THE REFLECTANCE MAP AND SHAPE-FROM-SHADING

REFLECTANCE MODELS

LAMBERTIAN MODEL

\[ E = L \rho \cos \theta \]

albedo

PHONG MODEL

\[ E = L (a \cos \theta + b \cos^n \alpha) \]

Diffuse albedo

Specular albedo

\[ a = 0.3, \ b = 0.7, \ n = 2 \]

\[ a = 0.7, \ b = 0.3, \ n \approx 0.5 \]
REFLECTANCE MODELS

• Description of how light energy incident on an object is transferred from the object to the camera sensor
REFLECTANCE MAP IS A VIEWER-CENTERED REPRESENTATION OF REFLECTANCE

\[(f_x, f_y, -1) = (0,1,f_x) \times (1,0,f_y)\]
REFLECTANCE MAP IS A VIEWER-CENTERED REPRESENTATION OF REFLECTANCE

\[(f_x, f_y, -1) = (p, q, -1)\]

p, q comprise a gradient or gradient space representation for local surface orientation.

Reflectance map expresses the reflectance of a material directly in terms of viewer-centered representation of local surface orientation.
LAMBERTIAN REFLECTANCE MAP

LAMBERTIAN MODEL

\[ E = L \rho \cos \theta \]

\[ \cos \theta = \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2} \sqrt{1 + p_s^2 + q_s^2}} \]
Grouping L and $\rho$ as a constant, local surface orientations that produce equivalent intensities under the Lambertian reflectance map are quadratic conic section contours in gradient space.

$$E = L\rho \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2} \sqrt{1 + p_s^2 + q_s^2}}$$

$$I = \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2} \sqrt{1 + p_s^2 + q_s^2}}$$
LAMBERTIAN REFLECTANCE MAP

\[ p_s = 0 \quad q_s = 0 \]
LAMBERTIAN REFLECTANCE MAP

\[ p_s = 0.7 \quad q_s = 0.3 \]
LAMBERTIAN REFLECTANCE MAP

\[ p_s = -2 \quad q_s = -1 \]
PHOTOMETRIC STEREO

Derivation of local surface normal at each pixel creates the derived normal map.
NORMAL MAP vs. DEPTH MAP

Surface Orientation

Depth

IMAGE PLANE
NORMAL MAP vs. DEPTH MAP

- Can determine Depth Map from Normal Map by integrating over gradients p,q across the image.
- Not all Normal Maps have a unique Depth Map. This happens when Depth Map produces different results depending upon image plane direction used to sum over gradients.
- Particularly a problem when there are errors in the Normal Map.
NORMAL MAP vs. DEPTH MAP

• A Normal Map that produces a unique Depth Map independent of image plane direction used to sum over gradients is called integrable.

• Integrability is enforced when the following condition holds:

\[
\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}
\]
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GREEN’S THEOREM

\[
\iint (\frac{\partial p}{\partial y} - \frac{\partial q}{\partial x}) \, dx \, dy = \oint (p \, dx + q \, dy)
\]
NORMAL MAP vs. DEPTH MAP

VIOLATION OF INTEGRABILITY

Penrose Staircase
SHAPE FROM SHADING

From a monocular view with a single distant light source of known incident orientation upon an object with known reflectance map, solve for the normal map.
SHAPE FROM SHADING

• Formulate as solving the Image Irradiance equation for surface orientation variables p,q:

\[ I(x,y) = R(p,q) \]

• Since this is underconstrained we can’t solve this equation directly

• What do we do ??.
SHAPE FROM SHADING
(Calculus of Variations Approach)

• First Attempt: Minimize error in agreement with Image Irradiance Equation over the region of interest:

\[ \iint_{object} (I(x, y) - R(p, q))^2 \, dx \, dy \]
SHAPE FROM SHADING
(Calculus of Variations Approach)

• Better Attempt: Regularize the Minimization of error in agreement with Image Irradiance Equation over the region of interest:

$$\iint_{\text{object}} p_x^2 + p_y^2 + q_x^2 + q_y^2 + \lambda (I(x, y) - R(p, q))^2 \, dx \, dy$$