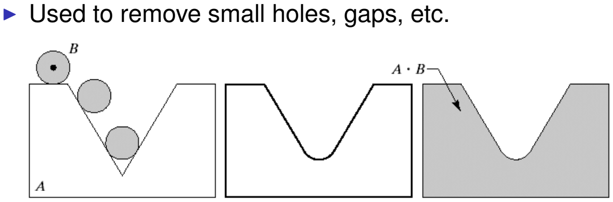
FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Closing

- A *closing* operation is a dilation followed by an erosion:

$$A \bullet B = (A \oplus B) \ominus B$$

- Used to remove small holes, gaps, etc.



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

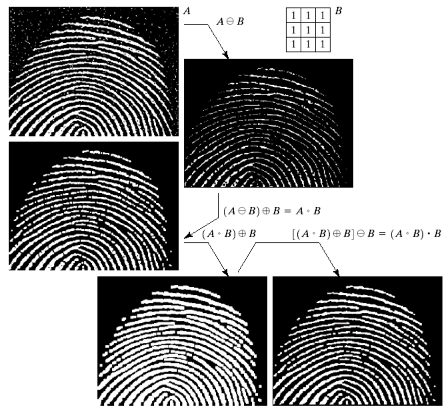
Duality of Opening and Closing

- Opening and closing are also duals:

$$(A \circ B)^c = A^c \bullet \check{B}$$

- Intuition: opening to remove small foreground objects is the same as closing small holes in the background

Example: Opening and Closing for Noise Removal



a b
d c
e f

FIGURE 9.11
(a) Noisy image. (c) Eroded image. (d) Opening of *A*. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Idempotence of Opening and Closing

- An operation is *idempotent* if applying it once is as good as applying it many times
- Opening and closing are idempotent

$$(A \circ B) \circ B = A \circ B$$

$$(A \bullet B) \bullet B = A \bullet B$$