Part Chapter 2: Cameras “Lenses”

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(slides modified from Marc Pollefeys, UNC Chapel Hill)
Pinhole size / aperture

How does the size of the aperture affect the image we’d get?

Larger

Smaller

Fig. 5.96  The pinhole camera. Note the variation in image clarity as the hole diameter decreases. [Photos courtesy Dr. N. Joel, UNESCO.]
Pinhole vs. lens
Adding a lens

A lens focuses light onto the film
- Rays passing through the center are not deviated
- All parallel rays converge to one point on a plane located at the *focal length* $f$
Cameras with lenses

- A lens focuses parallel rays onto a single focal point
- Gather more light, while keeping focus; make pinhole perspective projection practical
Snell’s law

\[ n_1 \sin \alpha_1 = n_2 \sin \alpha_2 \]
Paraxial (or first-order) optics

Snell’s law:

\[ n_1 \sin \alpha_1 = n_2 \sin \alpha_2 \]

Small angles:

\[ n_1 \alpha_1 \approx n_2 \alpha_2 \]

\[ \alpha_1 = \beta_1 + \gamma \approx \frac{h}{d_1} + \frac{h}{R} \]

\[ \alpha_2 = \gamma - \beta_2 \approx \frac{h}{R} - \frac{h}{d_2} \]

\[ n_1 \left( \frac{h}{d_1} + \frac{h}{R} \right) = n_2 \left( \frac{h}{R} - \frac{h}{d_2} \right) \]

\[ \frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R} \]
Thin Lenses

spherical lens surfaces; thickness << radii; same refractive index on both sides; all rays emerging from P and passing through the lens are focused at P'. Let $n_1=1$ (vacuum) and $n_2=n$.

\[
\frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R}
\]

\[
\frac{1}{Z} + \frac{n}{Z'} = \frac{n-1}{R}
\]

\[
\frac{n}{Z'} + \frac{1}{Z'} = \frac{1-n}{R}
\]

\[
\frac{n}{Z'} = \frac{n-1}{R} - \frac{1}{Z}
\]

\[
\frac{n}{Z'} = \frac{1-n}{R} - \frac{1}{Z'}
\]

\[
\frac{n-1}{R} - \frac{1-n}{Z} = \frac{1}{Z'}
\]

\[
\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}
\]

and

\[
f = \frac{R}{2(n-1)}
\]
Thin Lenses

spherical lens surfaces; thickness \(<<\) radii; same refractive index on both sides; all rays emerging from \(P\) and passing through the lens are focused at \(P'\). Let \(n_1=1\) (vaccum) and \(n_2=n\).

\[
\begin{align*}
x' &= z' \frac{x}{z} \\
y' &= z' \frac{y}{z}
\end{align*}
\]

where

\[
\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}
\]

and

\[
f = \frac{R}{2(n-1)}
\]

http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html
Thick Lens
Focus and depth of field

Image credit: cambridgeincolour.com
The depth-of-field
Focus and depth of field

- Depth of field: distance between image planes where blur is tolerable

Thin lens: scene points at distinct depths come in focus at different image planes.

(Real camera lens systems have greater depth of field.)
Focus and depth of field

- How does the aperture affect the depth of field?

- A smaller aperture increases the range in which the object is approximately in focus

Slide from S. Seitz
The depth-of-field
The depth-of-field

\[ \Delta Z_0^- = Z_0 - Z_0^- = \frac{Z_0(Z_0 - f)}{Z_0 + f \frac{d}{b - f}} \]

decreases with \( d^+ \), increases with \( Z_0^+ \)

strike a balance between incoming light and sharp depth range
Deviations from the lens model

3 assumptions:

1. all rays from a point are focused onto 1 image point
2. all image points in a single plane
3. magnification is constant

deviations from this ideal are *aberrations*
Aberrations

2 types:

1. geometrical

2. chromatic

*geometrical*: small for paraxial rays

\[
\sin(\theta) \approx \theta - \frac{\theta^3}{6}
\]

*chromatic*: refractive index function of wavelength
Geometrical aberrations

- spherical aberration
- astigmatism
- distortion
- coma

Aberrations are reduced by combining lenses
Spherical aberration

rays parallel to the axis do not converge

outer portions of the lens yield smaller focal lengths
Astigmatism

Different focal length for inclined rays
Distortion

magnification/focal length different for different angles of inclination

Can be corrected! (if parameters are known)
Coma
point off the axis depicted as comet shaped blob
Chromatic aberration

rays of different wavelengths focused in different planes

cannot be removed completely

sometimes *achromatization* is achieved for more than 2 wavelengths
Lens materials

reference wavelengths:

\[ \lambda_F = 486.13\,nm \]
\[ \lambda_d = 587.56\,nm \]
\[ \lambda_C = 656.28\,nm \]

lens characteristics:

1. refractive index \( n_d \)
2. Abbe number \( V_d = \frac{n_d - 1}{n_F - n_C} \)

typically, both should be high
allows small components with sufficient refraction

notation: e.g. glass BK7(517642)
\[ n_d = 1.517 \] and \[ V_d = 64.2 \]
Lens materials

- Crown Glass
- Fused Quartz & Fused Silica
- Calcium Fluoride
- Germanium
- Zinc Selenide
- Saphire
- Plastic (PMMA)

additional considerations:
humidity and temperature resistance, weight, price,...
Vignetting