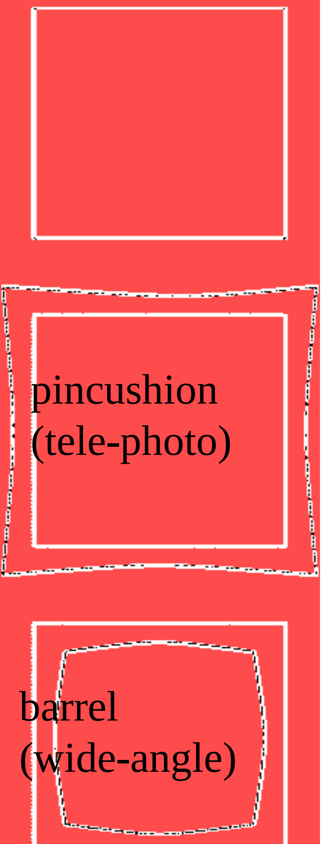
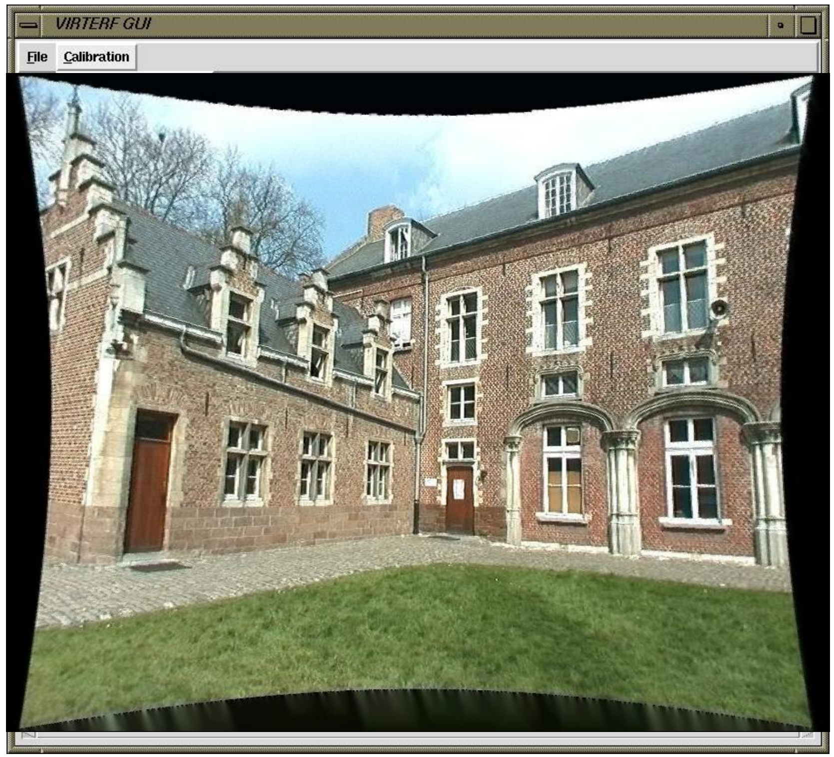
Radial Distortion



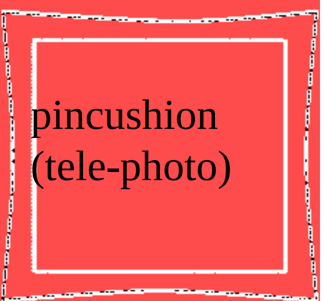
magnification/focal length different
for different angles of inclination

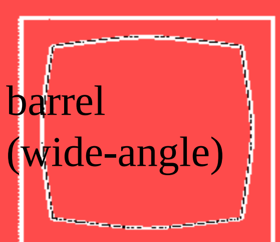


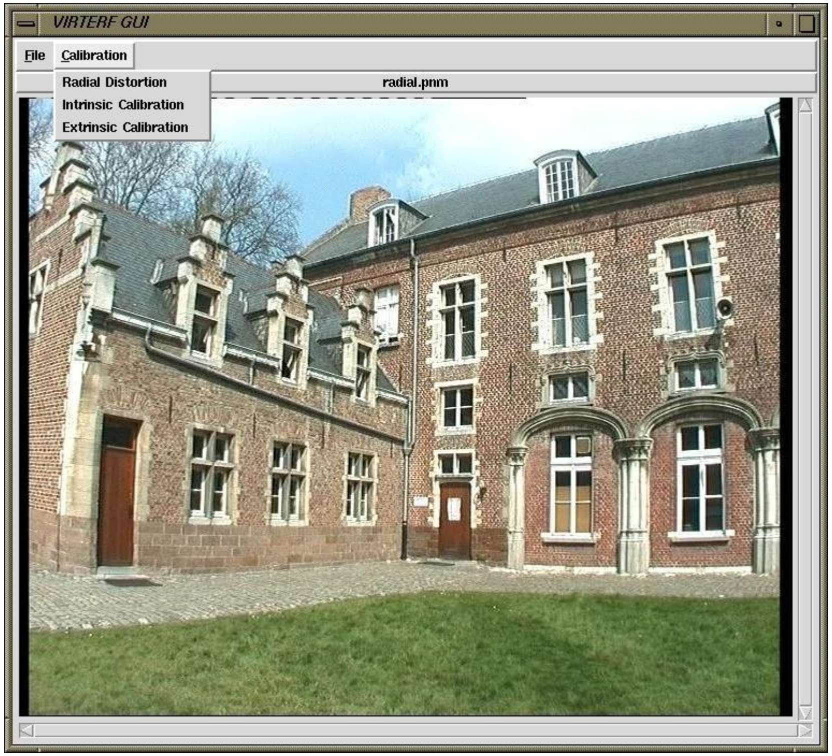
Can be corrected! (if parameters are know)

Radial Distortion

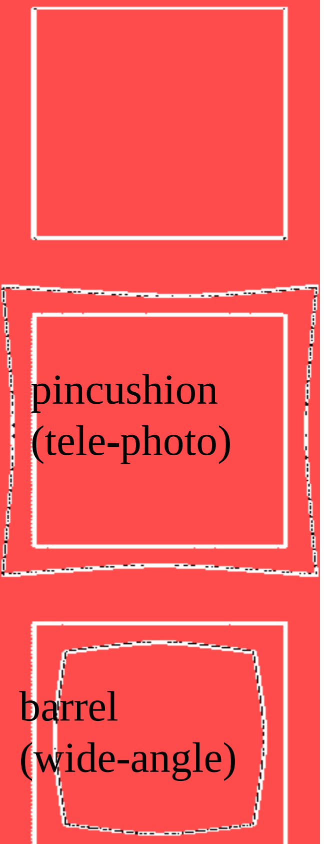
magnification/focal length different
for different angles of inclination







Radial Distortion



magnification/focal length different
for different angles of inclination

Can be corrected! (if parameters are know)

Radial Distortion

straight lines are not straight anymore

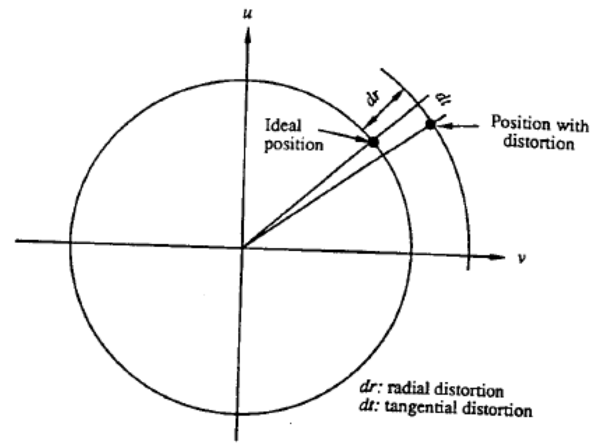


Fig. 2. Radial and tangential distortions.

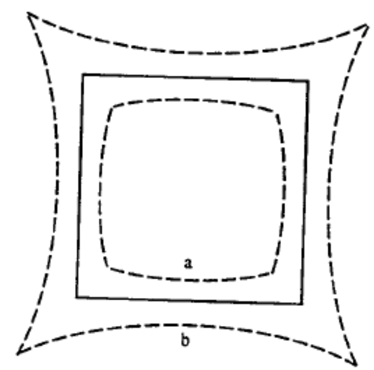
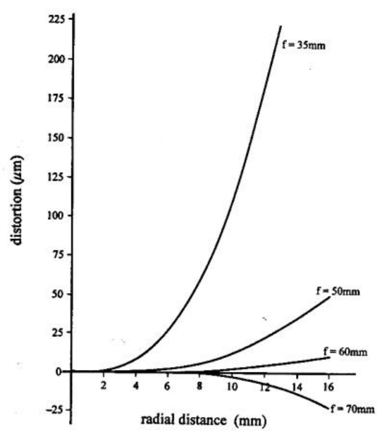


Fig. 3. Effect of radial distortion. Solid lines: no distortion; dashed lines: with radial distortion (a: negative, b: positive).



barrel dist.

pincushion dist.

http://foto.hut.fi/opetus/260/luennot/11/atkinson_6-11_radial_distortion_zoom_lenses.jpg

Radial distortion

- Due to spherical lenses (cheap/wide angle)
- Model:(following Tsai 1987 et al.):

$$\vec{p} = \boxed{R^{-1}} * \frac{1}{z} K \begin{pmatrix} {}^{C}_{W}R & {}^{C}_{W}\vec{t} \\ 0,0,0 & 1 \end{pmatrix} {}^{W}\vec{p}$$

$$\mathrm{R}(x,y) = (1 + K_1(x^2 + y^2) + K_2(x^4 + y^4) + \ldots) \begin{bmatrix} x^{rad} \\ y^{rad} \end{bmatrix}$$

$$\boldsymbol{p} = \begin{pmatrix} 1/\lambda & 0 & 0 \\ 0 & 1/\lambda & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathcal{M}\boldsymbol{P}$$

λ is a polynomial function of $\hat{r}^2 \stackrel{\text{def}}{=} \hat{u}^2 + \hat{v}^2$, i.e., $\lambda = 1 + \kappa_1 \hat{r}^2 + \kappa_2 \hat{r}^4 + \ldots$.

Radial distortion example

Radial distortion example

Radial distortion example

3.3.1 Estimation of Projection Matrix

Geometrically, radial distortion changes the distance between the image center and the image point $\boldsymbol{p}$ but it does not affect the direction of the vector joining these two points. This is called the *radial alignment constraint* by Tsai, and it can be expressed algebraically by writing

$$\lambda \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \dfrac{\boldsymbol{m}_1 \cdot \boldsymbol{P}}{\boldsymbol{m}_3 \cdot \boldsymbol{P}} \\ \dfrac{\boldsymbol{m}_2 \cdot \boldsymbol{P}}{\boldsymbol{m}_3 \cdot \boldsymbol{P}} \end{pmatrix} \Longrightarrow v(\boldsymbol{m}_1 \cdot \boldsymbol{P}) - u(\boldsymbol{m}_2 \cdot \boldsymbol{P}) = 0.$$

This is a linear constraint on the vectors $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$. Given n fiducial points we obtain n equations in the eight coefficients of the vectors $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$, namely

$$\mathcal{Q}\boldsymbol{n} = 0, \quad \text{where} \quad \mathcal{Q} \stackrel{\text{def}}{=} \begin{pmatrix} v_1 \boldsymbol{P}_1^T & -u_1 \boldsymbol{P}_1^T \\ \dots & \dots \\ v_n \boldsymbol{P}_n^T & -u_n \boldsymbol{P}_n^T \end{pmatrix} \quad \text{and} \quad \boldsymbol{n} = \begin{pmatrix} \boldsymbol{m}_1 \\ \boldsymbol{m}_2 \end{pmatrix}. \qquad (6.3.9)$$

Note the similarity with the previous case. When $n \geq 8$, the system of equations (6.3.9) is in general overconstrained, and a solution with unit norm can be found using linear least squares.

F&P page 48 ff

Useful Links

Demo calibration (some links broken):

- http://mitpress.mit.edu/e-journals/Videre/001/articles/Zhang/CalibEnv/CalibEnv.html

Bouget camera calibration SW:

- http://www.vision.caltech.edu/bouguetj/calib_doc/

CVonline: Monocular Camera calibration:

- http://homepages.inf.ed.ac.uk/cgi/rbf/CVONLINE/entries.pl?TAG250