Multi-View Geometry: Small Motion and Epipolar Constraints

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Motion Models (Review)

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \approx \begin{bmatrix} 1 & -\gamma & \beta \\ \gamma & 1 & -\alpha \\ -\beta & \alpha & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

3D Rigid Motion

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \approx \left(\begin{bmatrix} 0 & -\gamma & \beta \\ \gamma & 0 & -\alpha \\ -\beta & \alpha & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

$$\begin{bmatrix} X' - X \\ Y' - Y \\ Z' - Z \end{bmatrix} \approx \begin{bmatrix} 0 & -\gamma & \beta \\ \gamma & 0 & -\alpha \\ -\beta & \alpha & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

$$\begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix} \approx \begin{bmatrix} 0 & -\omega_Z & \omega_Y \\ \omega_Z & 0 & -\omega_X \\ -\omega_Y & \omega_X & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} V_{T_x} \\ V_{T_Y} \\ V_{T_Z} \end{bmatrix}$$

$\begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix} =$ Velocity Vector

$\begin{bmatrix} V_{T_X} \\ V_{T_Y} \\ V_{T_Z} \end{bmatrix} =$ Translational Component of Velocit

$\begin{bmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{bmatrix} =$ Angular Velocity

Small Motions

$$\dot{p} = \begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix} = \text{Velocity Vector}$$

$$t = \delta t\, v$$

$$v = \begin{bmatrix} V_{T_X} \\ V_{T_Y} \\ V_{T_Z} \end{bmatrix} = \text{Translational Component of Velocity}$$

$$R = I + \delta t [\omega]_\times$$

$$p' = p + \delta t \dot{p}$$

$$\omega = \begin{bmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{bmatrix} = \text{Angular Velocity}$$

$$p^T \mathcal{E} p' = 0$$

$$p^T [v]_\times \left(I + \delta t [\omega]_\times\right)\left(p + \delta t \dot{p}\right) = 0$$

$$p^T \left([v]_\times [\omega]_\times\right) p - \left(p \times \dot{p}\right) \cdot v = 0$$

Exercise 7.2

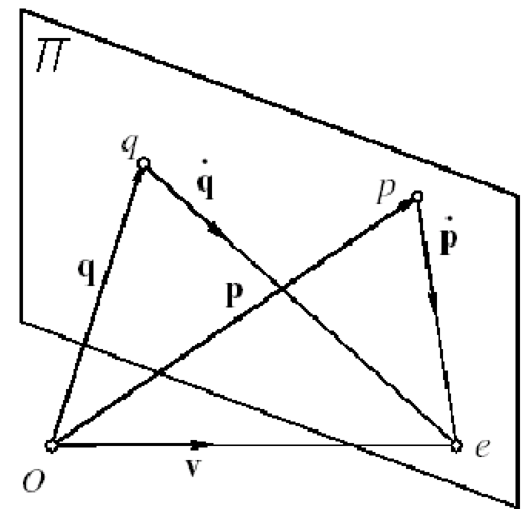
Translating Camera

$$p^T([v_\times][w_\times])p - (p \times \dot{p}) \cdot v =$$

$$w = 0$$

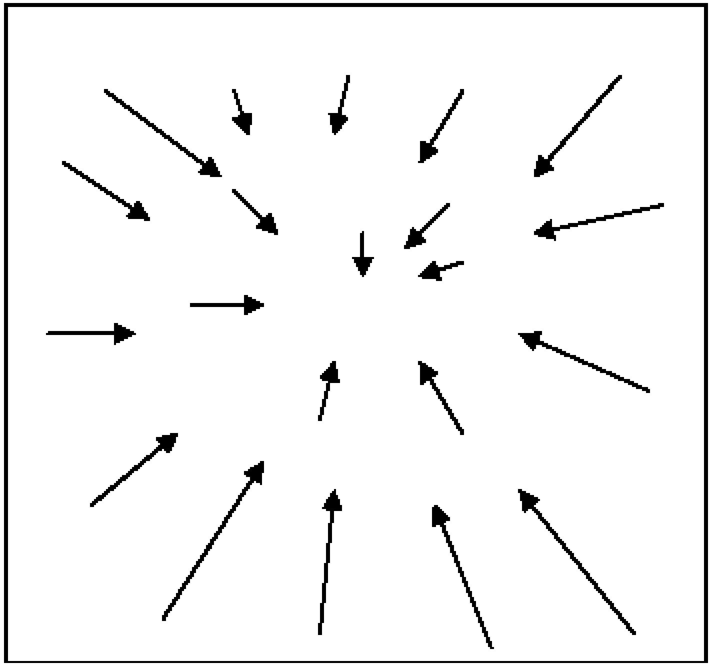
$$(p \times \dot{p}) \cdot v = 0$$

p, $\dot{p}$, and v are coplanar



Focus of expansion (FOE): Under pure translation, the motion field at every point in the image points toward the focus of expansion.

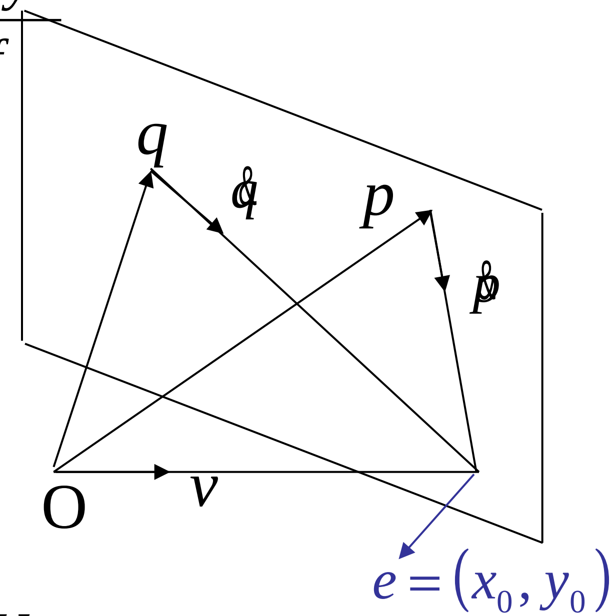
FOE for Translating Camera



FOE from Basic Equations of Motion (see later optical flow)

$$\dot{p}_x = \frac{V_{T_Z} x - V_{T_X} f}{Z} - \omega_Y f + \omega_Z y + \frac{\omega_X xy}{f} - \frac{\omega_Y x^2}{f}$$

$$\dot{p}_y = \frac{V_{T_Z} y - V_{T_Y} f}{Z} + \omega_X f - \omega_Z x - \frac{\omega_Y xy}{f} + \frac{\omega_X y^2}{f}$$



$$\omega = 0$$

$$\dot{p}_x = \frac{V_{T_Z} x - V_{T_X} f}{Z}$$

$$\dot{p}_y = \frac{V_{T_Z} y - V_{T_Y} f}{Z}$$

$$x_0 = f \frac{V_{T_X}}{V_{T_Z}}$$

$$y_0 = f \frac{V_{T_Y}}{V_{T_Z}}$$

$$\Longrightarrow$$

$$\dot{p}_x = (x - x_0)\frac{V_{T_Z}}{Z}$$

$$\dot{p}_y = (y - y_0)\frac{V_{T_Z}}{Z}$$