# Optical Flow II

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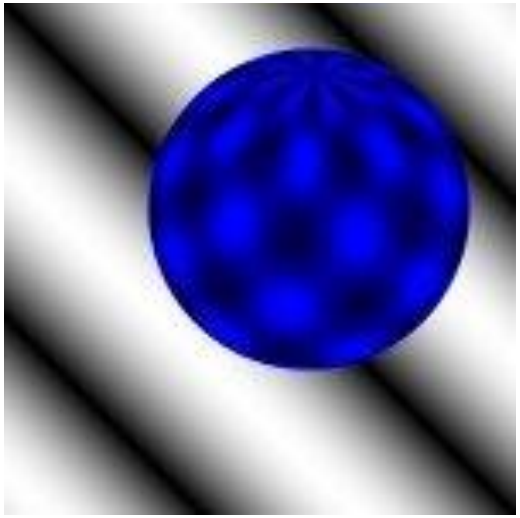
CS 6320, Spring 2013

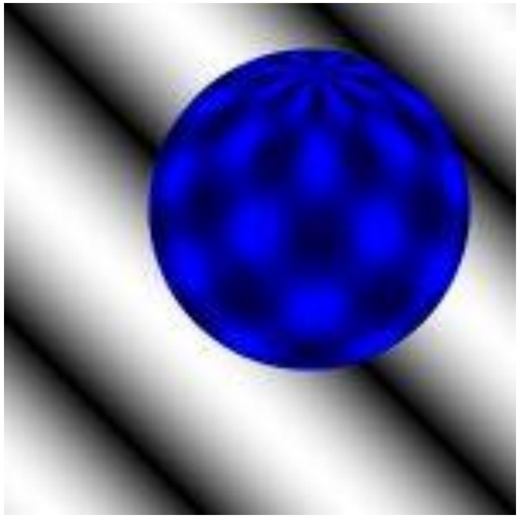
(credits:Pollefeys Comp 256, UNC, Trucco & Verri, Chapter 8, R. Szelisky, CS 223 Fall 2005)

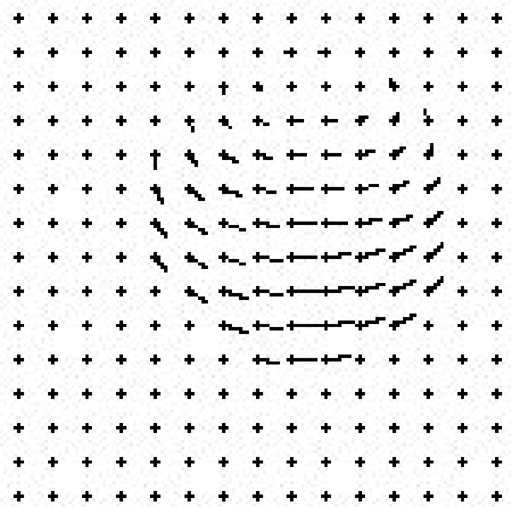
# Material

- R. Szelisky Computer Vision: Chapter 7.1-7.2, Chapter 8
- Trucco & Verri Chapter 8 (handout, pdf)
- Hand-written notes G. Gerig (pdf)
- Horn & Schunck Chapter 9
- Pollefeys CV course (ETH/UNC)
- Richard Szeliski, CS223B Fall 2005

# Structure from Motion?

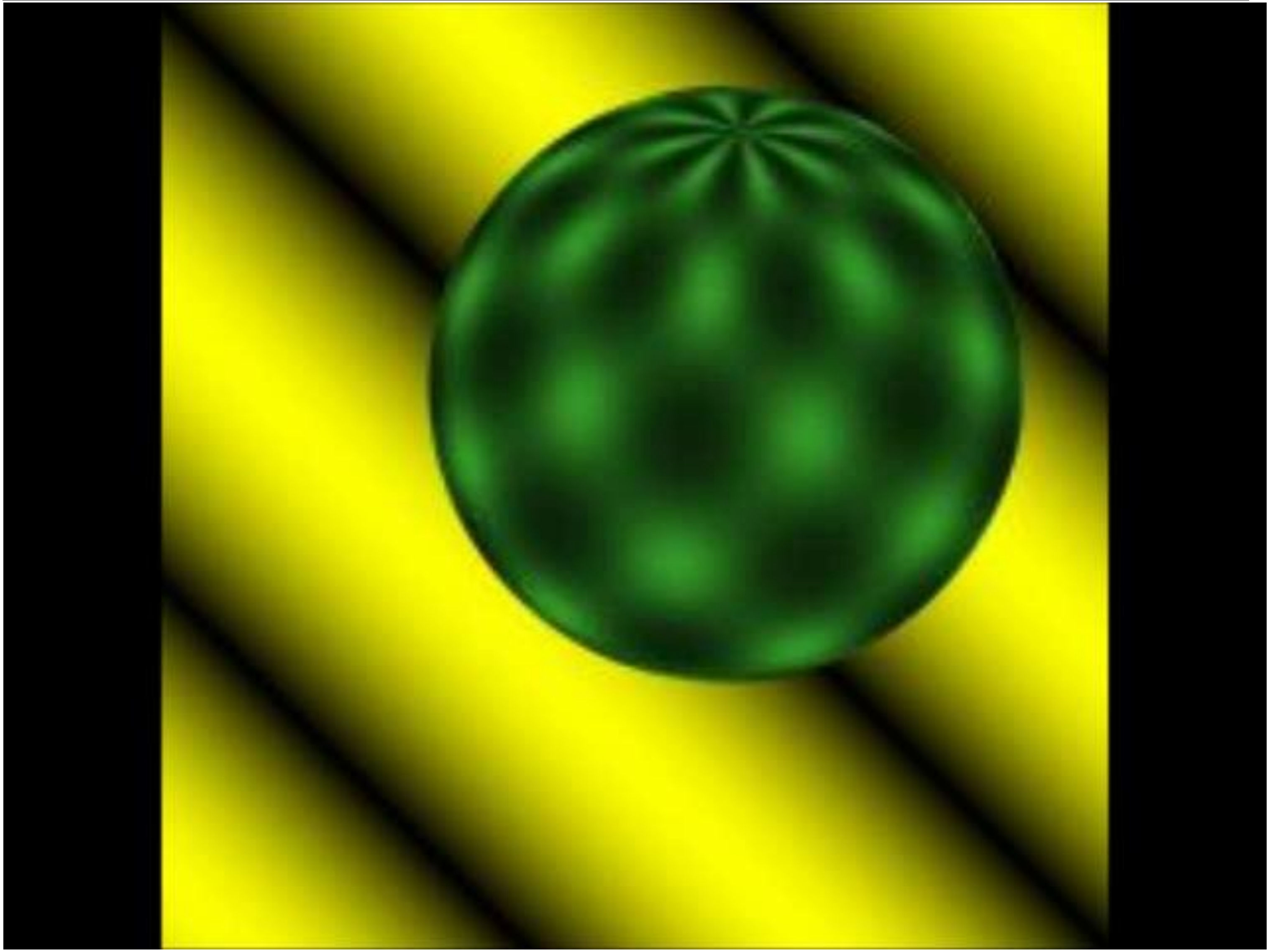






- Known: optical flow (instantaneous velocity)
- Motion of camera / object?

# Structure from Motion?

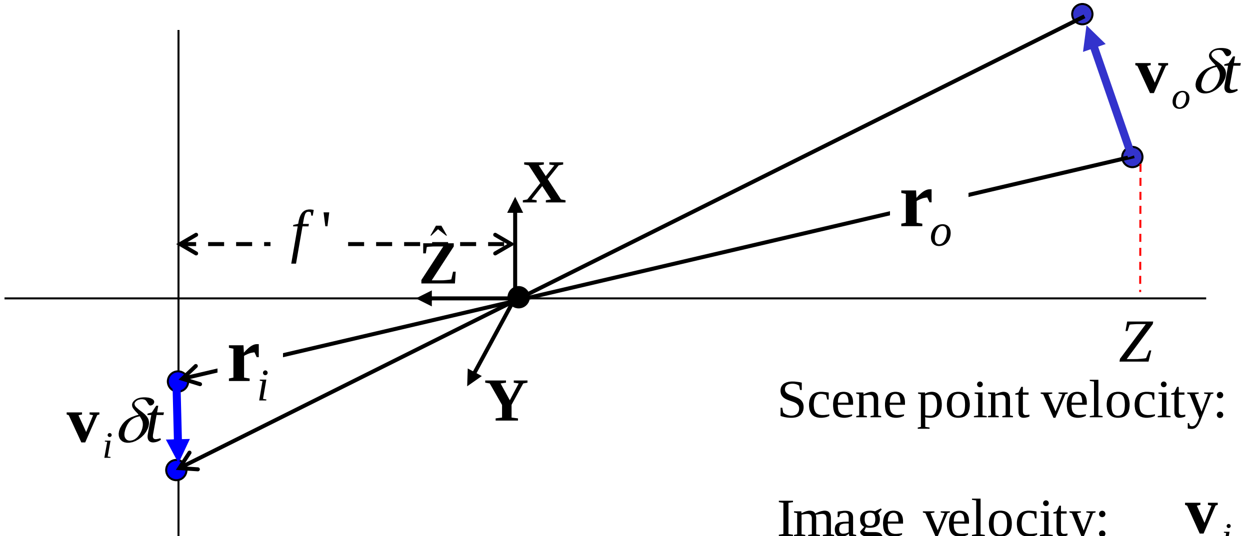


# Optical Flow

- Brightness Constancy
- The Aperture problem
- Regularization
- Lucas-Kanade
- Coarse-to-fine
- Parametric motion models
- Direct depth
- SSD tracking
- Robust flow
- Bayesian flow

# Motion Field: Perspective Projection

## Image velocity of a point moving in the scene




Scene point velocity: $\mathbf{v}_o = \dfrac{d\mathbf{r}_o}{dt}$

Image velocity: $\mathbf{v}_i = \dfrac{d\mathbf{r}_i}{dt}$

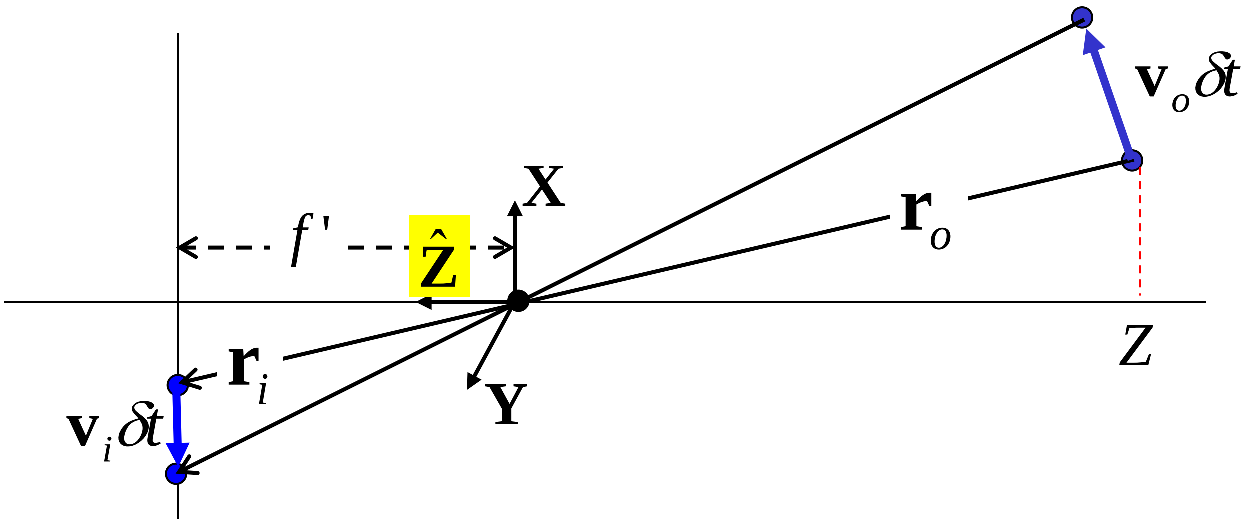
Perspective projection: $\dfrac{1}{f'}\mathbf{r}_i = \dfrac{\mathbf{r}_o}{\mathbf{r}_o \cdot \hat{\mathbf{Z}}} = \dfrac{\mathbf{r}_o}{Z}$

Motion field

$$\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt} = f' \frac{(\mathbf{r}_o \cdot \mathbf{Z})\mathbf{v}_o - (\mathbf{v}_o \cdot \mathbf{Z})\mathbf{r}_o}{(\mathbf{r}_o \cdot \mathbf{Z})^2} = f' \frac{(\mathbf{r}_o \times \mathbf{v}_o) \times \mathbf{Z}}{(\mathbf{r}_o \cdot \mathbf{Z})^2}$$

# Motion Field: Perspective Projection

Image velocity of a point moving in the scene




Motion field

$$\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt} = f' \frac{(\mathbf{r}_o \cdot \hat{\mathbf{Z}})\mathbf{v}_o - (\mathbf{v}_o \cdot \hat{\mathbf{Z}})\mathbf{r}_o}{(\mathbf{r}_o \cdot \hat{\mathbf{Z}})^2} = f' \frac{(\mathbf{r}_o \times \mathbf{v}_o) \times \hat{\mathbf{Z}}}{(\mathbf{r}_o \cdot \hat{\mathbf{Z}})^2}$$

**Discussion**: $\mathbf{v}_i$ is orthogonal to $(\mathbf{r}_o \times \mathbf{v}_o)$ and $\hat{Z} \rightarrow$ in image plane

# Motion Field: Perspective Projection

Motion field

$$\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt} = f' \frac{(\mathbf{r}_o \cdot \hat{\mathbf{Z}})\mathbf{v}_o - (\mathbf{v}_o \cdot \hat{\mathbf{Z}})\mathbf{r}_o}{(\mathbf{r}_o \cdot \hat{\mathbf{Z}})^2} = f' \frac{(\mathbf{r}_o \times \mathbf{v}_o) \times \hat{\mathbf{Z}}}{(\mathbf{r}_o \cdot \hat{\mathbf{Z}})^2}$$

**Set** $\hat{Z} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{1} \end{pmatrix}$ and do the math (see handwritten notes G. Gerig):

$$v_{ix} = \frac{v_{ox} f}{Z} - \frac{x v_{oz}}{Z}$$

$$v_{iy} = \frac{v_{oy} f}{Z} - \frac{y v_{oz}}{Z}$$

# Motion Field: Perspective Projection

$$v_{ix} = \frac{v_{ox} f}{Z} - \frac{x v_{oz}}{Z}$$

$$v_{iy} = \frac{v_{oy} f}{Z} - \frac{y v_{oz}}{Z}$$

Discussion:

Component of optical flow in image only due to $v_x$ and $v_y$ , object motion parallel to image plane.

Component of optical flow in image only due to $v_z$ , object motion towards/away from camera.

# Motion Field: Perspective Projection

$$v_{ix} = \frac{v_{ox} f}{Z} - \frac{x v_{oz}}{Z}$$

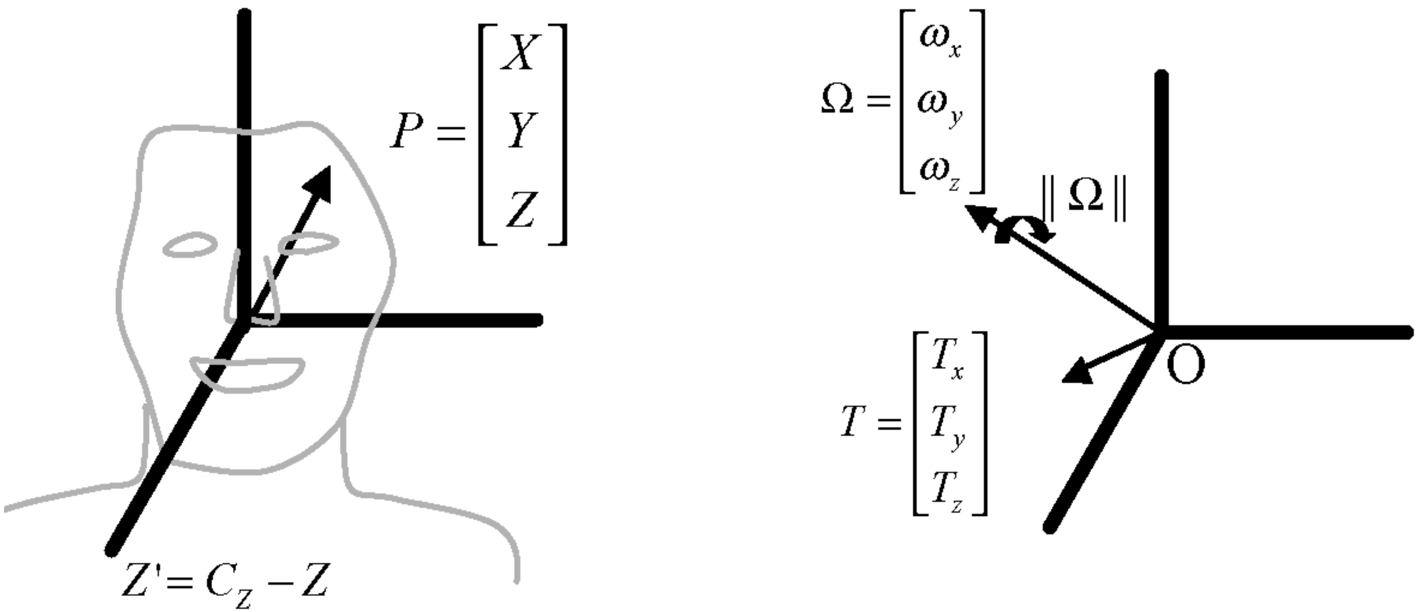
$$v_{iy} = \frac{v_{oy} f}{Z} - \frac{y v_{oz}}{Z}$$

Reformulate: perspective projection of velocity:

$$\begin{bmatrix} v_{ix} \\ v_{iy} \end{bmatrix} = \begin{bmatrix} f & 0 & -x \\ 0 & f & -y \end{bmatrix} \frac{1}{Z} \begin{bmatrix} v_{ox} \\ v_{oy} \\ v_{oz} \end{bmatrix}$$

# Rigid pose estimation

- Head pose model: 6 DOF




Please note notation: T stands for translational motion of object, Ω for rotational component.

# Optic flow for rigid motion

- 3-D velocity:

$$V = T + \Omega \times P = T - \hat{\mathbf{P}}\Omega = \begin{bmatrix} \mathbf{I} & -\hat{\mathbf{P}} \end{bmatrix} \begin{bmatrix} T \\ \Omega \end{bmatrix}$$

$$\hat{\mathbf{P}} = [\mathbf{P}_x]$$
(skew-sym.)

$$V = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & Z & -Y \\ 0 & 1 & 0 & -Z & 0 & X \\ 0 & 0 & 1 & Y & -X & 0 \end{bmatrix} \begin{bmatrix} T \\ \Omega \end{bmatrix}$$

# Optic flow for rigid motion

- Perspective projection

$$\begin{bmatrix} v_{ix} \\ v_{iy} \end{bmatrix} = \begin{bmatrix} f & 0 & -x \\ 0 & f & -y \end{bmatrix} \frac{1}{Z} \begin{bmatrix} v_{ox} \\ v_{oy} \\ v_{oz} \end{bmatrix}$$

# Optic flow for rigid motion

- Combine

$$V=\begin{bmatrix}V_x\\V_y\\V_z\end{bmatrix}=\begin{bmatrix}1&0&0&0&Z&-Y\\0&1&0&-Z&0&X\\0&0&1&Y&-X&0\end{bmatrix}\begin{bmatrix}T\\\Omega\end{bmatrix}$$

$$\begin{bmatrix}v_{ix}\\v_{iy}\end{bmatrix}=\begin{bmatrix}f&0&-x\\0&f&-y\end{bmatrix}\frac{1}{Z}\begin{bmatrix}v_{ox}\\v_{oy}\\v_{oz}\end{bmatrix}$$

# Optic flow for rigid motion

- Rigid Motion (for small v): $\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \mathbf{H} \begin{bmatrix} T \\ \Omega \end{bmatrix}$

$$\mathbf{H} = \begin{bmatrix} f & 0 & -x \\ 0 & f & -y \end{bmatrix} \frac{1}{Z'} \begin{bmatrix} 1 & 0 & 0 & 0 & Z & -Y \\ 0 & 1 & 0 & -Z & 0 & X \\ 0 & 0 & 1 & Y & -X & 0 \end{bmatrix}$$

Perspective projection of 3-D velocity

3-D velocity at point P

Hard to solve with just optic flow vectors! (but see Horn 17.3-17.5).

Convert from scene to image: $\bar{p} = f\,\frac{\bar{P}}{Z}$

# Flow field of rigid motion

In components, and using (8.5), (8.6) read

$$v_x = \frac{T_z x - T_x f}{Z} - \omega_y f + \omega_z y + \frac{\omega_x xy}{f} - \frac{\omega_y x^2}{f}$$

$$v_y = \frac{T_z y - T_y f}{Z} + \omega_x f - \omega_z x - \frac{\omega_y xy}{f} + \frac{\omega_x y^2}{f}.$$

(

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Notice that *the motion field is the sum of two components, one of which depends translation only, the other on rotation only*. In particular, the translational compone of the motion field are

$$v_x^T = \frac{T_z x - T_x f}{Z}$$

$$v_y^T = \frac{T_z y - T_y f}{Z}.$$

and the rotational components are

$$v_x^\omega = -\omega_y f + \omega_z y + \frac{\omega_x xy}{f} - \frac{\omega_y x^2}{f}$$

$$v_y^\omega = \omega_x f - \omega_z x - \frac{\omega_y xy}{f} + \frac{\omega_x y^2}{f}.$$

Trucco & Verri p. 184

# Flow field of rigid motion

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$$v_y^\omega = \omega_x f - \omega_z x - \frac{\omega_y x y}{f} + \frac{\omega_x y^2}{f}.$$

**Discussion**:

- Motion field of translational component depends on T and depth Z. For increasing Z, velocity becomes smaller.
- Motion field that depends on angular velocity does NOT carry information on depth Z!

Trucco & Verri p. 184

# Special Case: Pure Translation

$$v_x = \frac{T_z x - T_x f}{Z}$$

$$v_y = \frac{T_z y - T_y f}{Z}$$

Choose $x_0$ and $y_0$ so that v becomes 0

$$x_0 = f T_x / T_z$$

$$y_0 = f T_y / T_z,$$

$$\rightarrow \quad v_x = (x - x_0) \frac{T_z}{Z}$$

$$v_y = (y - y_0) \frac{T_z}{Z}.$$

Says that motion field of a pure translation is radial, it consists of vectors radiating from a common origin $p_0=(x_0,y_0)$, which is the vanishing point.

Trucco & Verri p. 184/185

See also F&P Chapter 10.1.3 p. 218

# Special Case: Pure Translation

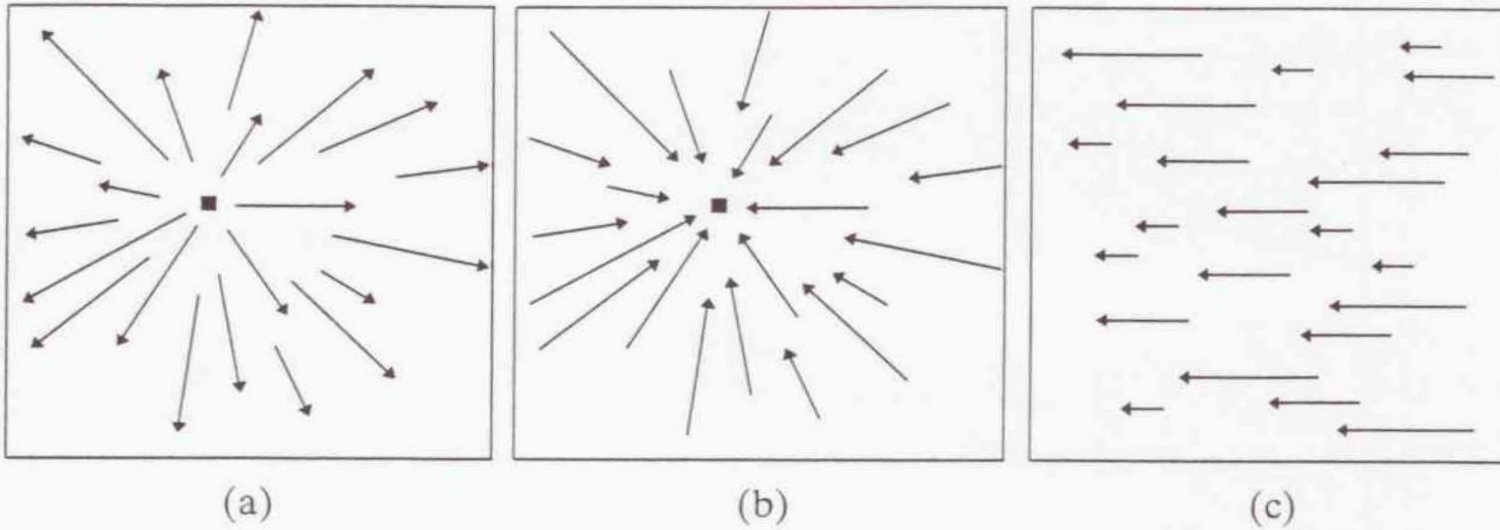



Figure 8.4 The three types of motion field generated by translational motion. The filled square marks the instantaneous epipole.

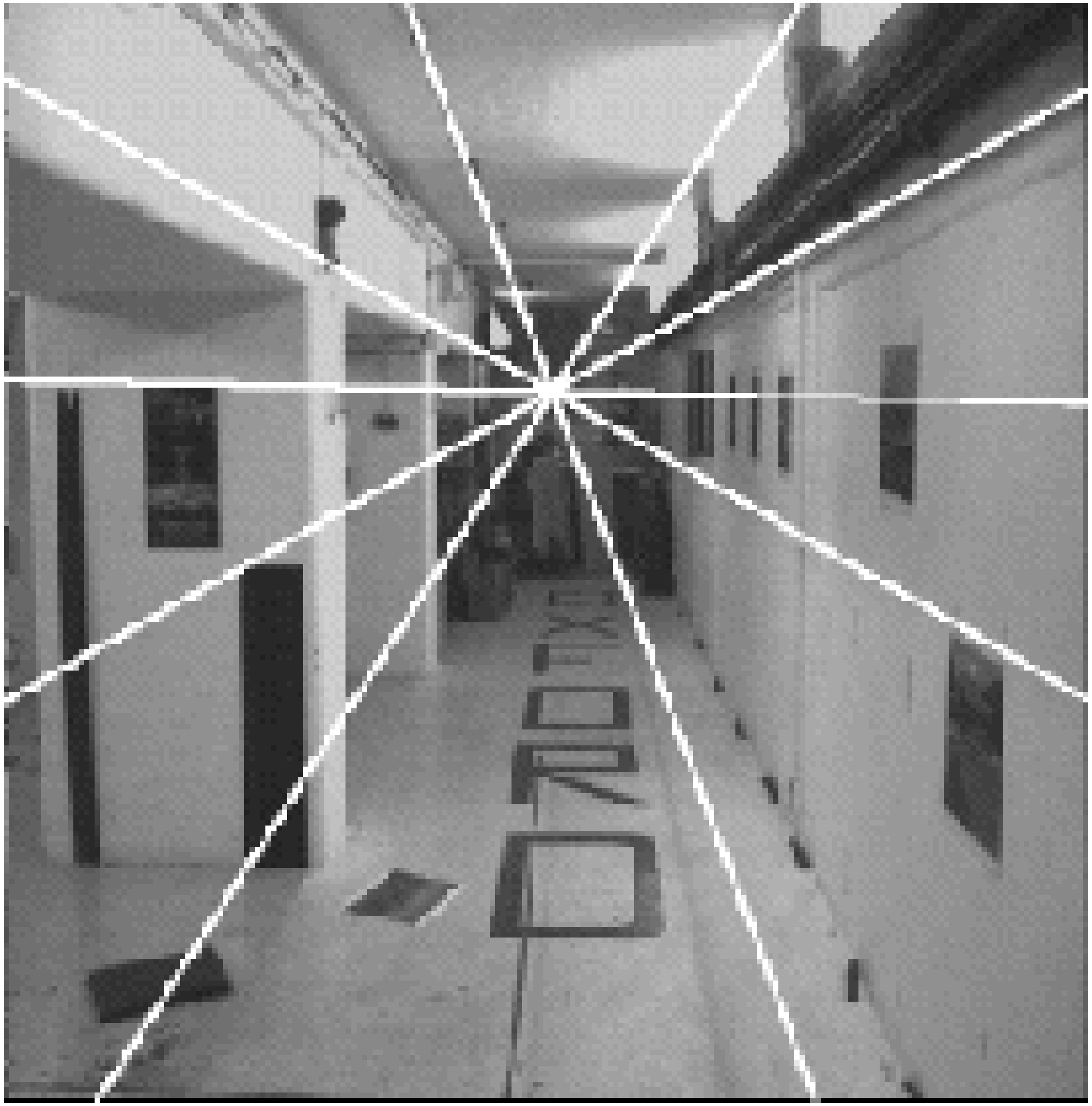
Focus of expansion/contraction:

$$x_0 = f T_x / T_z$$

$$y_0 = f T_y / T_z,$$

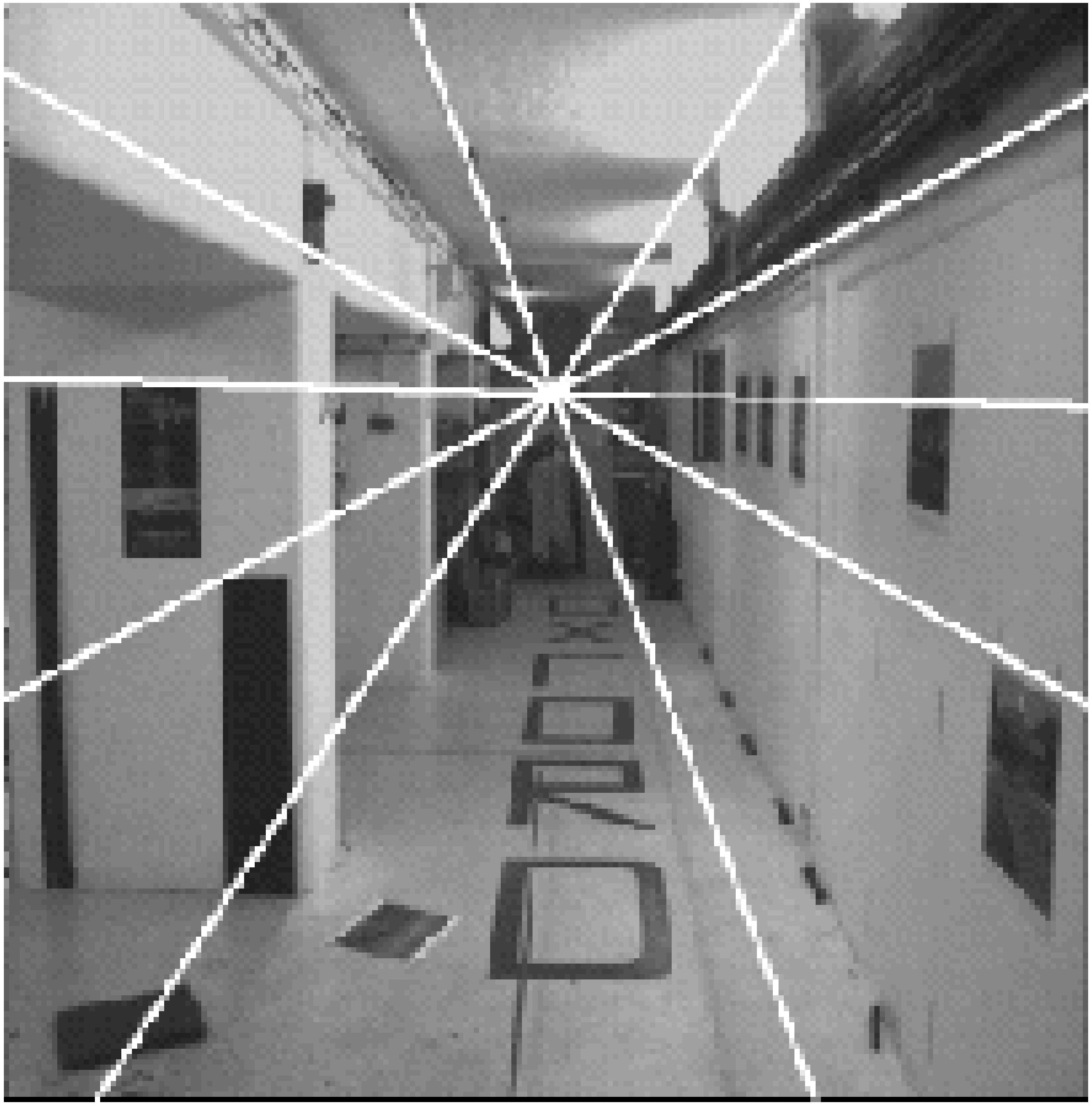
Trucco & Verri p. 184/185
See also F&P Chapter 10.1.3 p. 218

# Example: forward motion



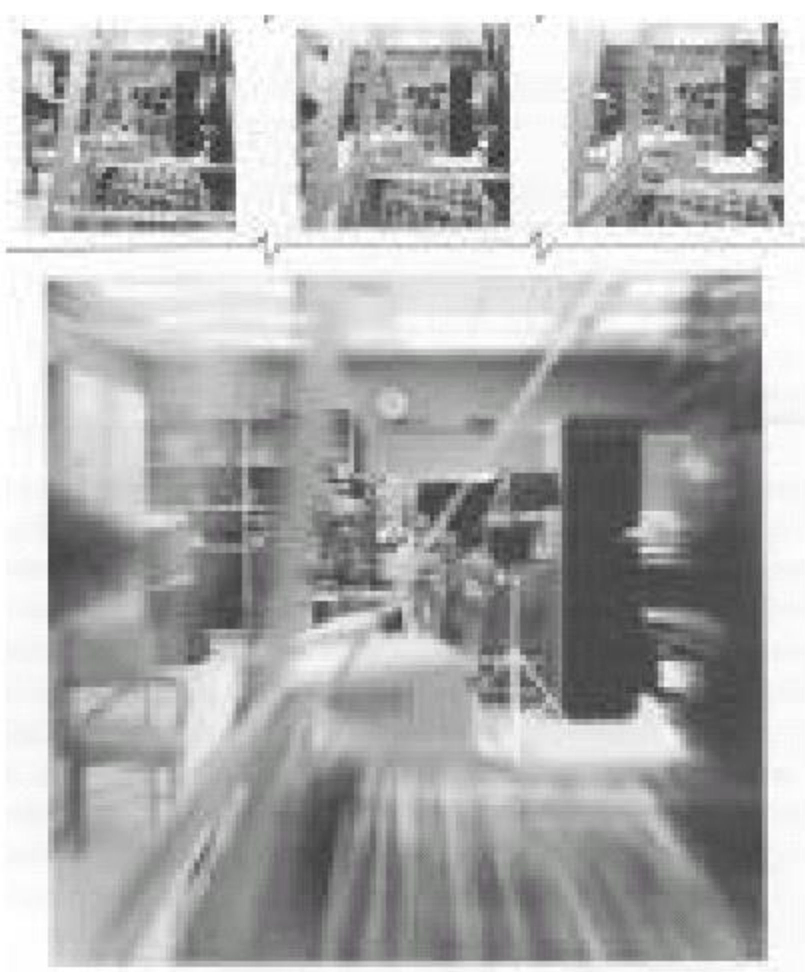
courtesy of Andrew Zisserman

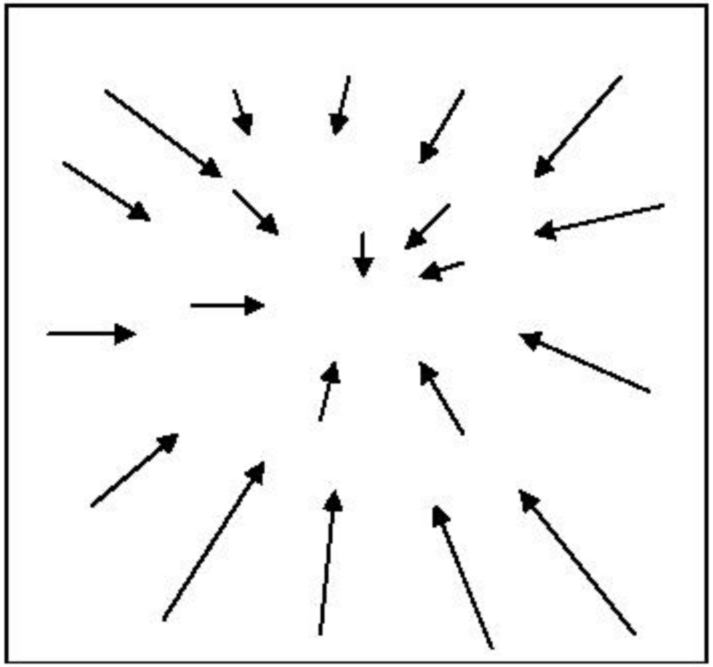
# Example: forward motion



courtesy of Andrew Zisserman

# FOE for Translating Camera





# Moving Plane (Trucco&Verri p.187)

$$v_x = \frac{1}{fd}(a_1 x^2 + a_2 xy + a_3 fx + a_4 fy + a_5 f^2)$$

$$v_y = \frac{1}{fd}(a_1 xy + a_2 y^2 + a_6 fy + a_7 fx + a_8 f^2)$$

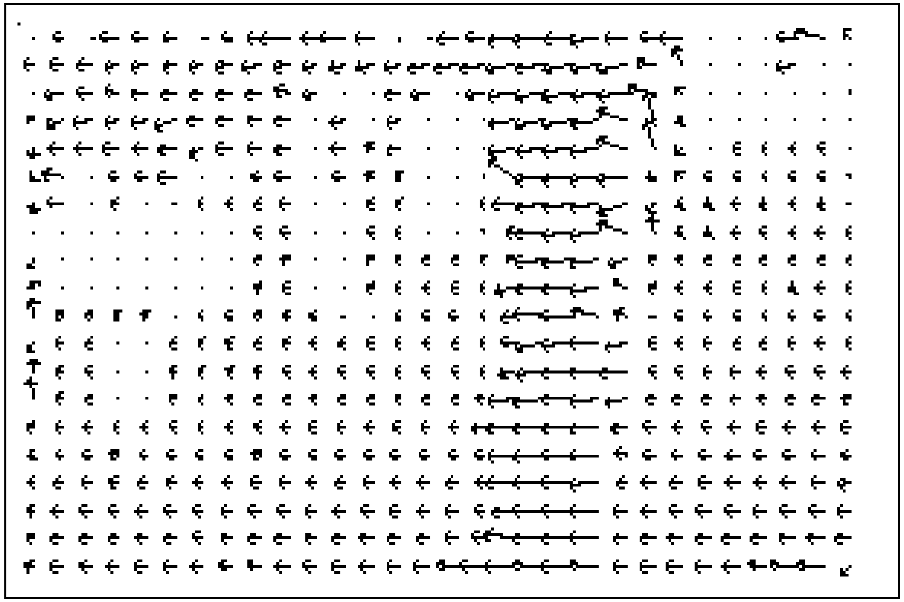
$$a_1 = -d\omega_y + T_z n_x, \qquad a_2 = d\omega_x + T_z n_y,$$
$$a_3 = T_z n_z - T_x n_x, \qquad a_4 = d\omega_z - T_x n_y,$$
$$a_5 = -d\omega_y - T_x n_z, \qquad a_6 = T_z n_z - T_y n_y,$$
$$a_7 = -d\omega_z - T_y n_x, \qquad a_8 = d\omega_x - T_y n_z.$$

- Motion field of planar surface is quadratic polynomial in (f,x,y)
- Same motion field produced by two different planes w. two different 3D motions
- Not unique: co-planar set of points (remember 8 point algorithm for calibration)

# Application (Szeklisky): Motion representations

- How can we describe this scene?

# Optical Flow Field

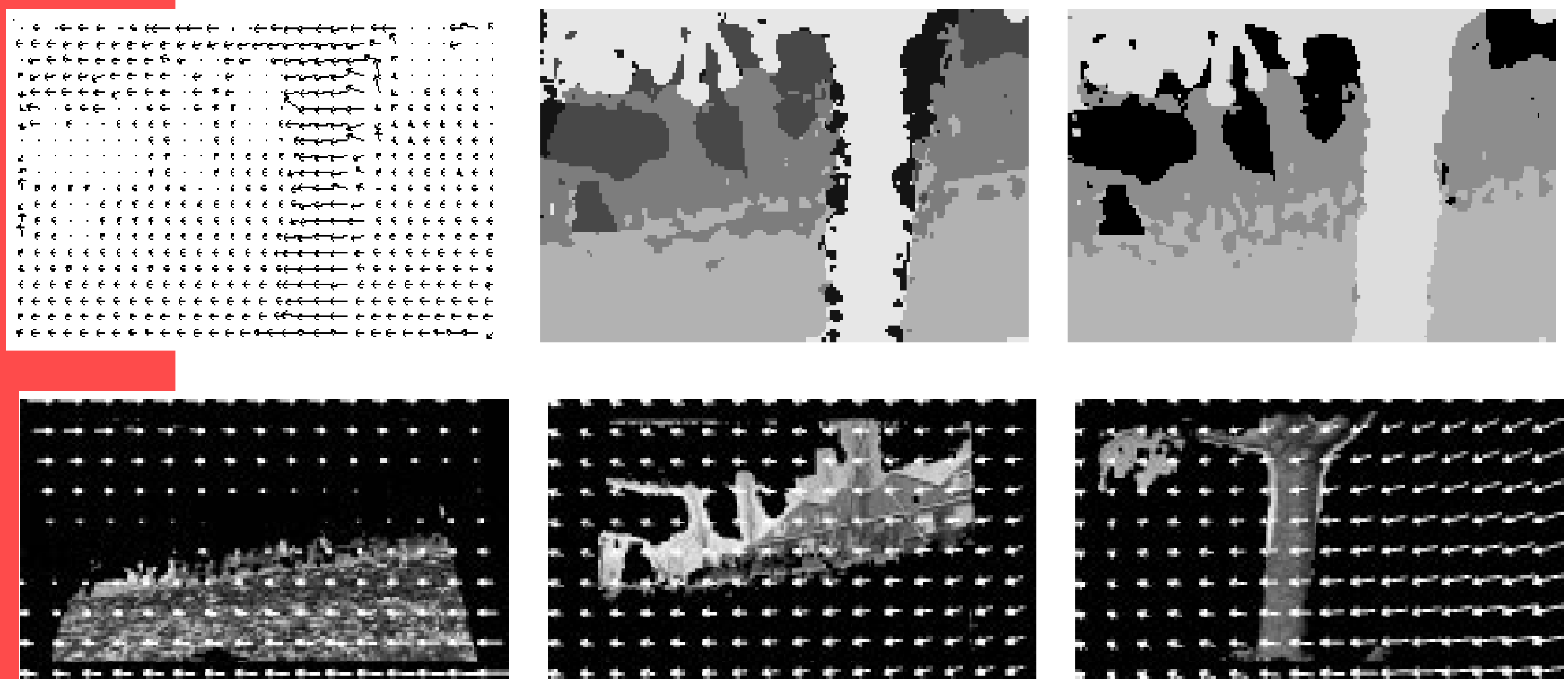


# Layered motion

- Break image sequence up into “layers”:
-  ÷  =

- Describe each layer’s motion

# Results



# Additional Slides, not discussed in class.

# Direct Motion Estimation

- One equation per pixel:

$$\left[-\frac{dI}{dt}\right]=\left[\frac{dI}{dx} \quad \frac{dI}{dy}\right]\begin{bmatrix} f & 0 & -x \\ 0 & f & -y \end{bmatrix}\frac{1}{Z'}\begin{bmatrix} 1 & 0 & 0 & 0 & Z & -yZ'/f \\ 0 & 1 & 0 & -Z & 0 & xZ'/f \\ 0 & 0 & 1 & yZ'/f & -xZ'/f & 0 \end{bmatrix}\begin{bmatrix} T \\ \Omega \end{bmatrix}$$

- Still hard!
- Z unknown; assume surface shape…
  - Negahdaripour & Horn - Planar
  - Black and Yacoob - Affine
  - Basu and Pentland; Bregler and Malik - Ellipsoidal
  - Essa et al. - Polygonal approximation
  - ...

# Layers for video summarization

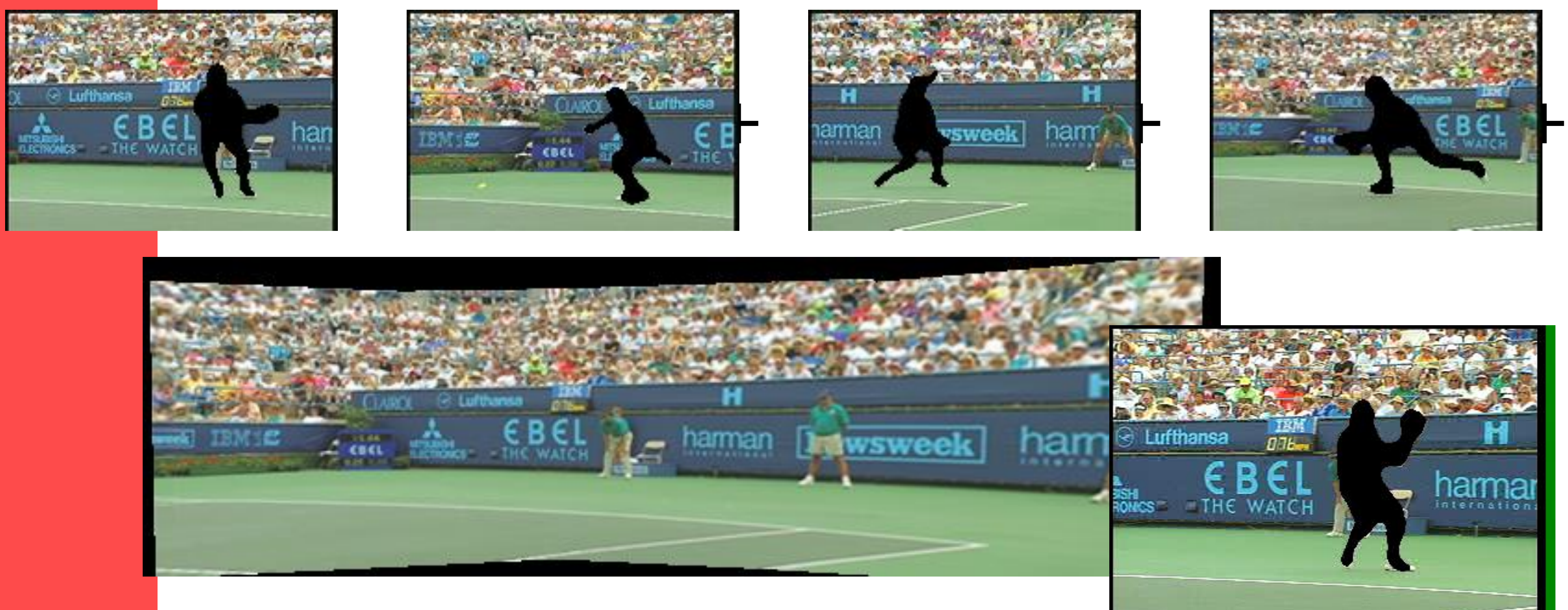

Frame 0 Frame 50 Frame 80

Background scene (players removed)

Complete synopsis of the video

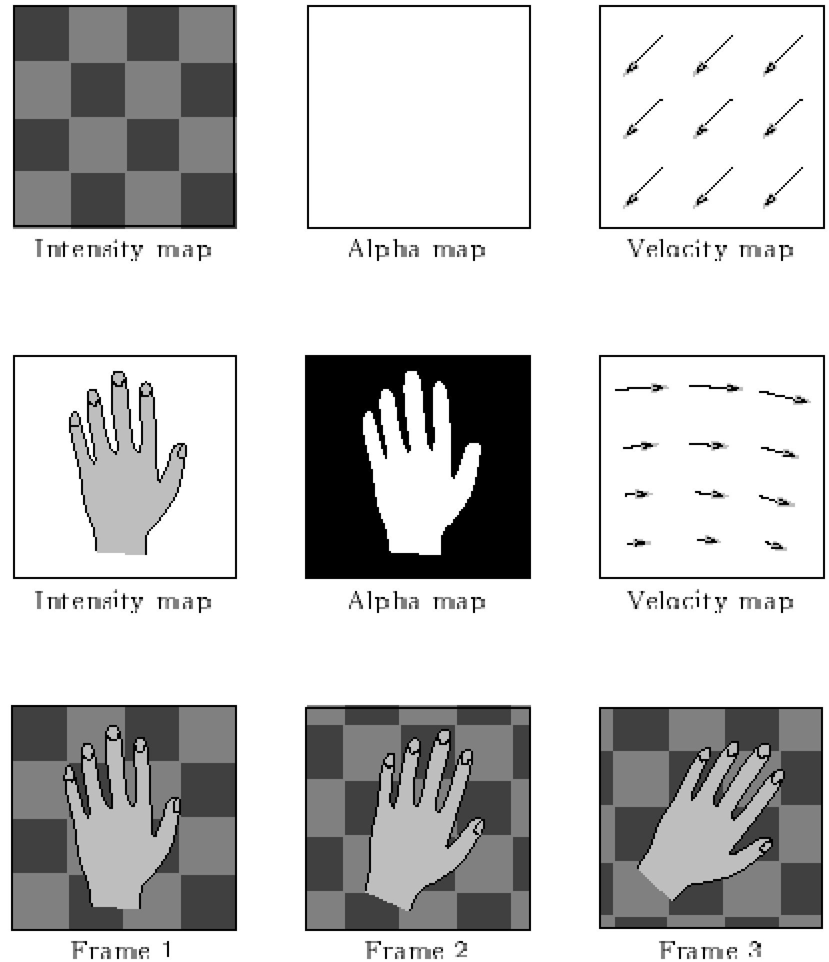
# Background modeling (MPEG-4)

- Convert masked images into a background sprite for layered video coding



# What are layers?

- [Wang & Adelson, 1994]
- intensities
- alphas
- velocities



# How do we form them?

(a) (b) (c)

Figure 7: (a) Frame 1 warped with an affine transformation to align the flowerbed region with that of frame 15. (b) Original frame 15 used as reference. (c) Frame 30 warped with an affine transformation to align the flowerbed region with that of frame 15.

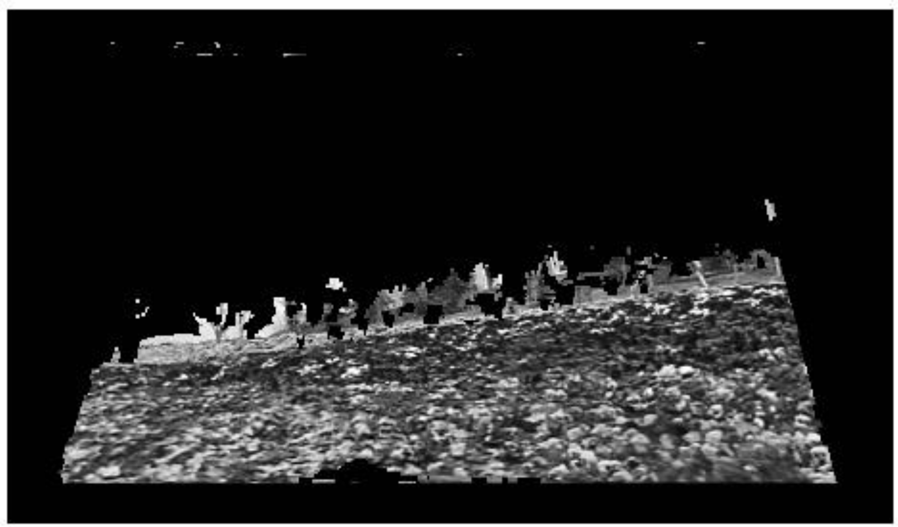


Figure 8: Accumulation of the flowerbed. Image intensities are obtained from a temporal median operation on the motion compensated images. Only the regions belonging to the flowerbed layer is accumulated in this image. Note also occluded regions are correctly recovered by accumulating data over many frames.