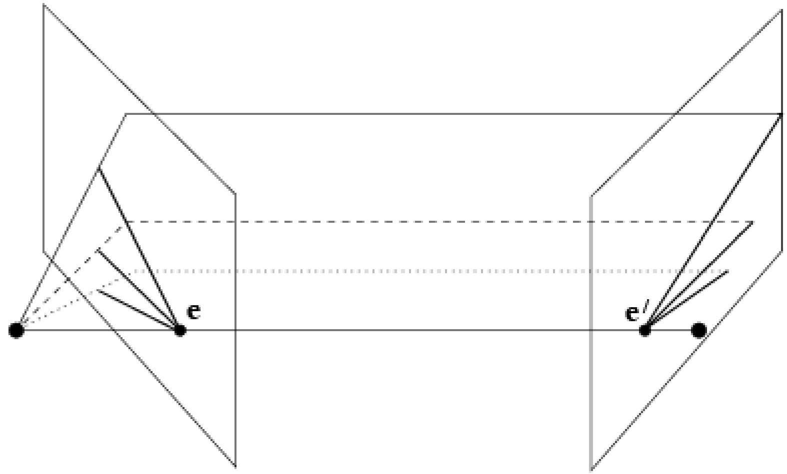
Image Rectification (Stereo) (New book: 7.2.1, old book: 11.1)

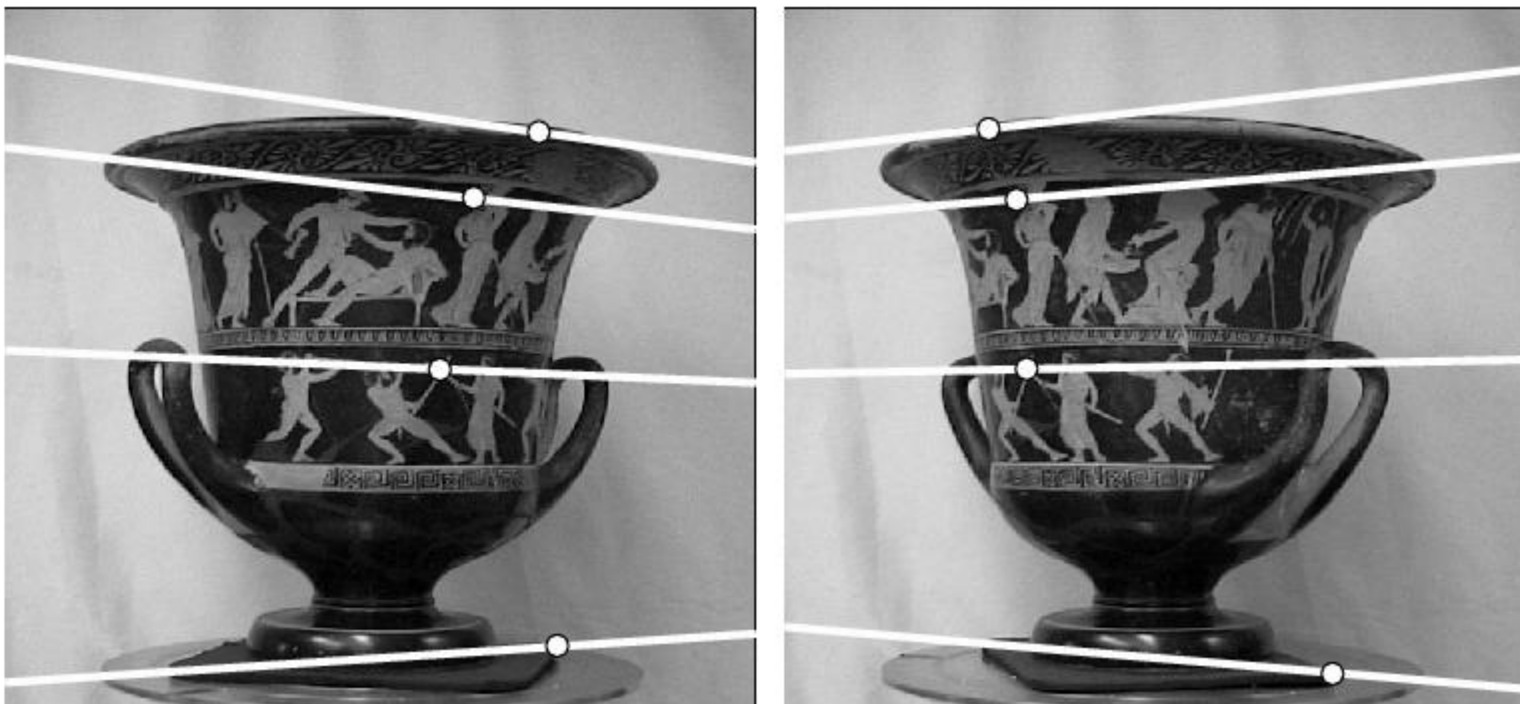
Guido Gerig

CS 6320 Spring 2013

Credits: Prof. Mubarak Shah, Course notes modified from: http://www.cs.ucf.edu/courses/cap6411/cap5415/, Lecture 25

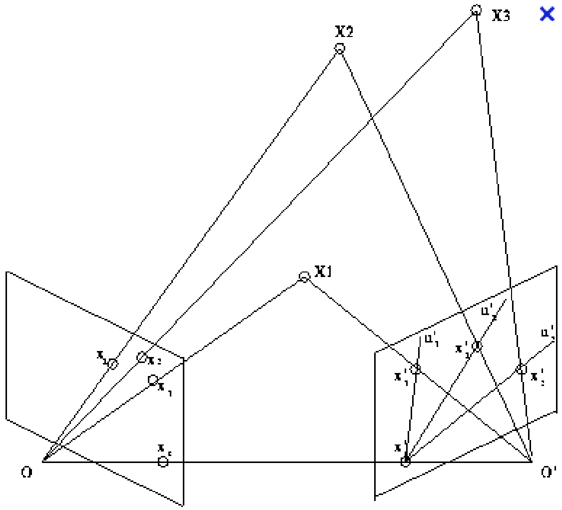
Example: converging cameras



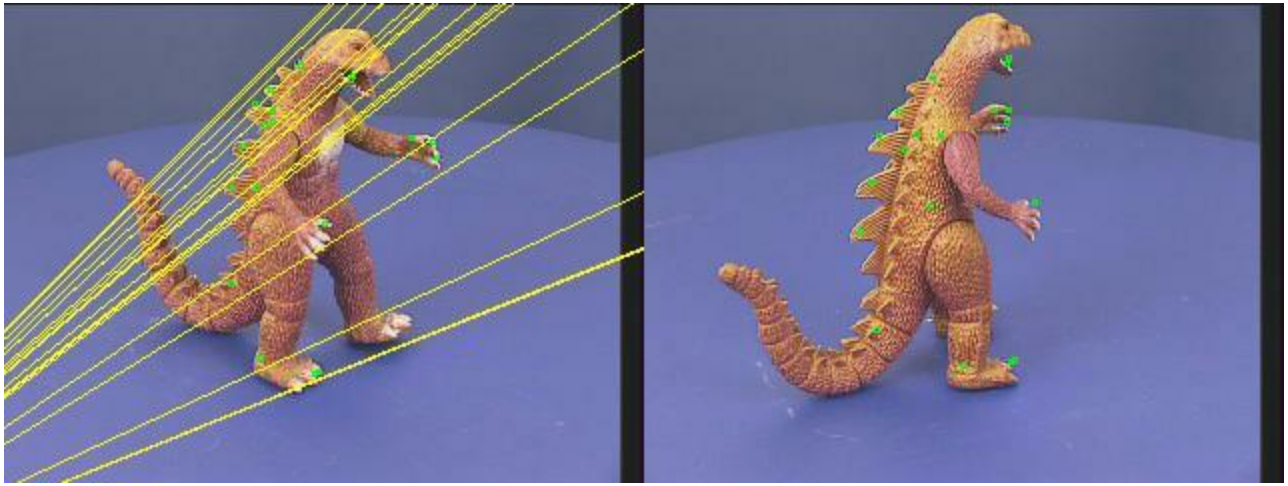


courtesy of Andrew Zisserman

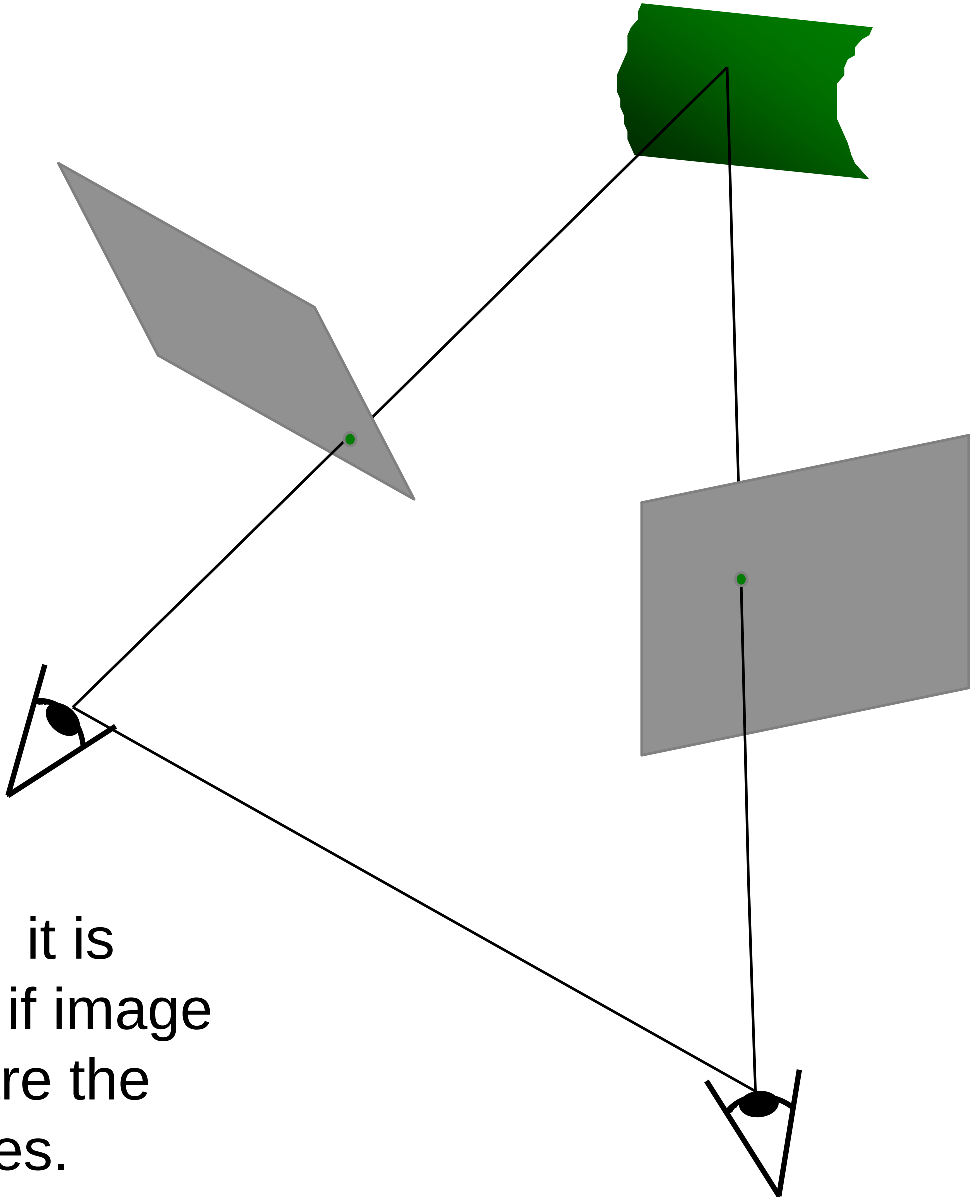
Epipolar Lines in Converging Cameras



Epipolar lines all intersect at epipoles.

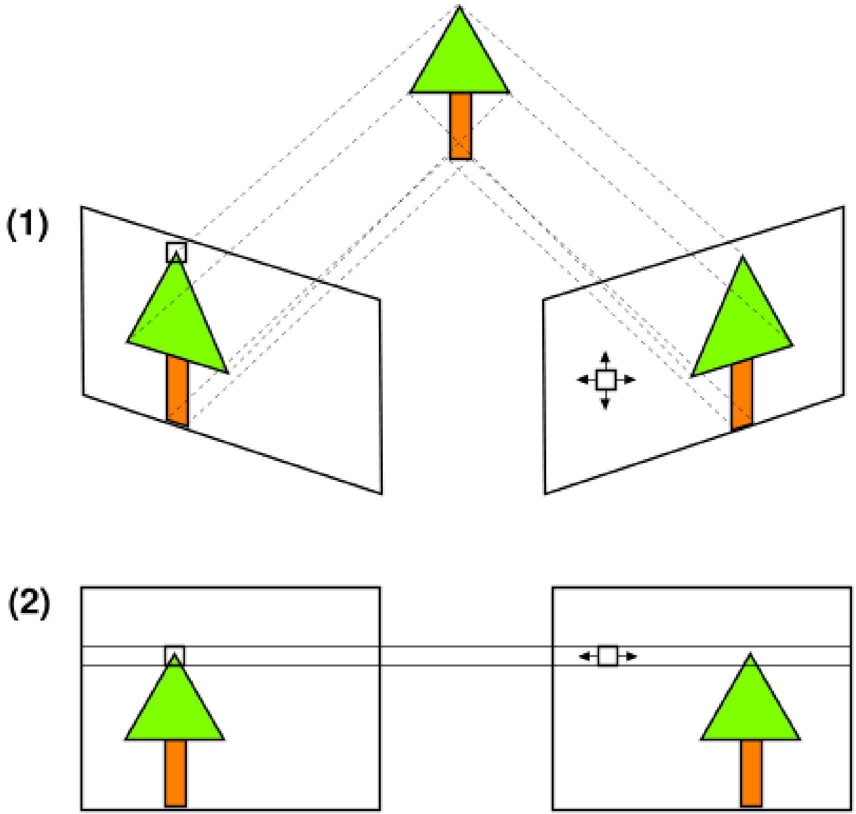


Stereo image rectification



In practice, it is convenient if image scanlines are the epipolar lines.

Image Rectification

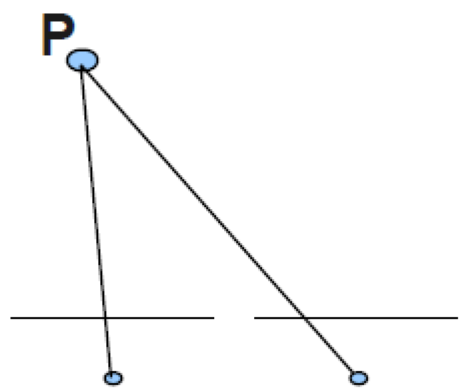


If the two cameras are aligned to be coplanar, the search is simplified to one dimension - a horizontal line parallel to the baseline between the cameras

http://en.wikipedia.org/wiki/Image_rectification

Intuitions on Rectification

- Let's look in 2D
- If I change the *position* of my camera, then the location of the point depends on depth



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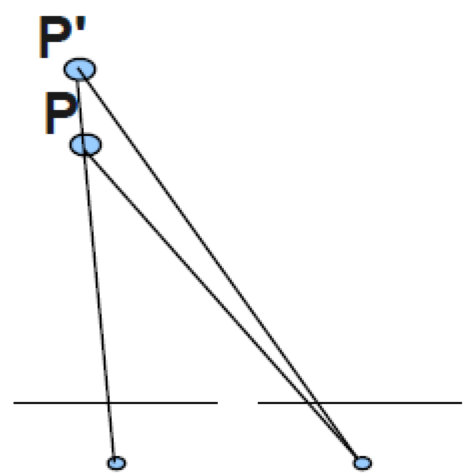


Intuitions on Rectification

- But if I just rotate the camera, then I don't need to know the depth
- Notice that P' and P still project to the same point in the rotated camera

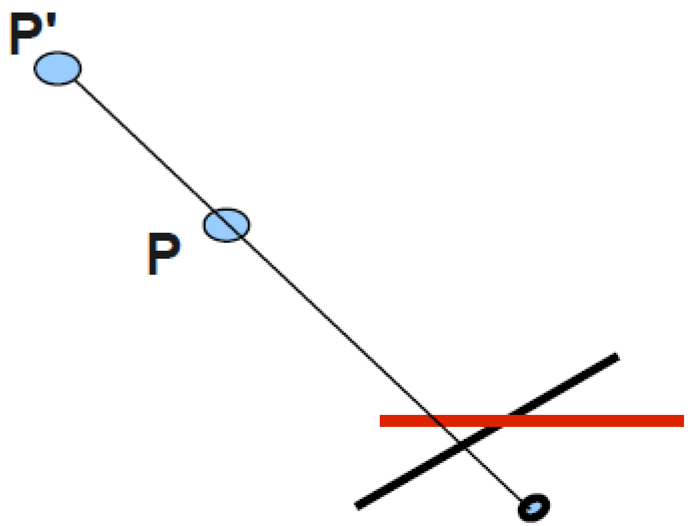
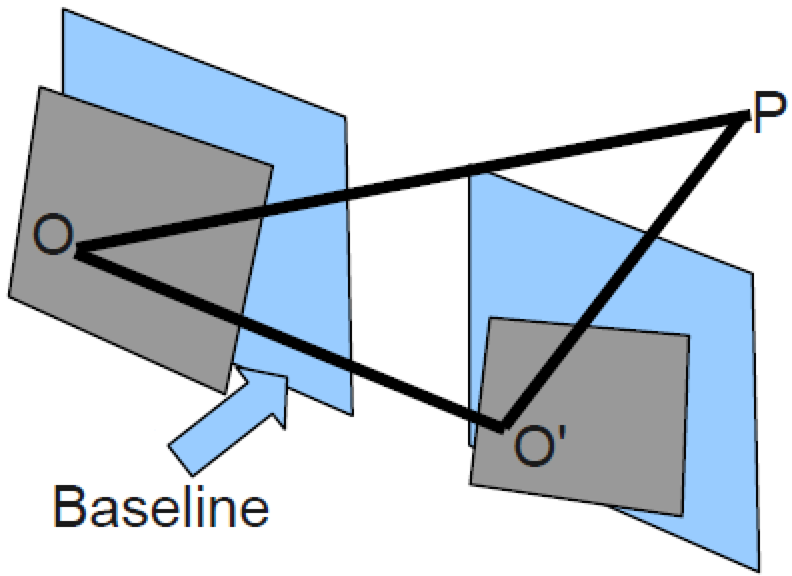
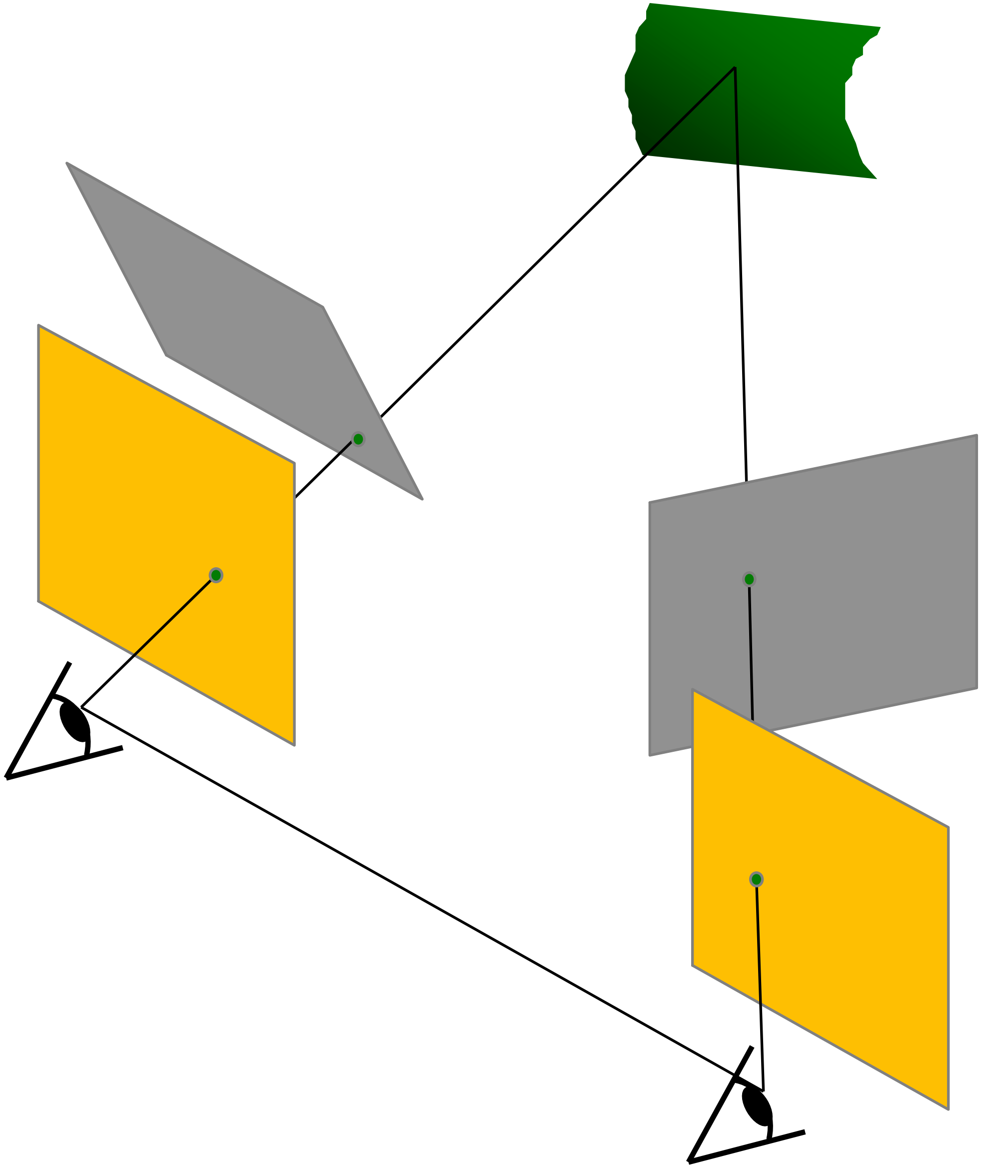


Image Rectification

- I can rotate views to be parallel to each other and baseline

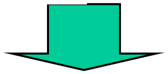


Stereo image rectification



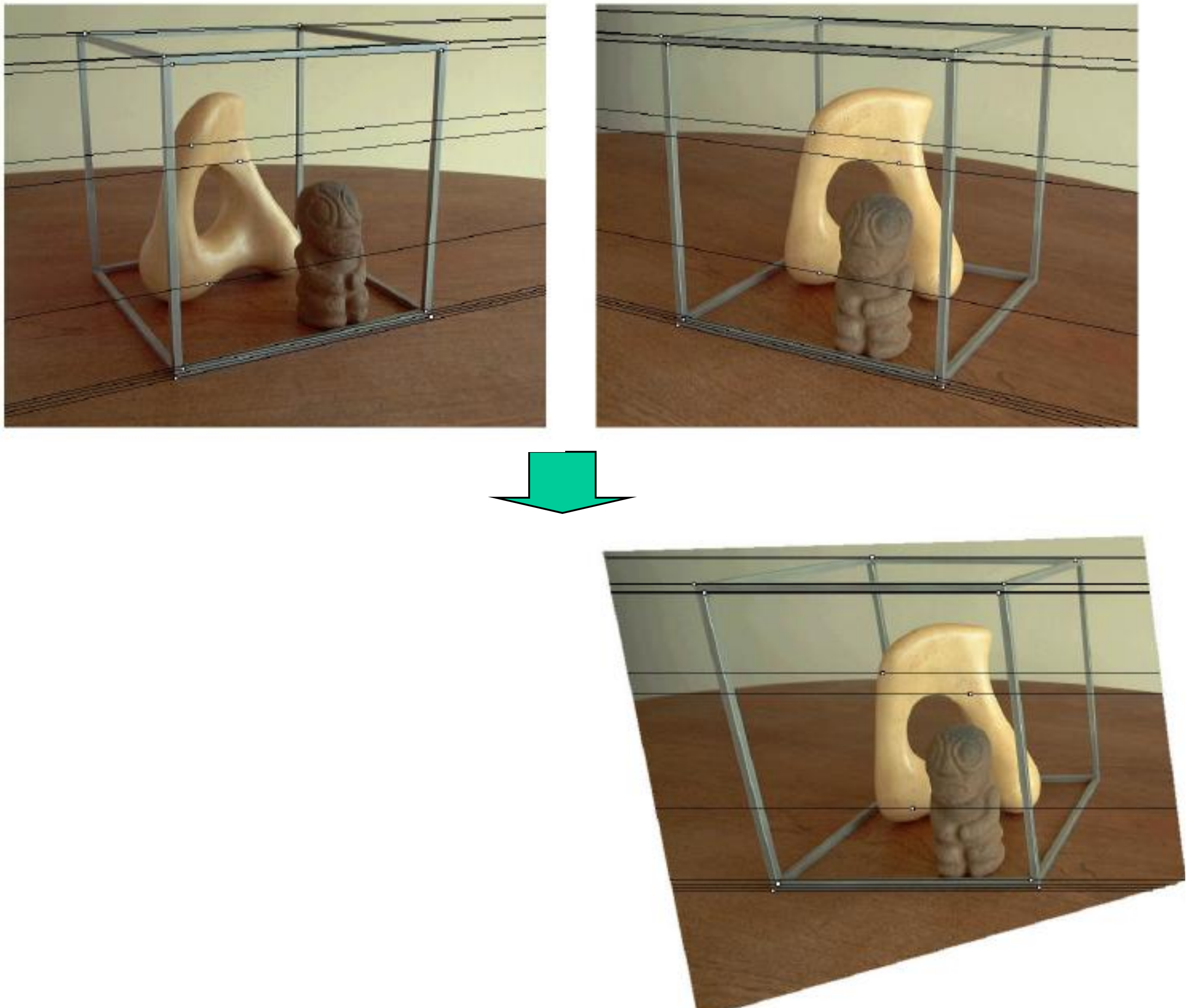
- Image Reprojection
  - reproject image planes onto common
    plane parallel to line between optical centers
  - a homography (3x3 transform)
    applied to both input images
  - pixel motion is horizontal after this transformation
  - C. Loop and Z. Zhang. Computing Rectifying Homographies for Stereo Vision. IEEE Conf. Computer Vision and Pattern Recognition, 1999.

Stereo image rectification: example



Source: Alyosha Efros

Stereo image rectification: example



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Stereo image rectification: example

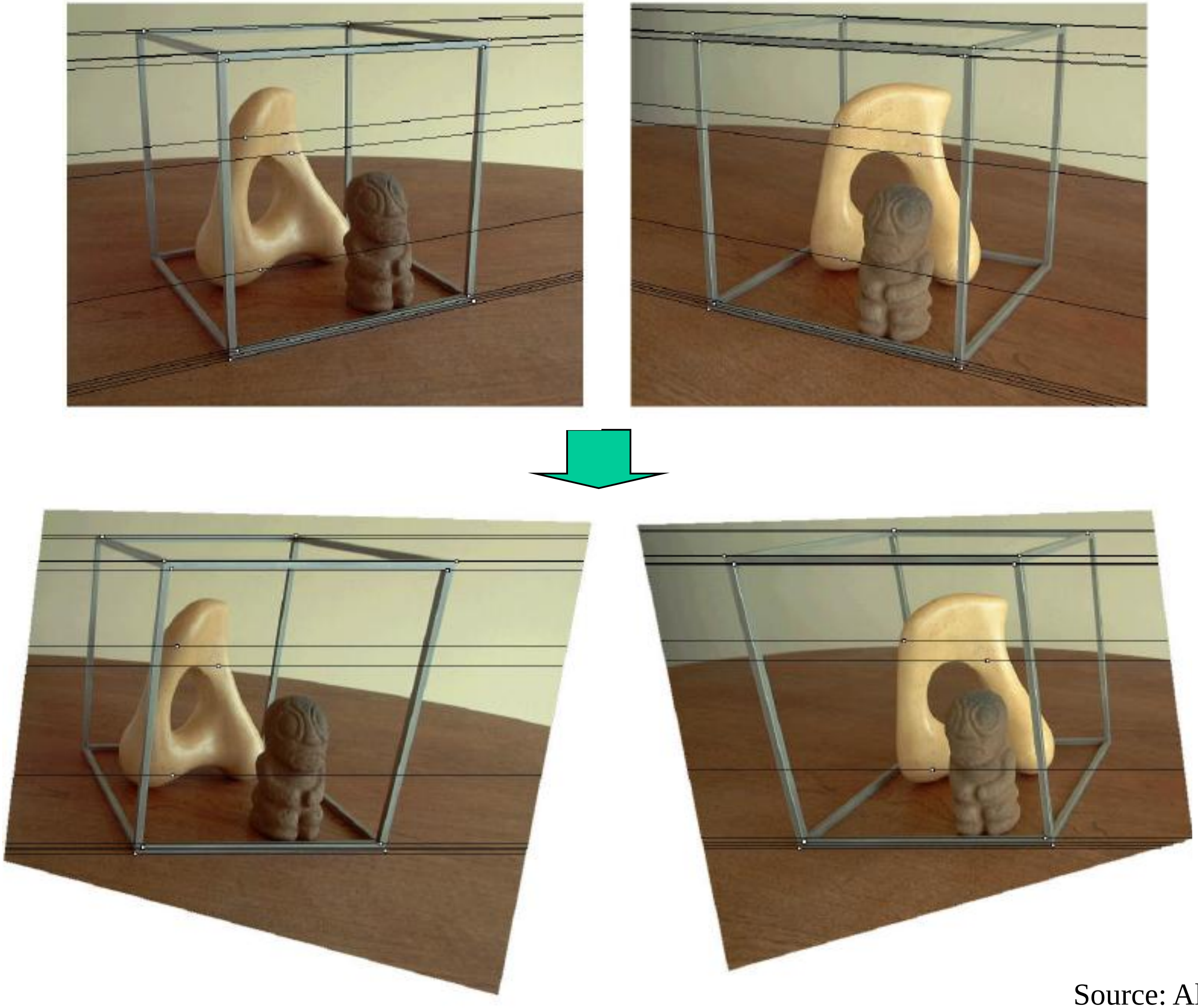

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Image Rectification

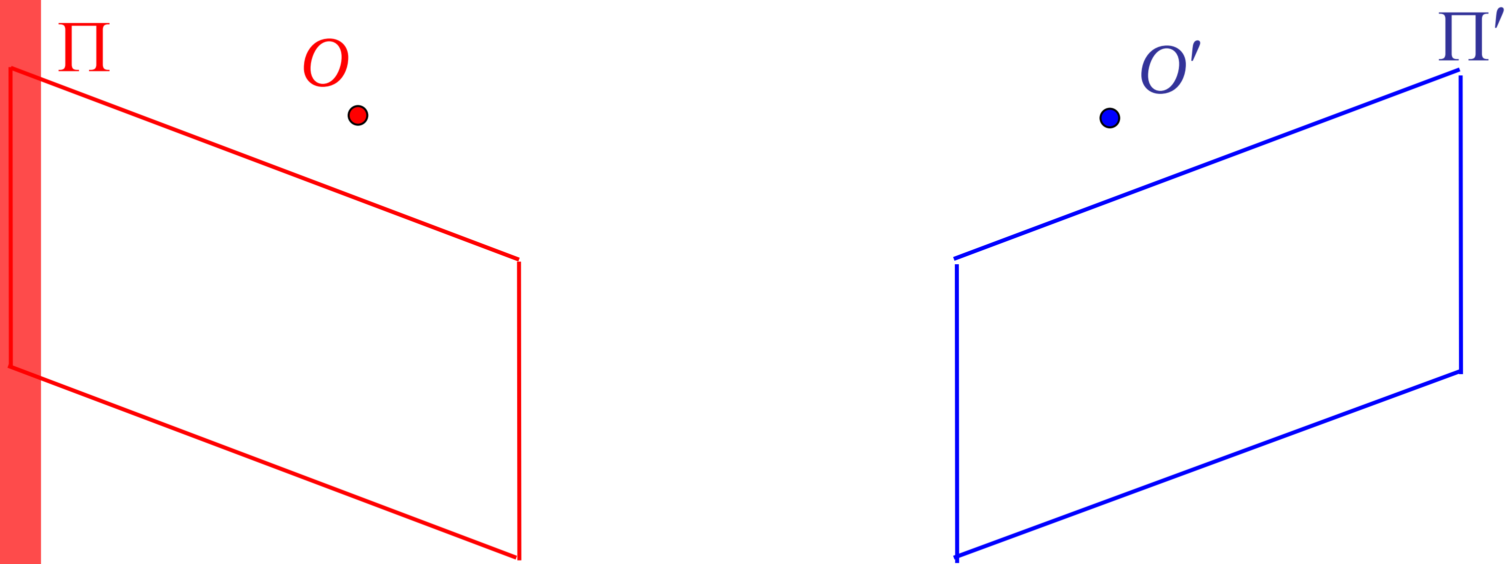


Image Rectification

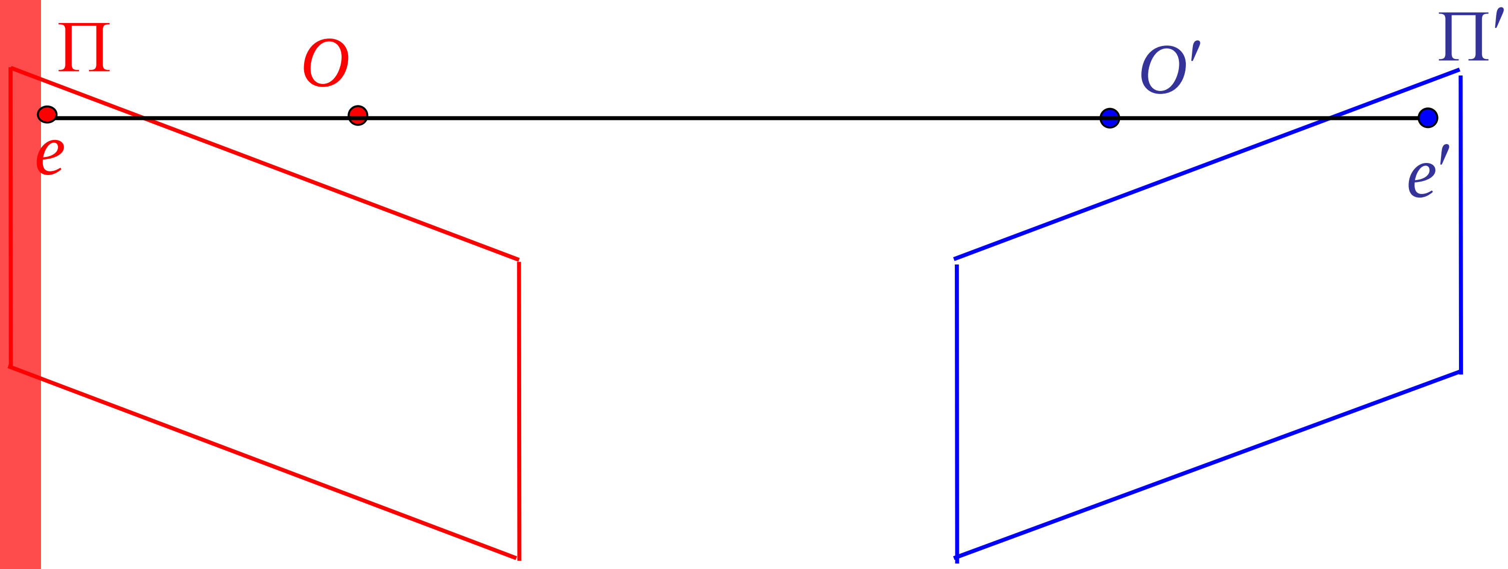
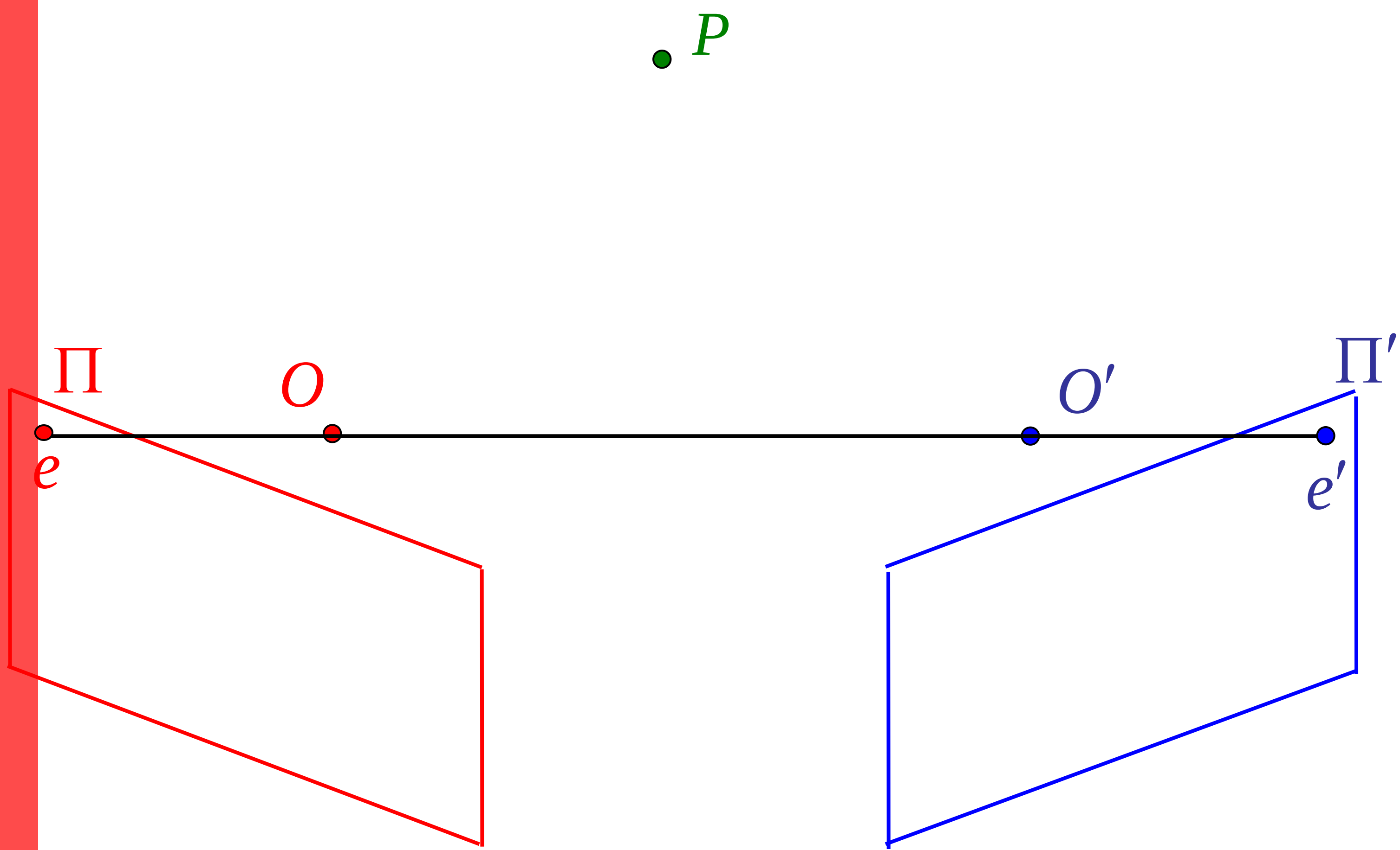


Image Rectification



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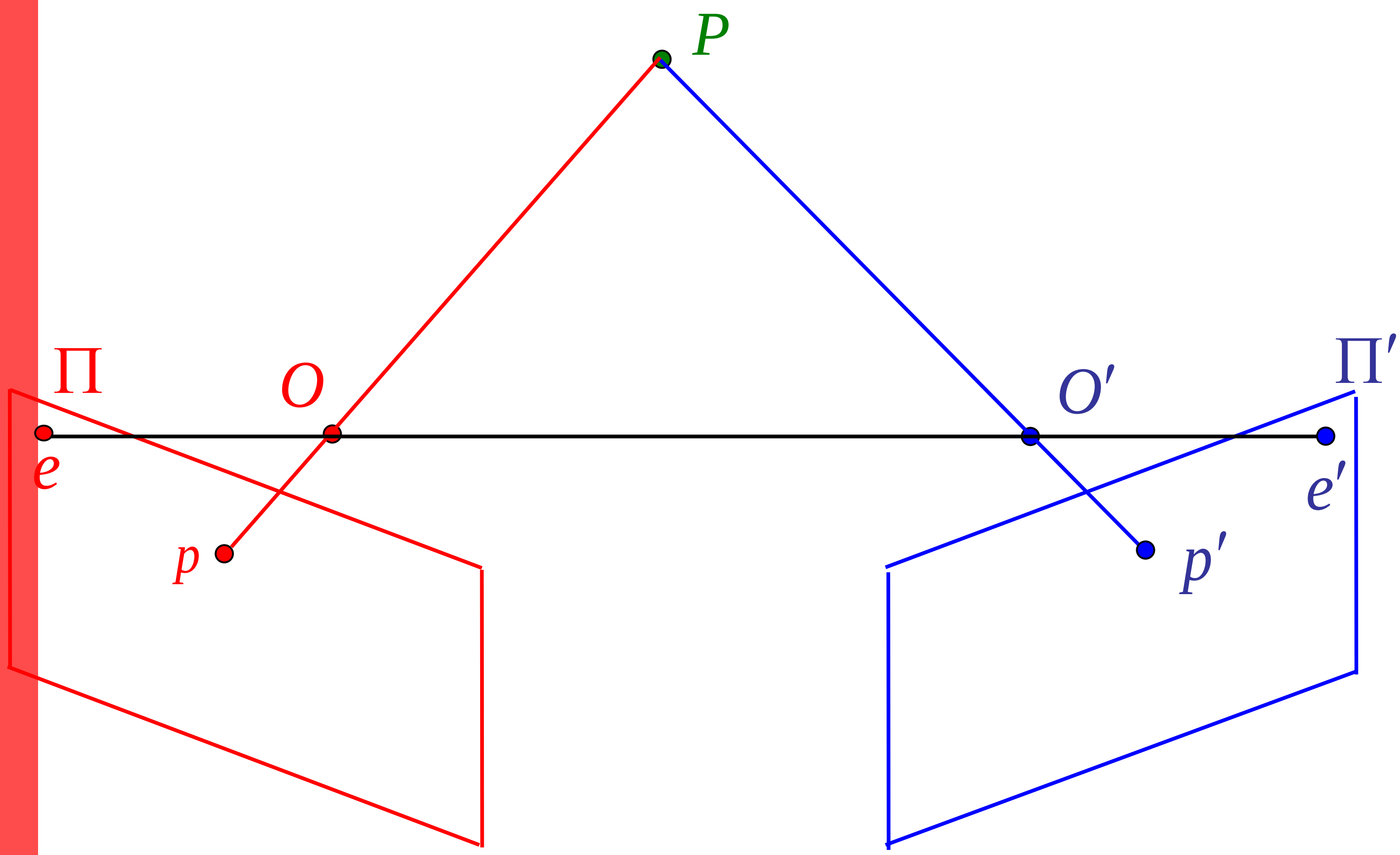

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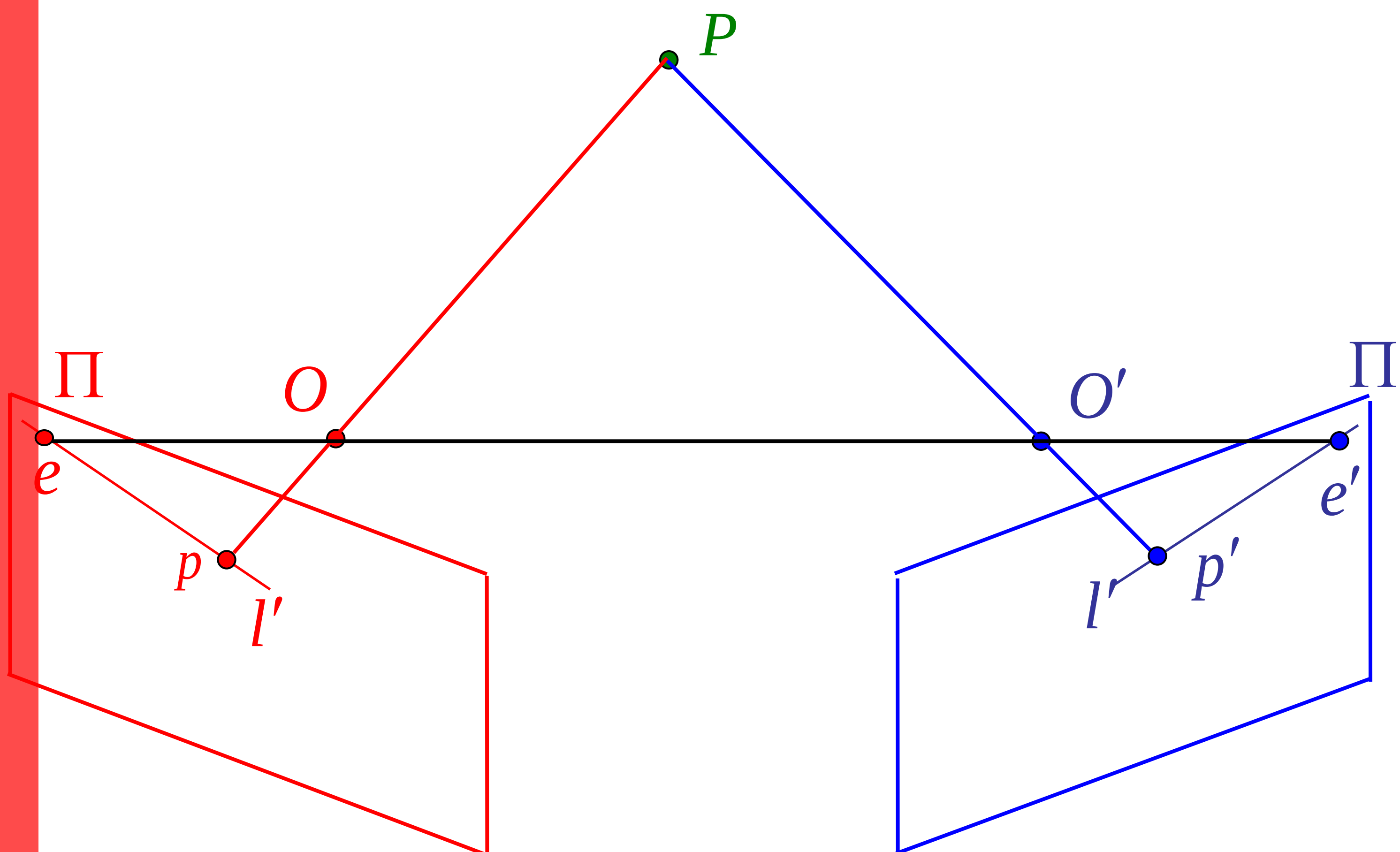

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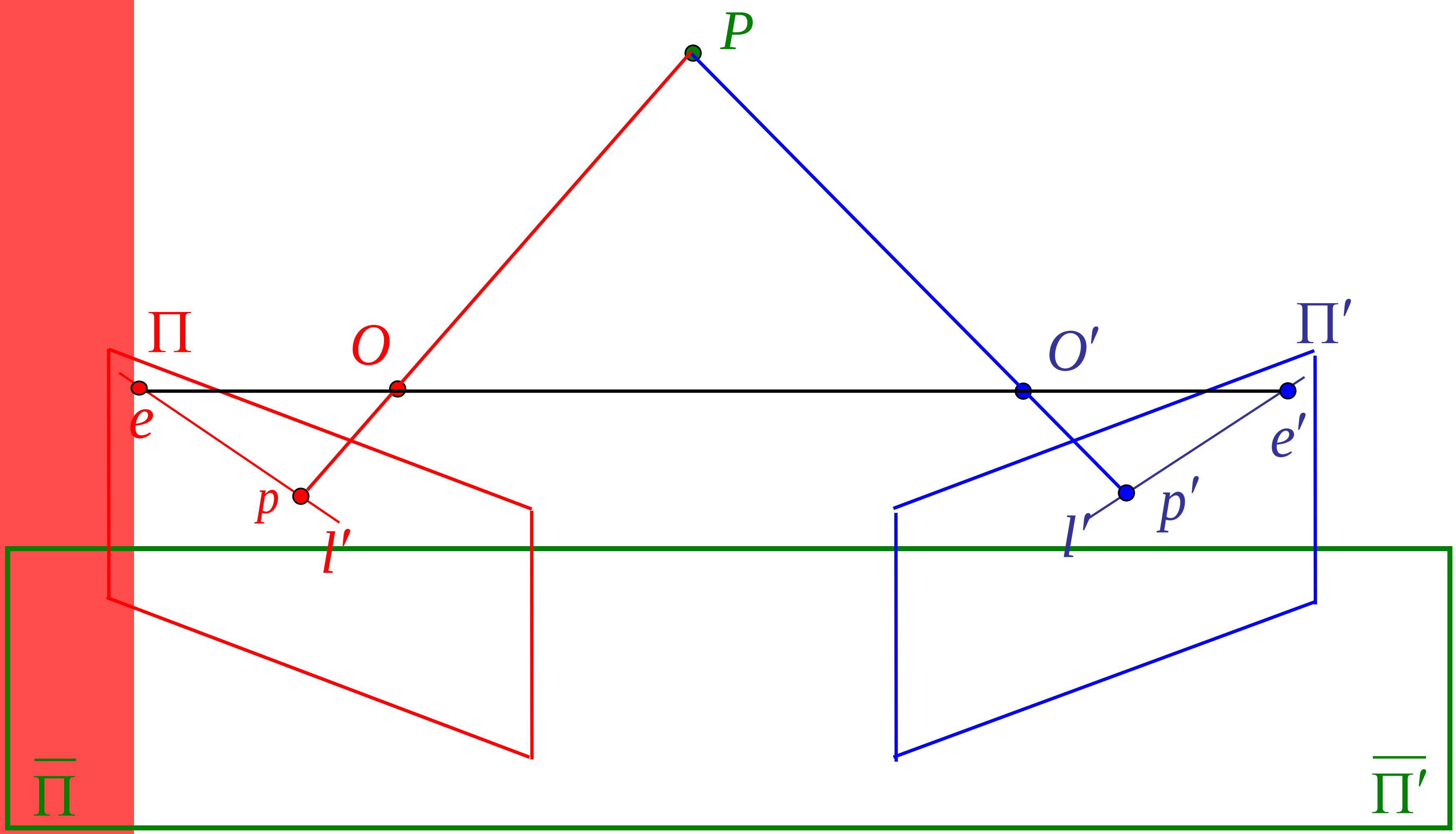

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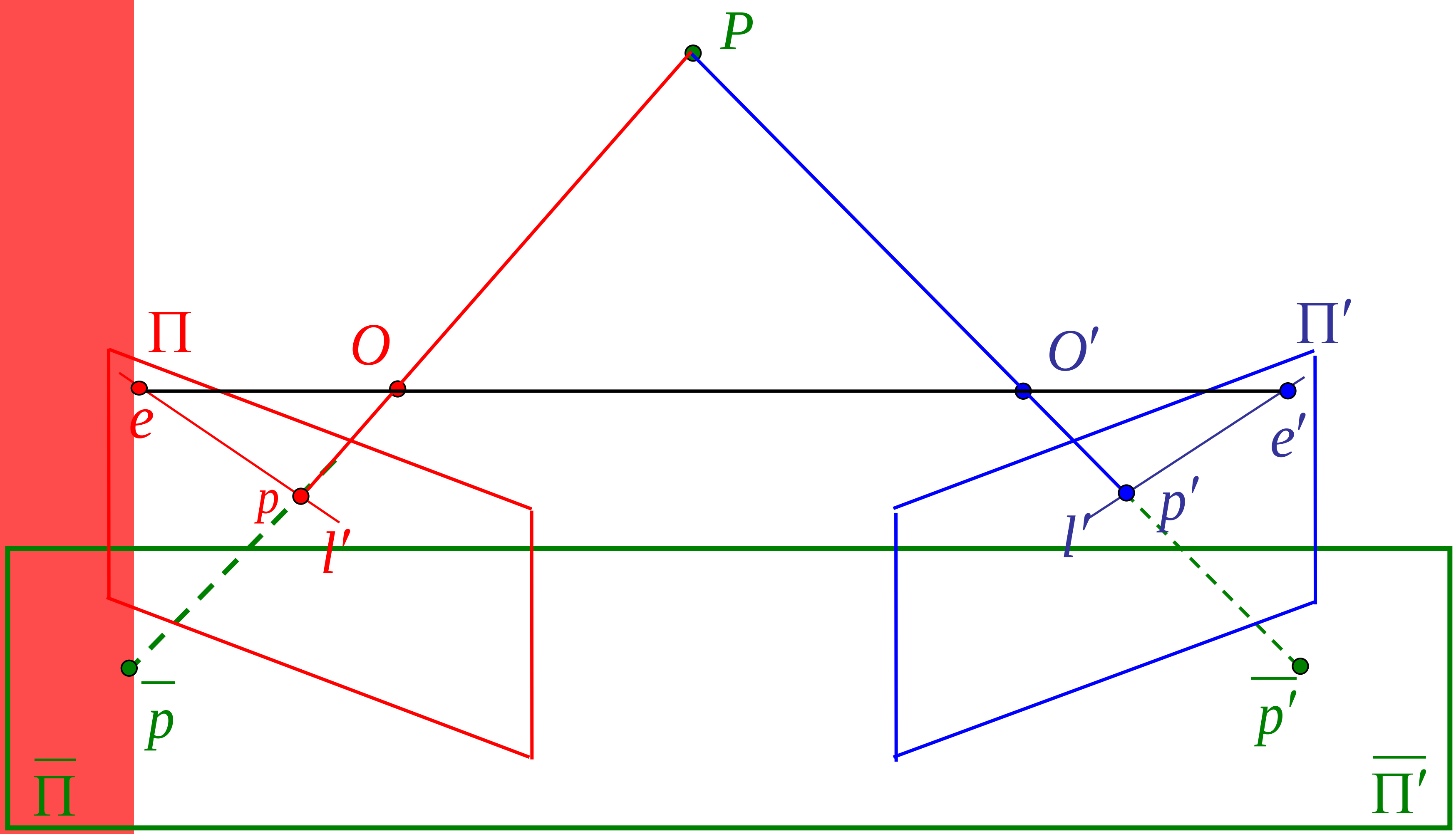


Image Rectification

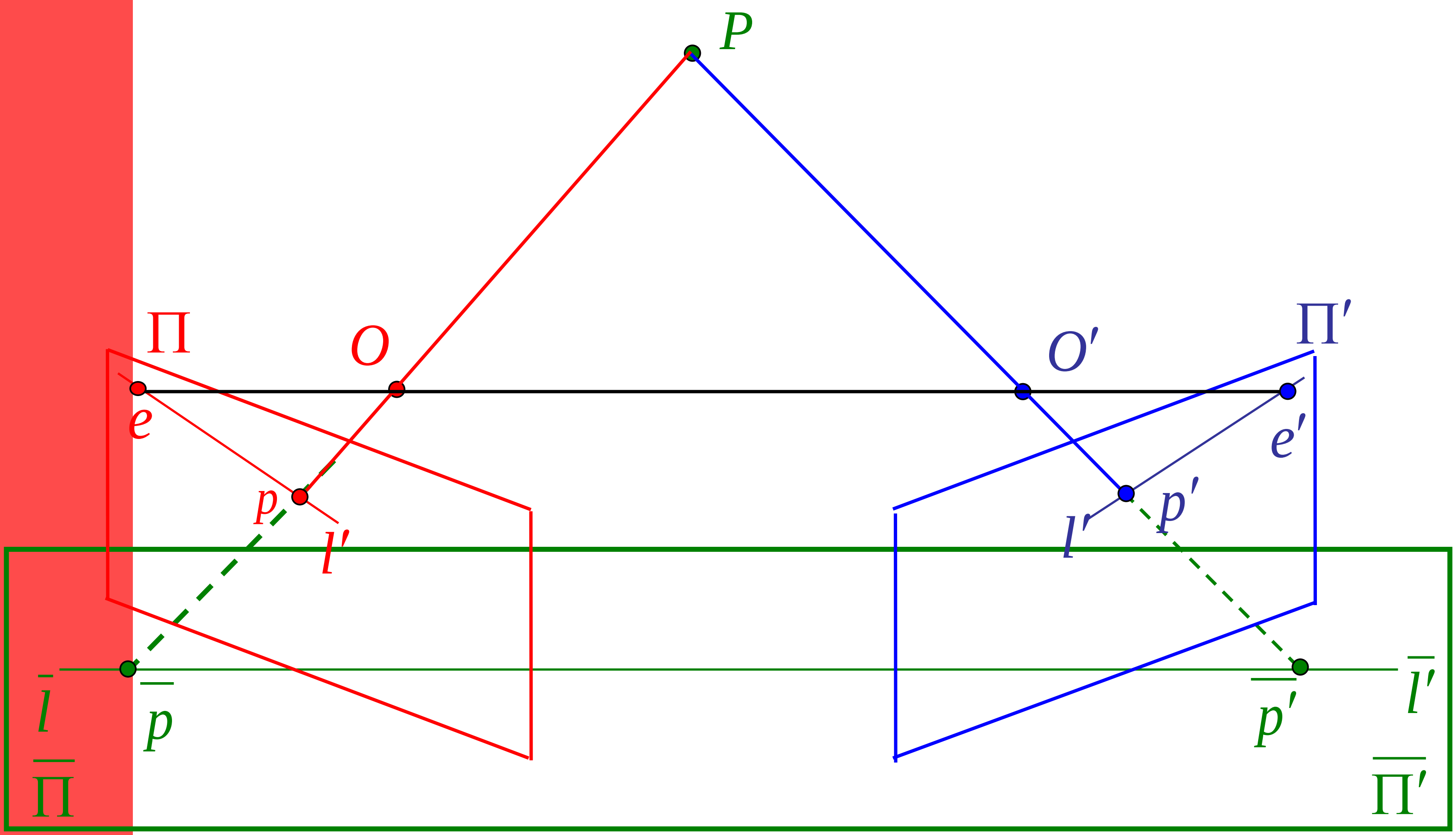
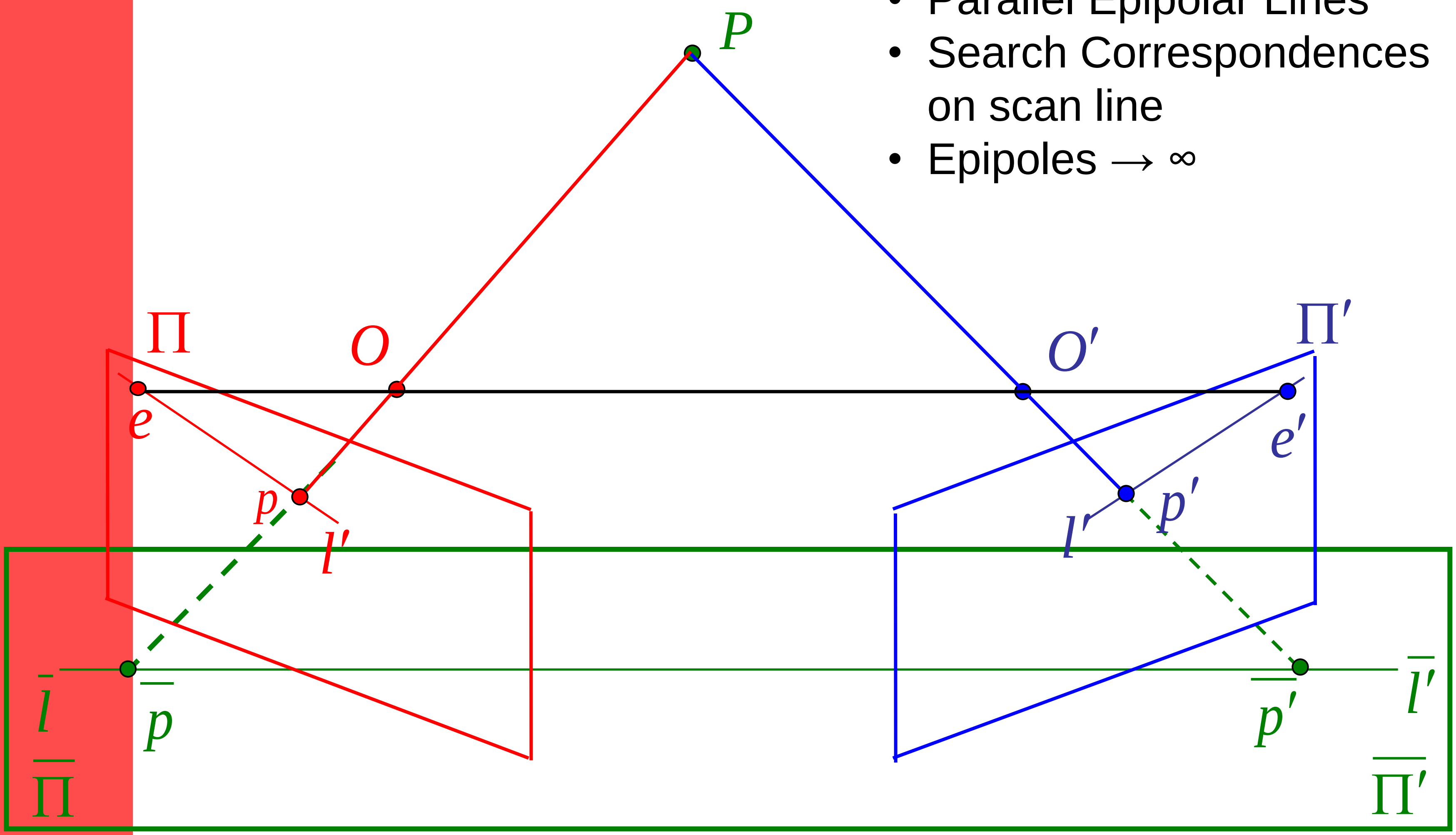


Image Rectification

- Common Image Plane
- Parallel Epipolar Lines
- Search Correspondences on scan line
- Epipoles $\rightarrow \infty$



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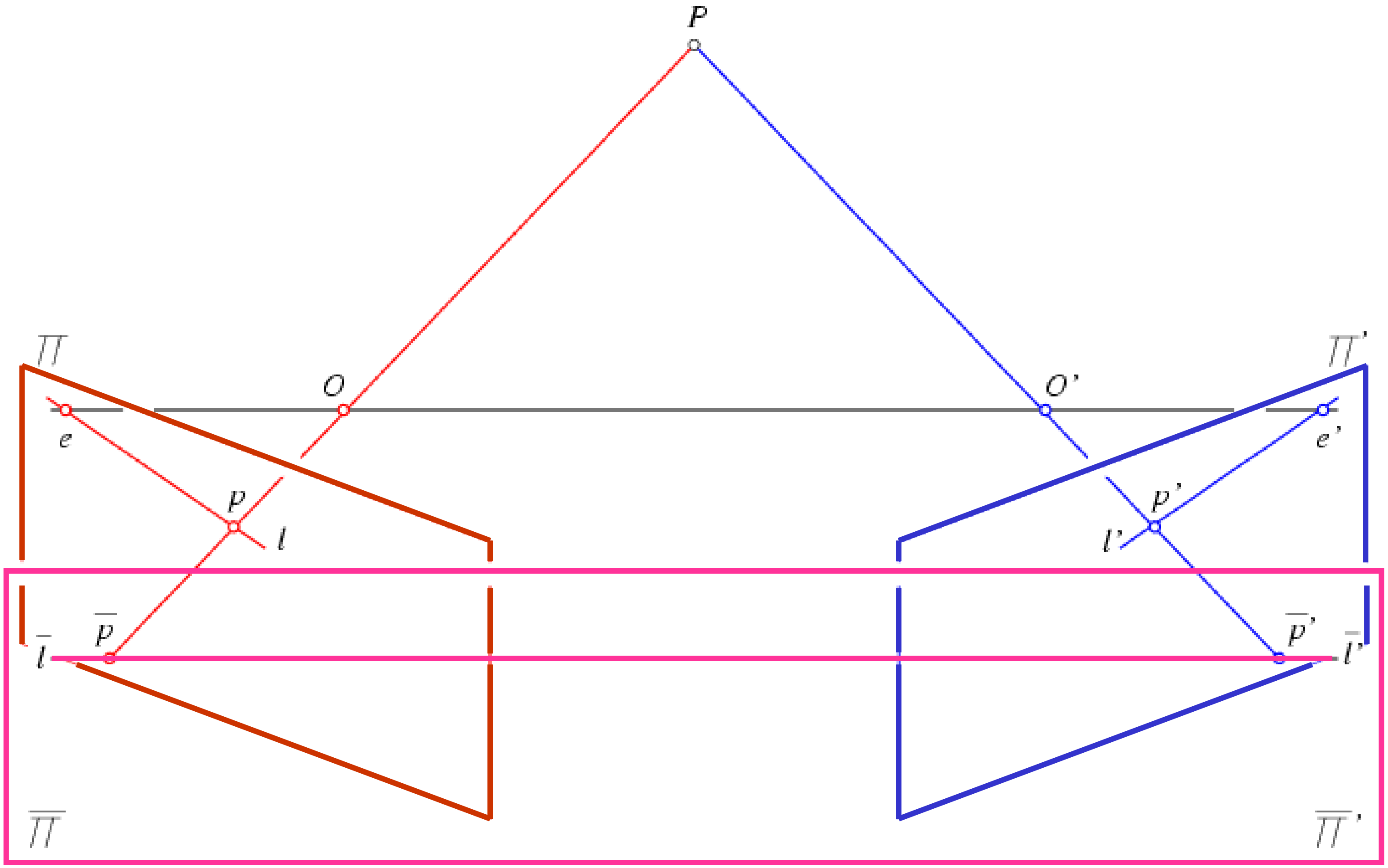
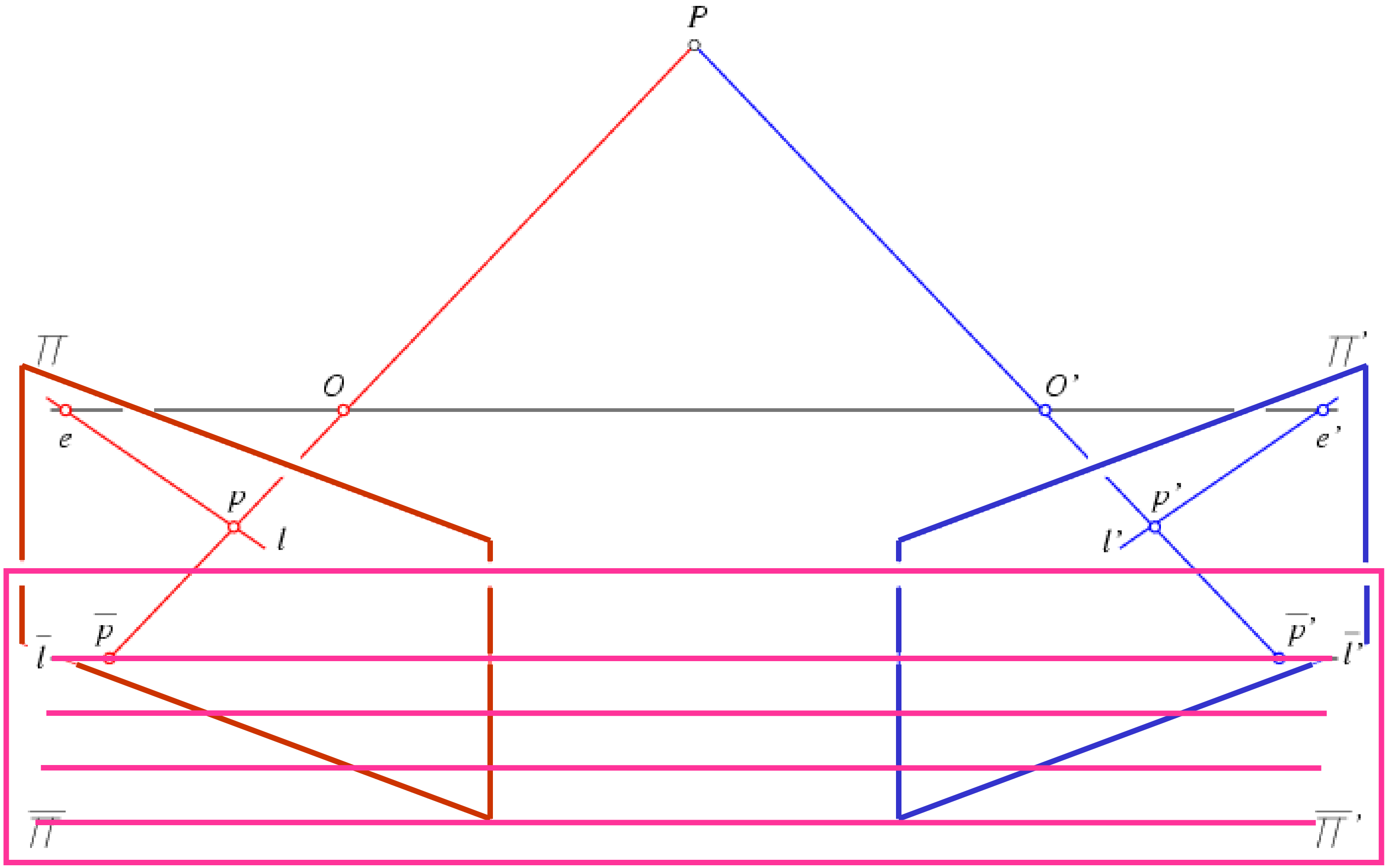


Image Rectification



All epipolar lines are parallel in the rectified image plane.

Image Rectification

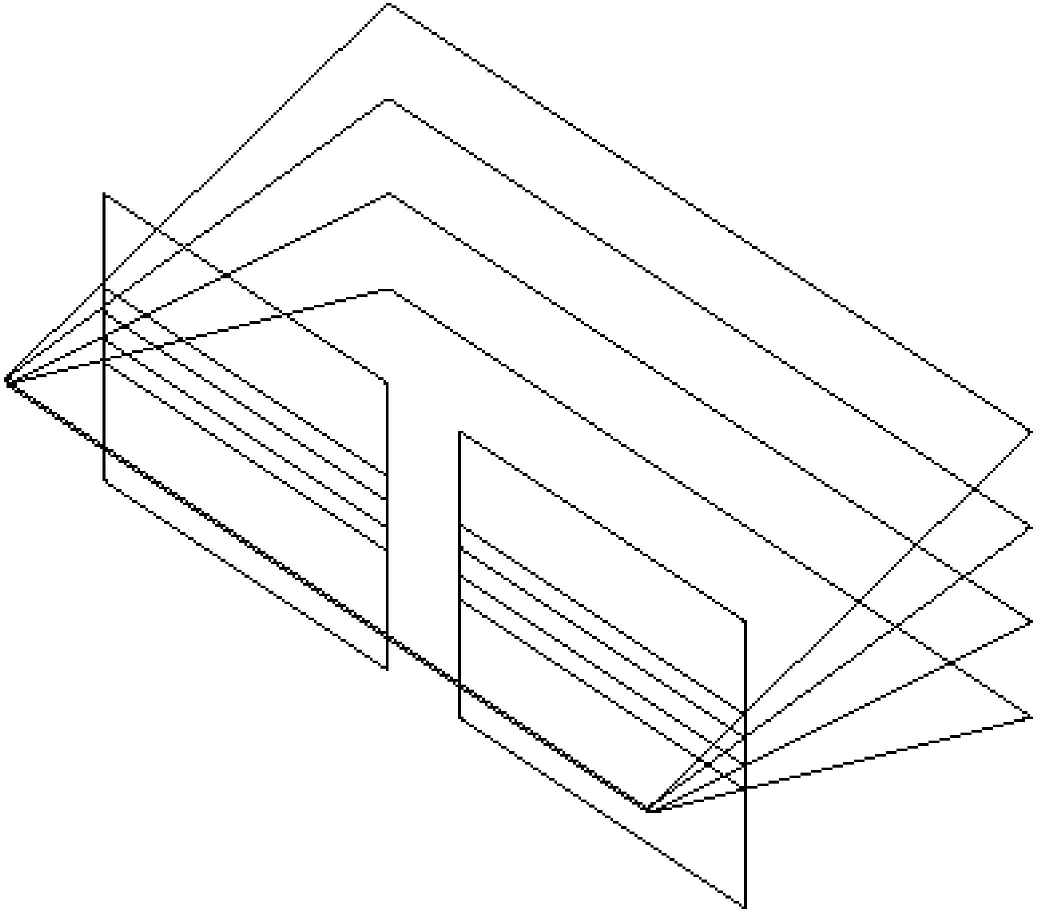
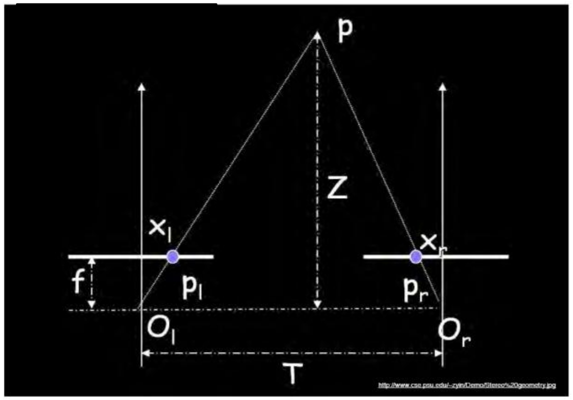


Figure 7.15: Standard stereo setup

Essential matrix example: parallel cameras



$$\mathbf{R} =$$

$$\mathbf{T} =$$

$$\mathbf{E} = [\mathbf{T_x}]\mathbf{R} =$$

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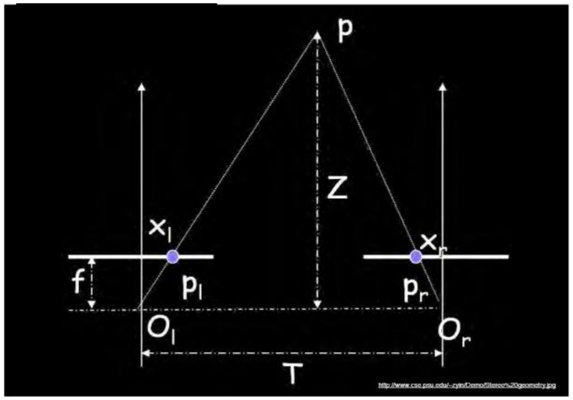
Essential matrix example: parallel cameras



$$\mathbf{R} = \mathbf{I}$$

$$\mathbf{T} =$$

$$\mathbf{E} = [\mathbf{T}_{\mathbf{x}}]\mathbf{R} =$$

Grauman

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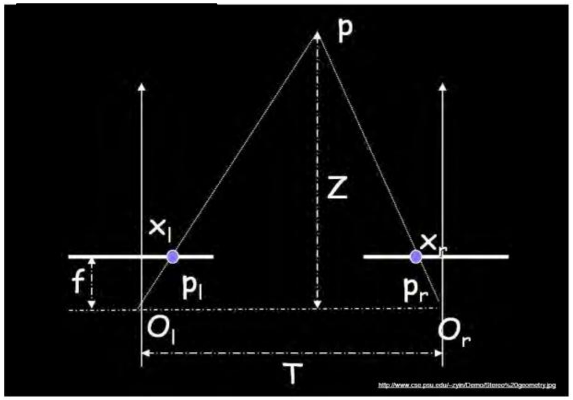


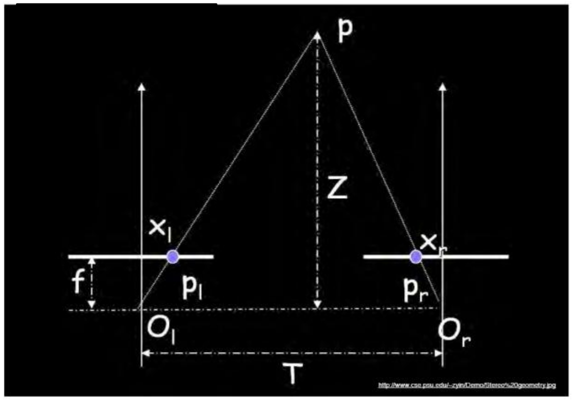
$$\mathbf{R} = \mathbf{I}$$

$$\mathbf{T} = [-d, 0, 0]^{\mathrm{T}}$$

$$\mathbf{E} = [\mathbf{T}_{\mathbf{x}}]\mathbf{R} =$$

Grauman

Essential matrix example: parallel cameras



$$\mathbf{R} = \mathbf{I}$$

$$\mathbf{T} = [-d, 0, 0]^{\mathrm{T}}$$

$$\mathbf{E} = [\mathbf{T_x}]\mathbf{R} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & d \\ 0 & -d & 0 \end{pmatrix}$$

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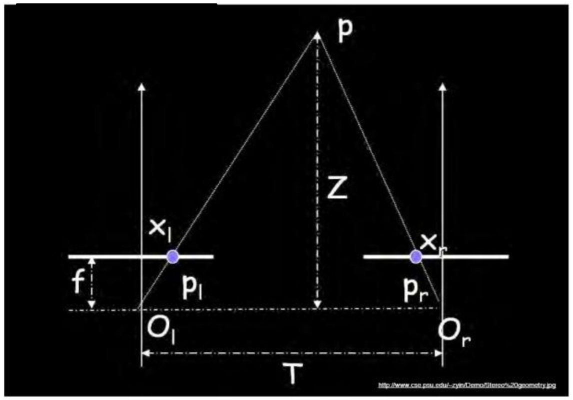

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$$\mathbf{p}'^{\mathrm{T}}\mathbf{E}\mathbf{p} = 0$$

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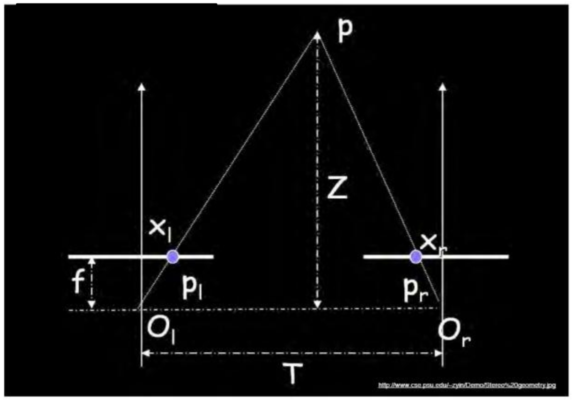

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$$\begin{bmatrix} x' & y' & f \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & d \\ 0 & -d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ f \end{bmatrix} = 0$$

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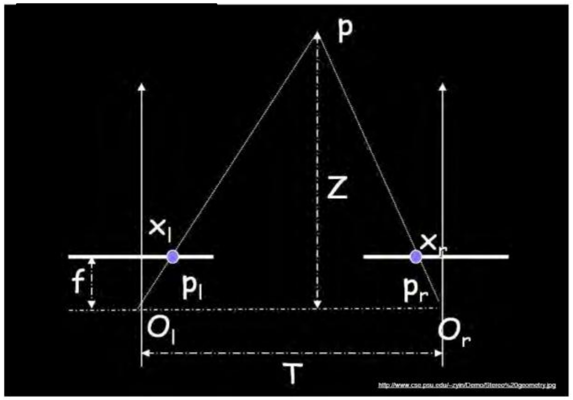

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$$\Leftrightarrow \begin{bmatrix} x' & y' & f \end{bmatrix} \begin{bmatrix} 0 \\ df \\ -dy \end{bmatrix} = 0$$

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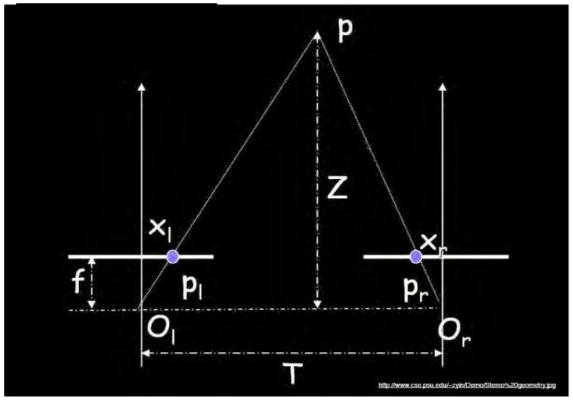

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$$\boxed{\Leftrightarrow y = y'}$$

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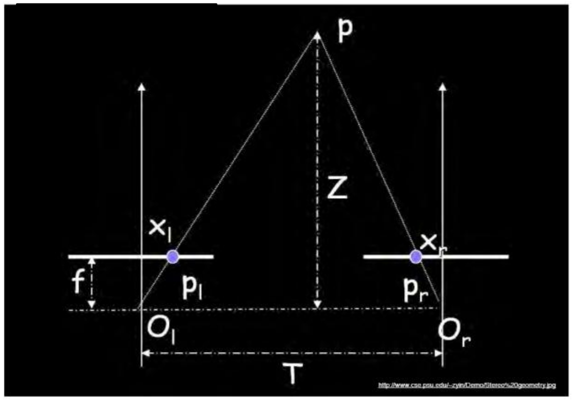
Essential matrix example: parallel cameras



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$$\boxed{\Leftrightarrow y = y'}$$

For the parallel cameras, image of any point must lie on same horizontal line in each image plane.

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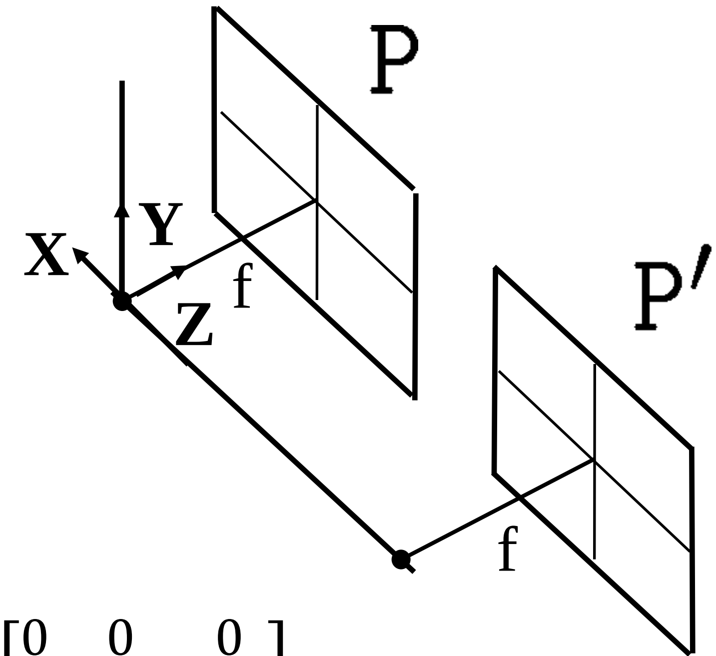
Example I: compute the fundamental matrix for a parallel camera stereo rig

$$\mathtt{P} = \mathtt{K}[\mathtt{I} \mid \mathbf{0}] \qquad \mathtt{P}' = \mathtt{K}'[\mathtt{R} \mid \mathbf{t}]$$



$$K = K' = \begin{bmatrix} \alpha & 0 & u0 \\ 0 & \beta & v0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathrm{R{=}I} \quad \mathrm{t{=}} \begin{pmatrix} tx \\ 0 \\ 0 \end{pmatrix}$$

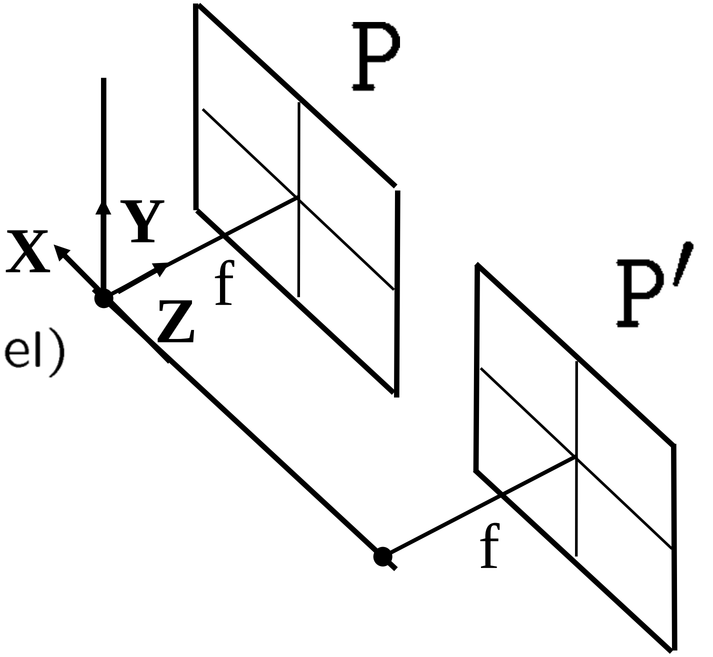
$$\mathtt{F} = \mathtt{K}'^{-\top}[\mathbf{t}]_\times \mathtt{R}\mathtt{K}^{-1}$$

$$= \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ -\frac{u0}{a} & -\frac{v0}{b} & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -tx \\ 0 & tx & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{a} & 0 & -\frac{u0}{a} \\ 0 & \frac{1}{b} & -\frac{v0}{b} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{tx}{b} \\ 0 & \frac{tx}{b} & 0 \end{bmatrix}$$

$$\mathbf{x}'^\top \mathbf{F} \mathbf{x} = \begin{pmatrix} x' & y' & 1 \end{pmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0$$

- reduces to $y = y'$, i.e. raster correspondence (horizontal scan-lines)

$\mathtt{F}$ is a rank 2 matrix



The epipole $\mathbf{e}$ is the null-space vector (kernel) of $\mathtt{F}$ (exercise), i.e. $\mathtt{F}\mathbf{e} = \mathbf{0}$

In this case

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \mathbf{0}$$

so that

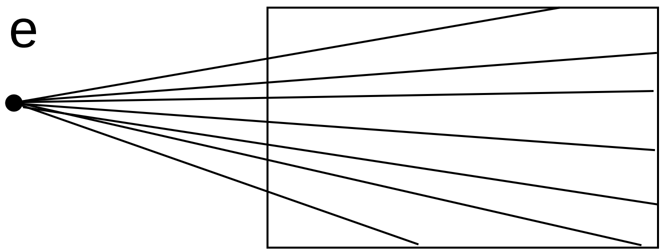
$$\mathbf{e} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Geometric interpretation ?

Image pair rectification

Goal: Simplify stereo matching by “warping” the images

Apply projective transformation so that epipolar lines correspond to horizontal scanlines



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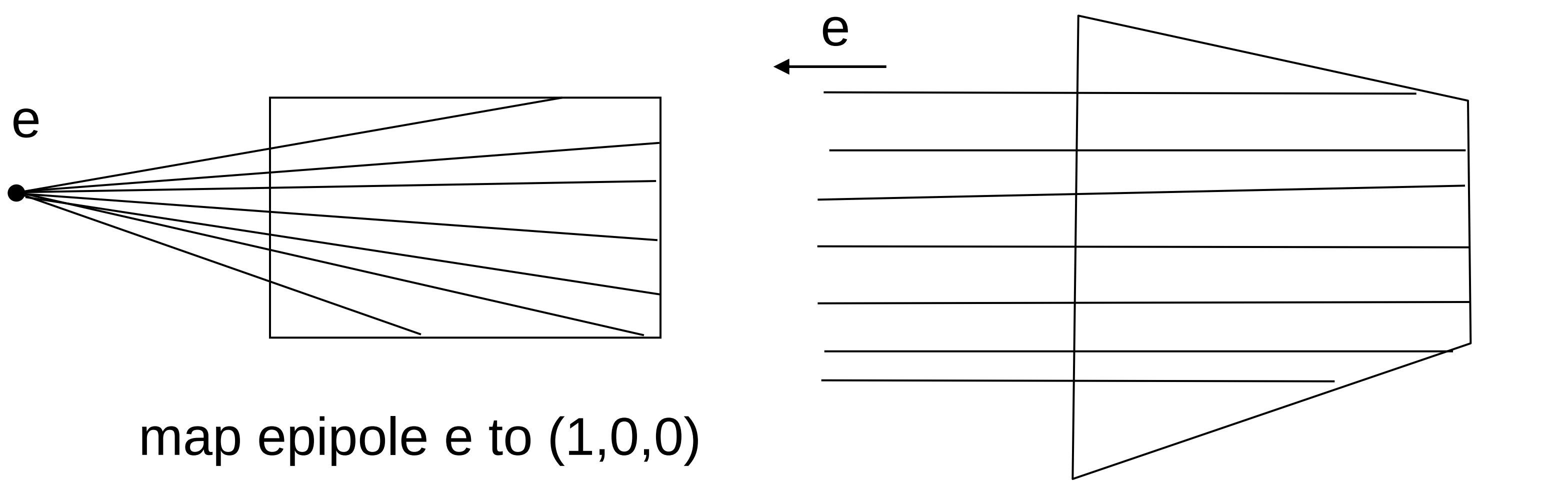

map epipole e to (1,0,0)

$$\begin{bmatrix}1\\0\\0\end{bmatrix} = He$$

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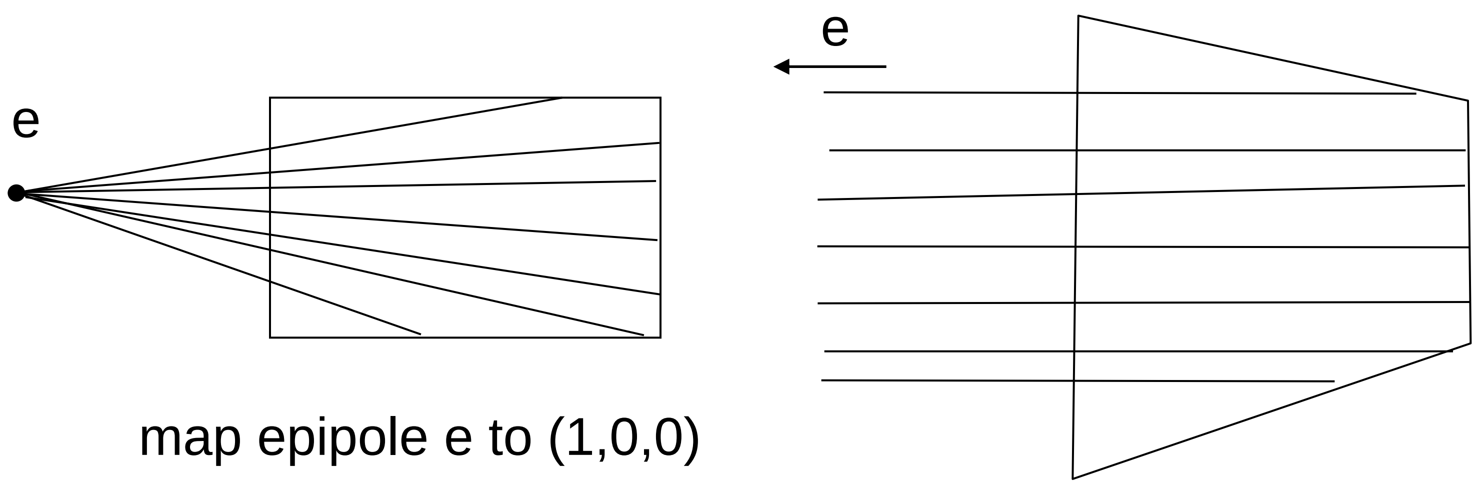

map epipole e to (1,0,0)

try to minimize image distortion

$$\begin{bmatrix}1\\0\\0\end{bmatrix} = He$$

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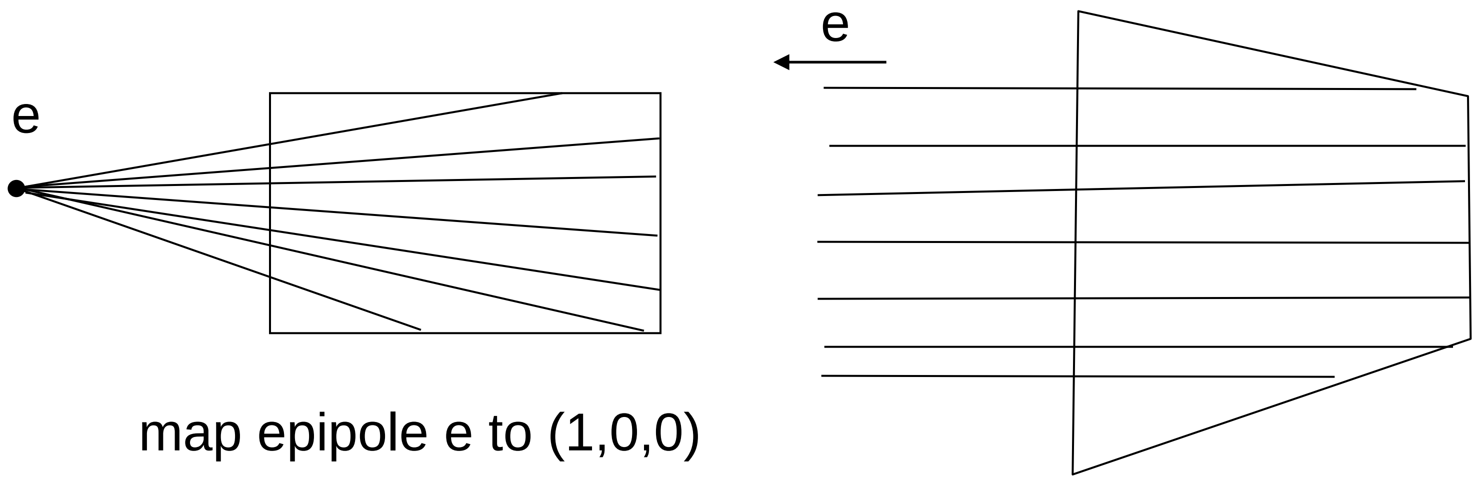


$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = He$$

map epipole e to (1,0,0)

try to minimize image distortion

problem when epipole in (or close to) the image

Planar rectification

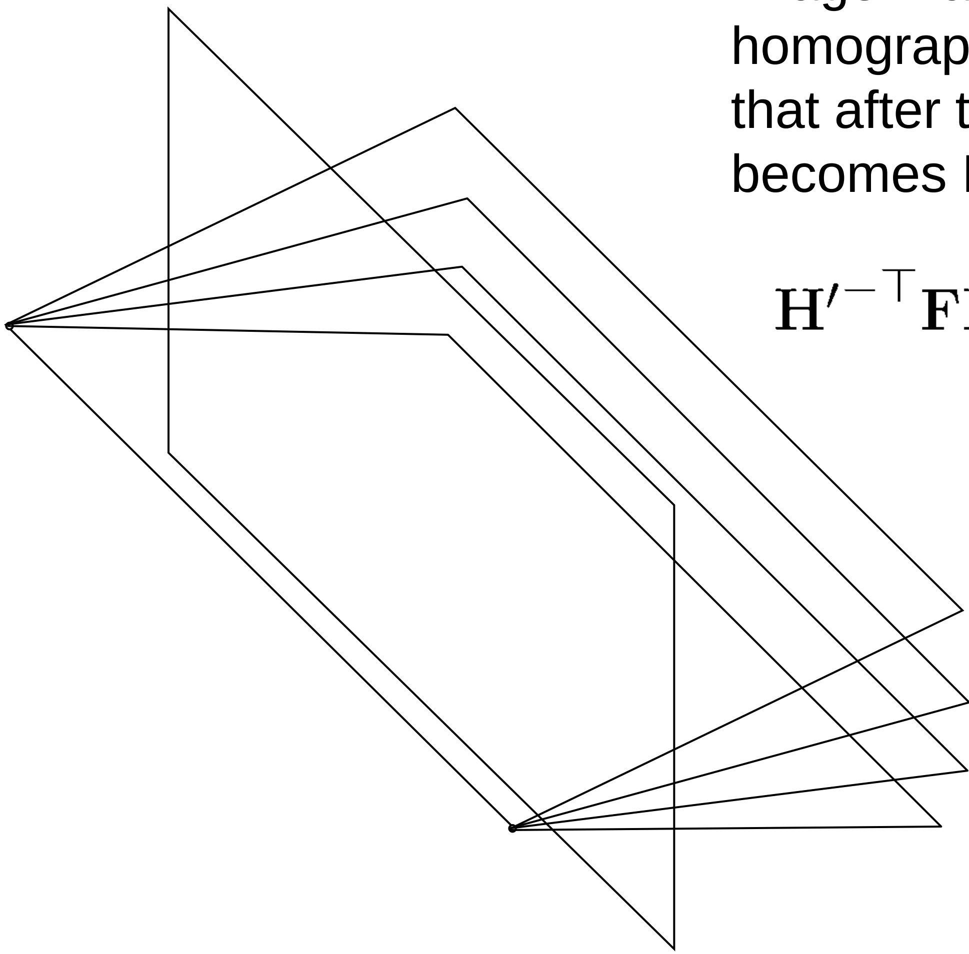
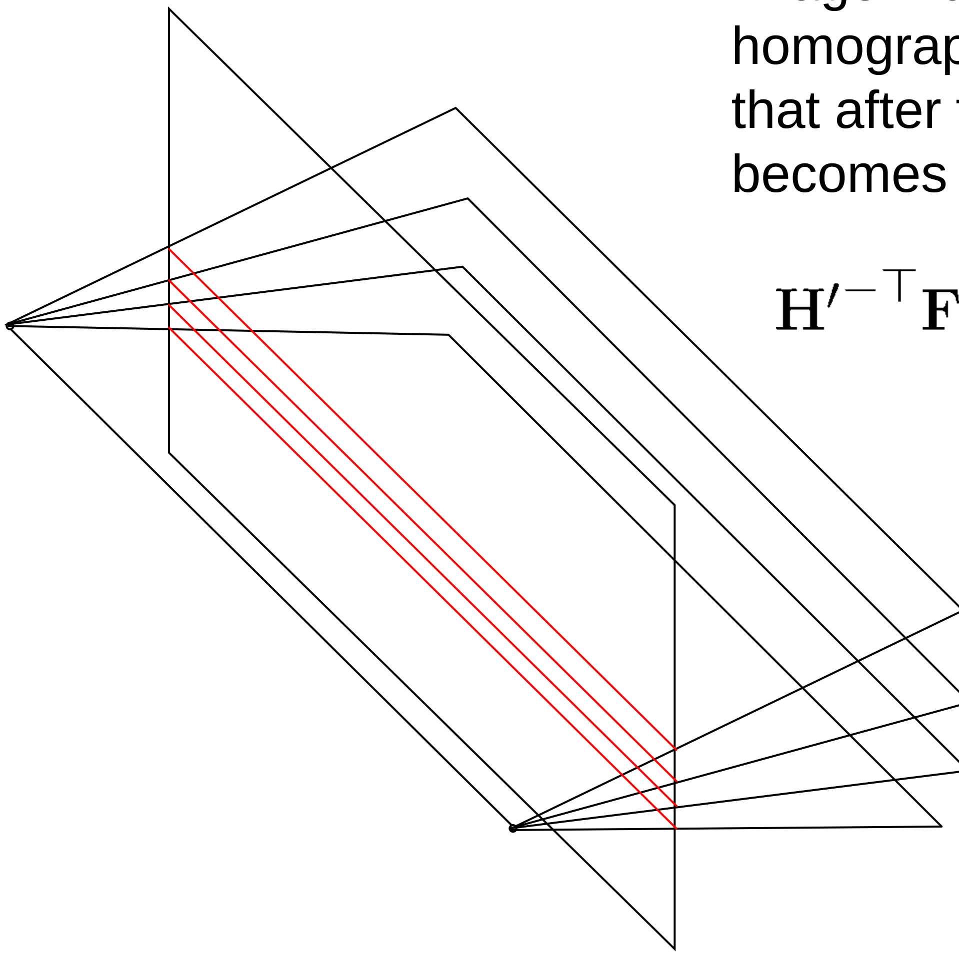


Image Transformations: Find homographies H' and H so that after transformations, F^n becomes F of parallel cameras:

$$\mathbf{H}'^{-\top}\mathbf{F}\mathbf{H}^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

Planar rectification



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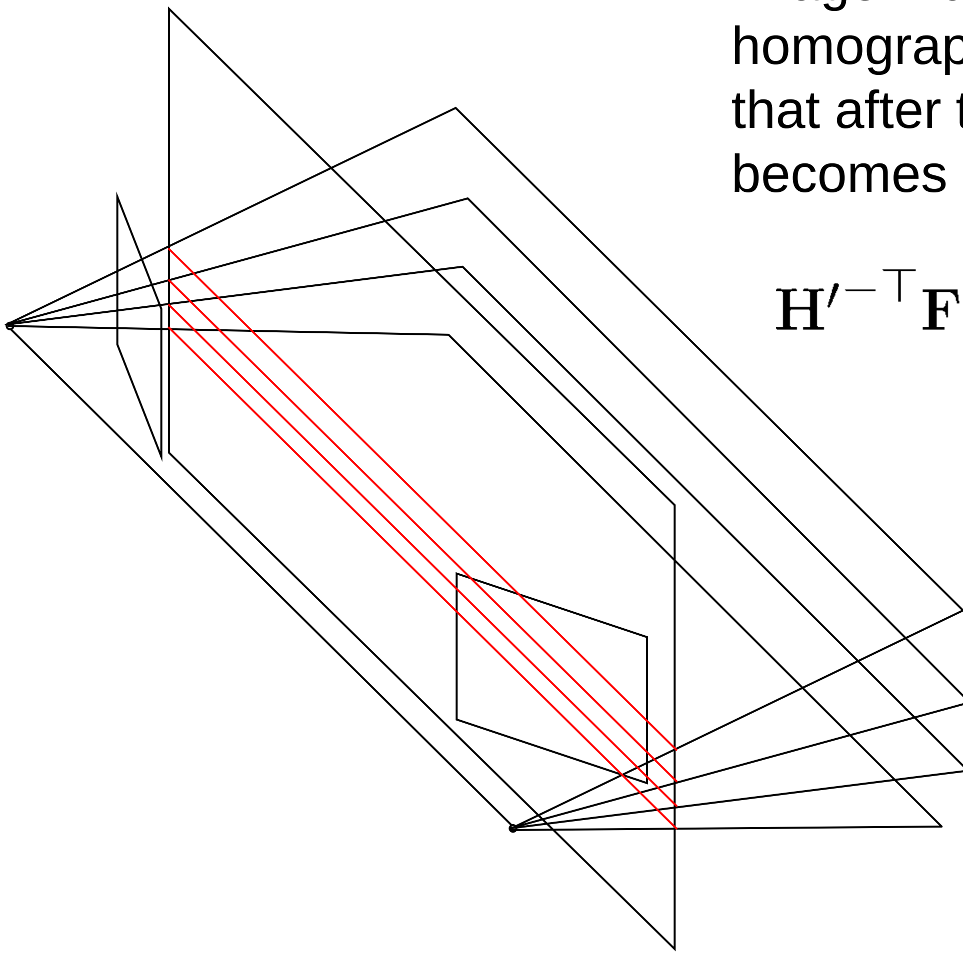

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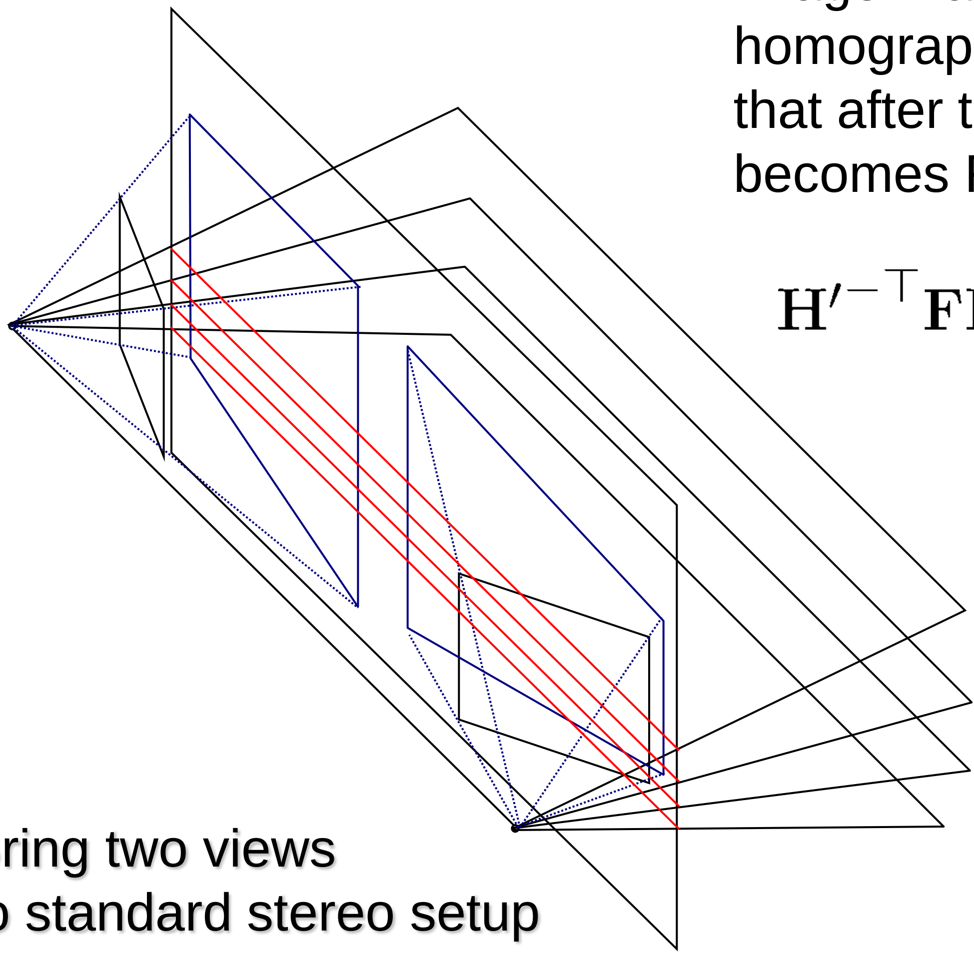


Bring two views
to standard stereo setup

(moves epipole to ∞)
(not possible when in/close to image)

Algorithm Rectification

Following Trucco & Verri book pp. 159

- known T and R between cameras
- Rotate left camera so that epipole e_l goes to infinity along horizontal axis
- Apply same rotation to right camera to recover geometry
- Rotate right camera by R^{-1}
- Adjust scale

The algorithm consists of four steps:

- Rotate the left camera so that the epipole goes to infinity along the horizontal axis.
- Apply the same rotation to the right camera to recover the original geometry.
- Rotate the right camera by R.
- Adjust the scale in both camera reference frames.

To carry out this method, we construct a triple of mutually orthogonal unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$. Since the problem is underconstrained, we are going to make an arbitrary choice. The first vector, $\mathbf{e}_1$, is given by the epipole; since the image center is in the origin, $\mathbf{e}_1$ coincides with the direction of translation, or

$$\mathbf{e}_1 = \frac{\mathbf{T}}{\|\mathbf{T}\|}.$$

The only constraint we have on the second vector, $\mathbf{e}_2$, is that it must be orthogonal to $\mathbf{e}_1$. To this purpose, we compute and normalize the cross product of $\mathbf{e}_1$ with the direction vector of the optical axis, to obtain

$$\mathbf{e}_2 = \frac{1}{\sqrt{T_x^2 + T_y^2}} \left[-T_y, T_x, 0\right]^\top .$$

The third unit vector is unambiguously determined as

$$\mathbf{e}_3 = \mathbf{e}_1 \times \mathbf{e}_2.$$

It is easy to check that the orthogonal matrix defined as

$$R_{rect} = \begin{pmatrix} \mathbf{e}_1^\top \\ \mathbf{e}_2^\top \\ \mathbf{e}_3^\top \end{pmatrix} \qquad (7.22)$$

rotates the left camera about the projection center in such a way that the epipolar lines become parallel to the horizontal axis. This implements the first step of the algorithm. Since the remaining steps are straightforward, we proceed to give the customary algorithm:

Algorithm RECTIFICATION

The input is formed by the intrinsic and extrinsic parameters of a stereo system and a set of points in each camera to be rectified (which could be the whole images). In addition, Assumptions 1 and 2 above hold.

1. Build the matrix R_{rect} as in (7.22);
2. Set $R_l = R_{rect}$ and $R_r = R R_{rect}$;
3. For each left-camera point, $\mathbf{p}_l = [x, y, f]^\top$ compute

$$R_l \mathbf{p}_l = [x', y', z']$$

and the coordinates of the corresponding rectified point, $\mathbf{p}'_l$, as

$$\mathbf{p}'_l = \frac{f}{z'}[x', y', z'].$$

4. Repeat the previous step for the right camera using R_r and $\mathbf{p}_r$.

The output is the pair of transformations to be applied to the two cameras in order to rectify the two input point sets, as well as the rectified sets of points.

From: Trucco & Verri, Introductory Techniques for 3-D Computer Vision, pp. 157-161

More elegant Solution

- Idea: Mapping epipole to infinity $\rightarrow [1,0,0]^T$
- Factorization of matrix F=SM, where S is skew symmetric and M representing the required homography (projective transformation).
- Use SVD:

$$\mathtt{F} = \mathtt{UDV}^\top = \mathtt{UWZD'V}^\top = \left(\mathtt{UWU}^\top\right)\left(\mathtt{UZD'V}^\top\right) = \mathtt{SM}$$

where

$$\mathtt{W} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \; ; \; \mathtt{Z} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

http://www.robots.ox.ac.uk/~vgg/hzbook/hzbook2/clarification_rectification.pdf

Stereo matching with general camera configuration

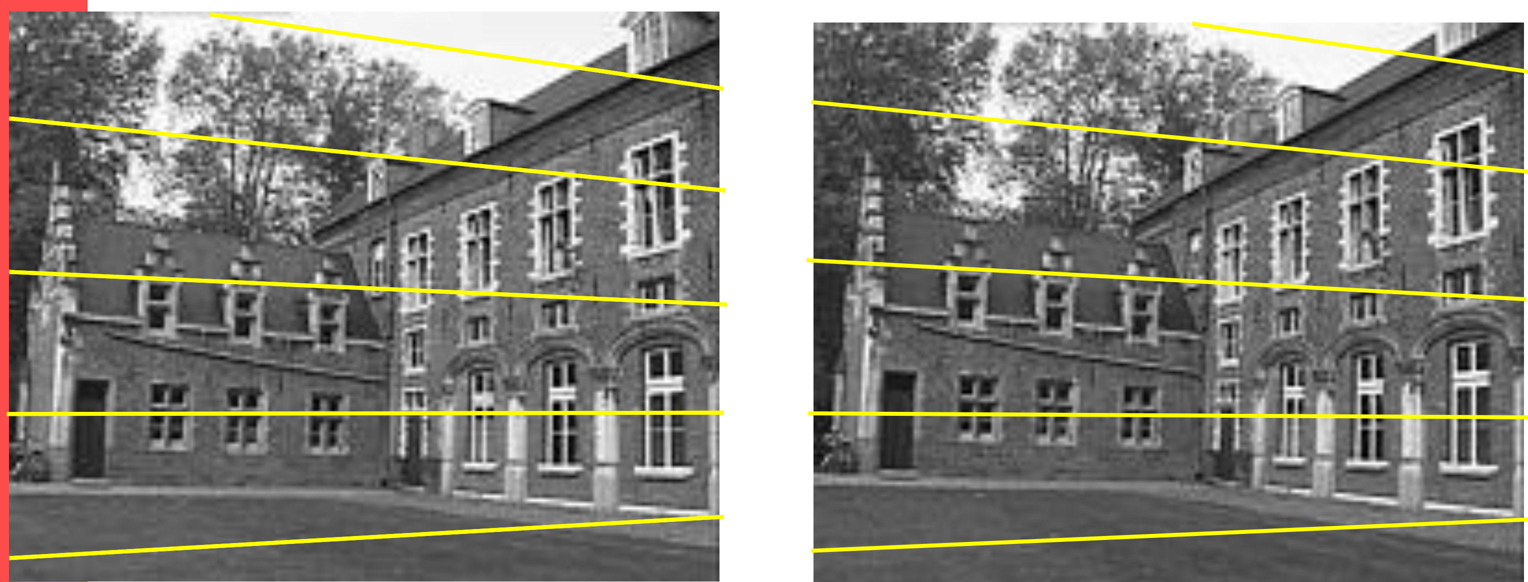
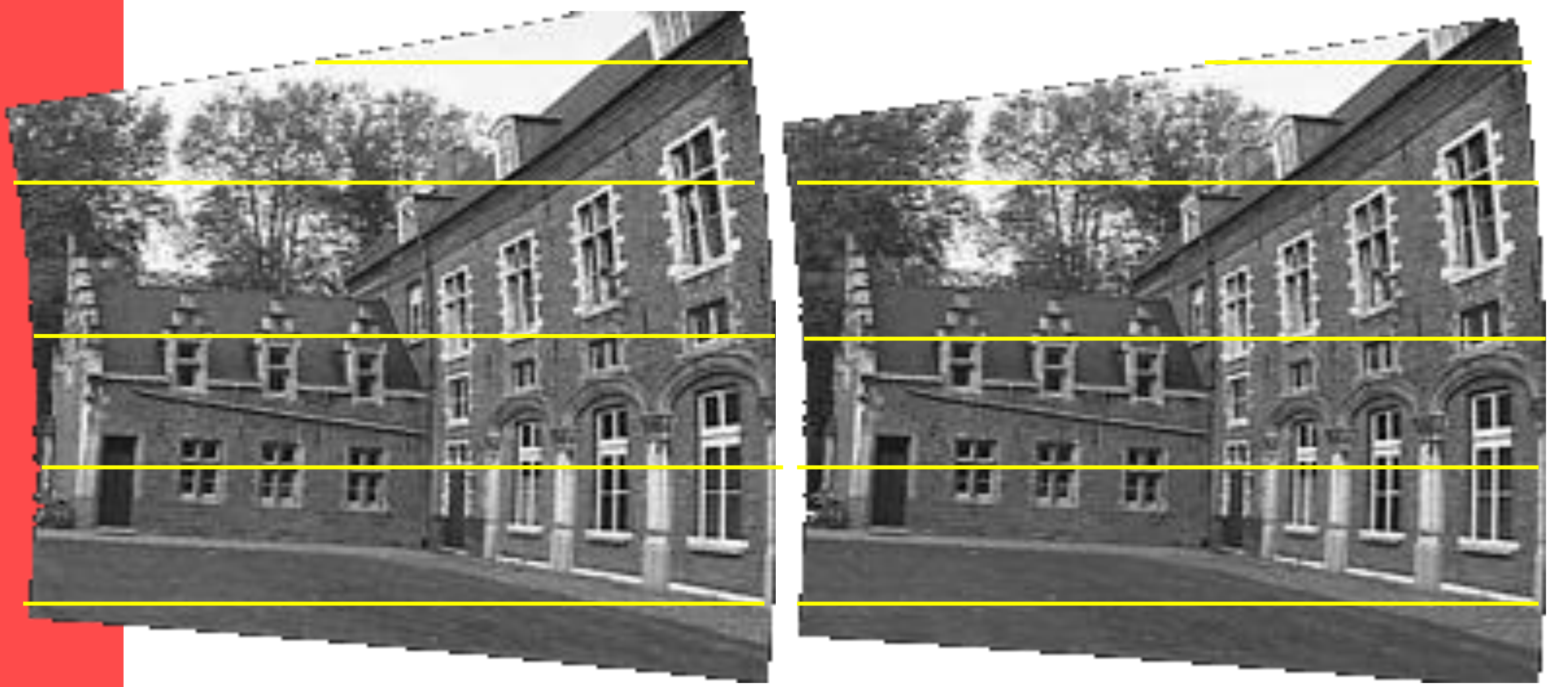
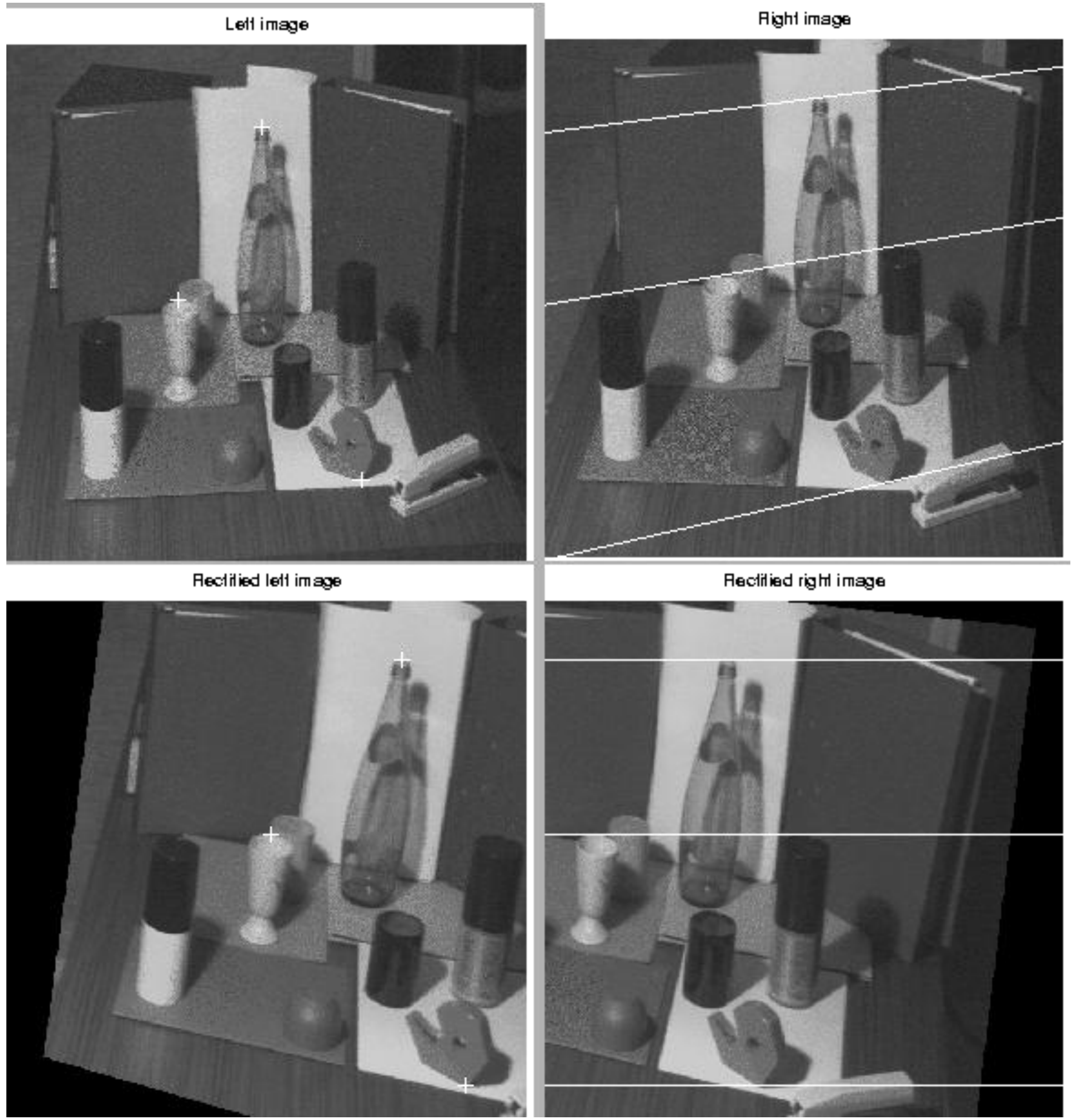
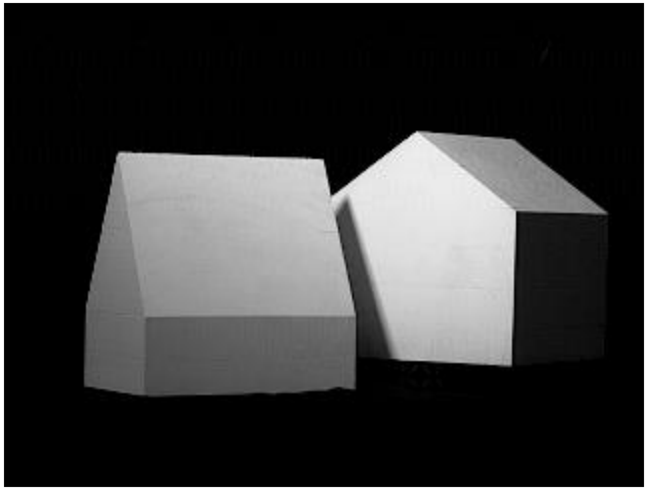
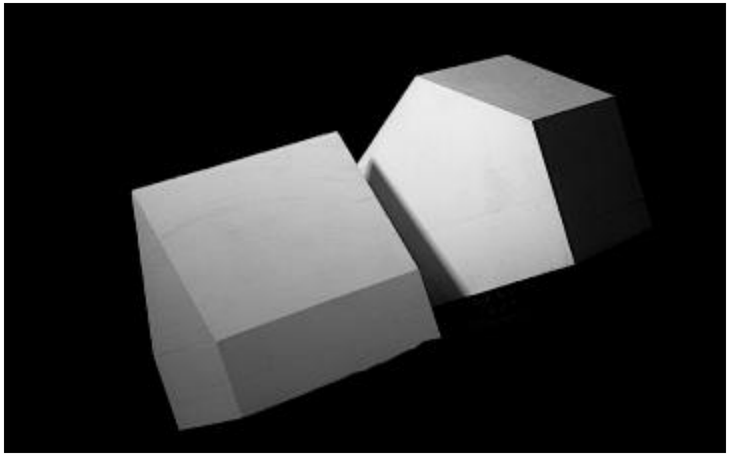


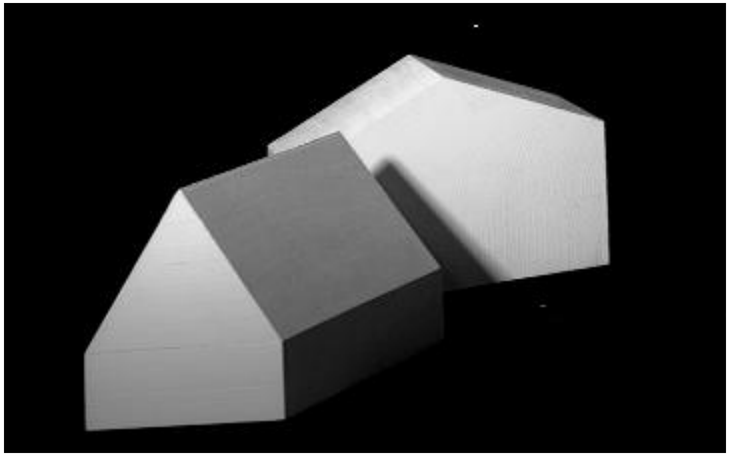
Image pair rectification




Left image
Right image
Rectified left image
Rectified right image







Other Material /Code

- Epipolar Geometry, Rectification: http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/FUSIELLO2/rectif_cvol.html

- Fusiello, Trucco & Verri:Tutorial, Matlab code etc: http://profs.sci.univr.it/~fusiello/demo/rect/

Run Example

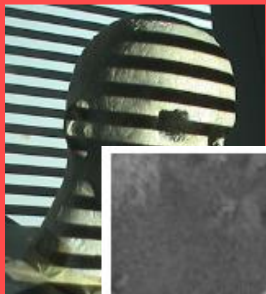
Demo for stereo reconstruction (out of date):
http://mitpress.mit.edu/e-journals/Videre/001/articles/Zhang/CalibEnv/CalibEnv.html

Updated Webpages:

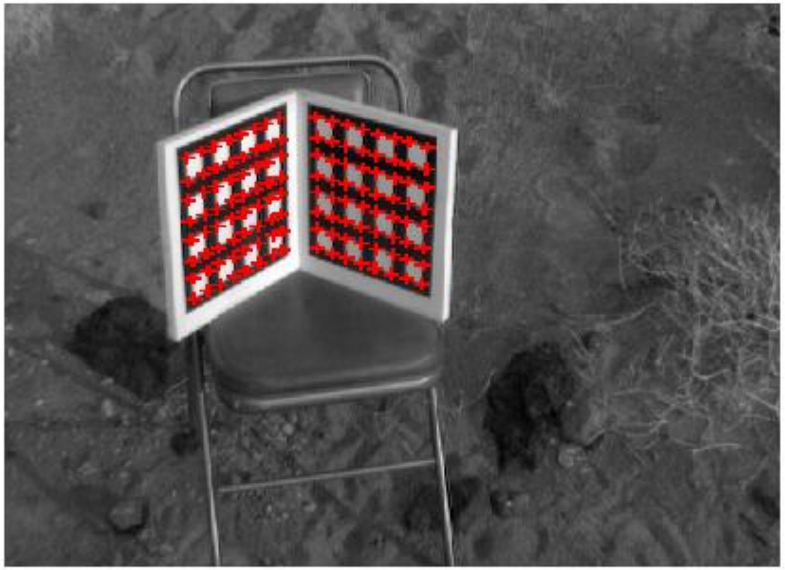
http://research.microsoft.com/en-us/um/people/zhang/INRIA/softwares.html

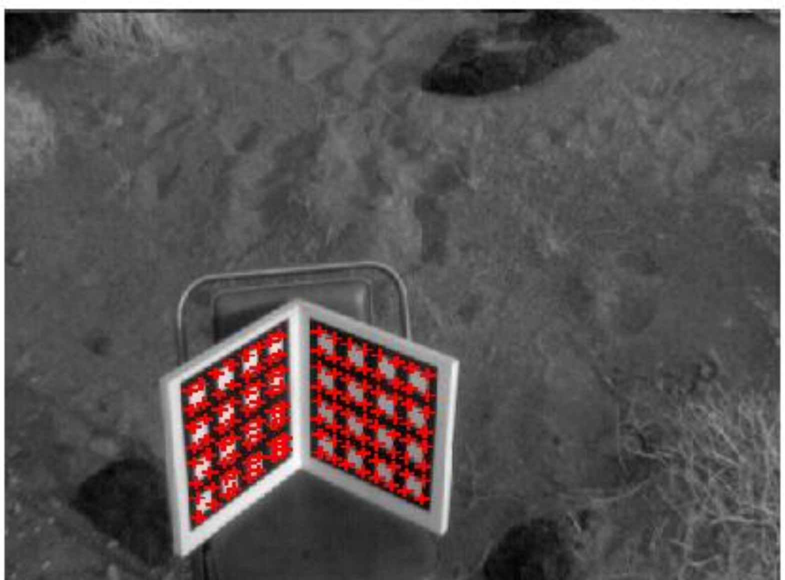
SFM Example:
http://research.microsoft.com/en-us/um/people/zhang/INRIA/SFM-Ex/SFM-Ex.html

Software:
http://research.microsoft.com/en-us/um/people/zhang/INRIA/softwares.html



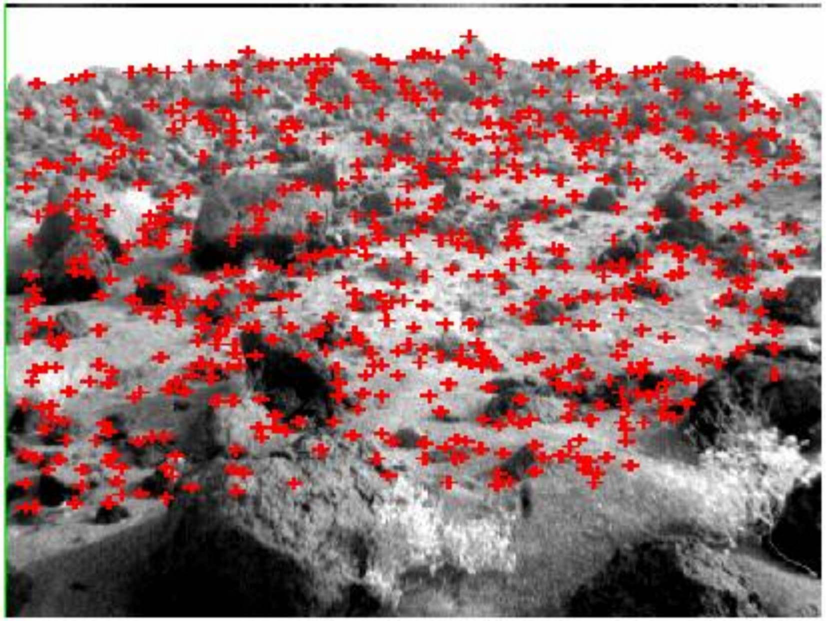
Example: **[Zhengyou Zhang]()**

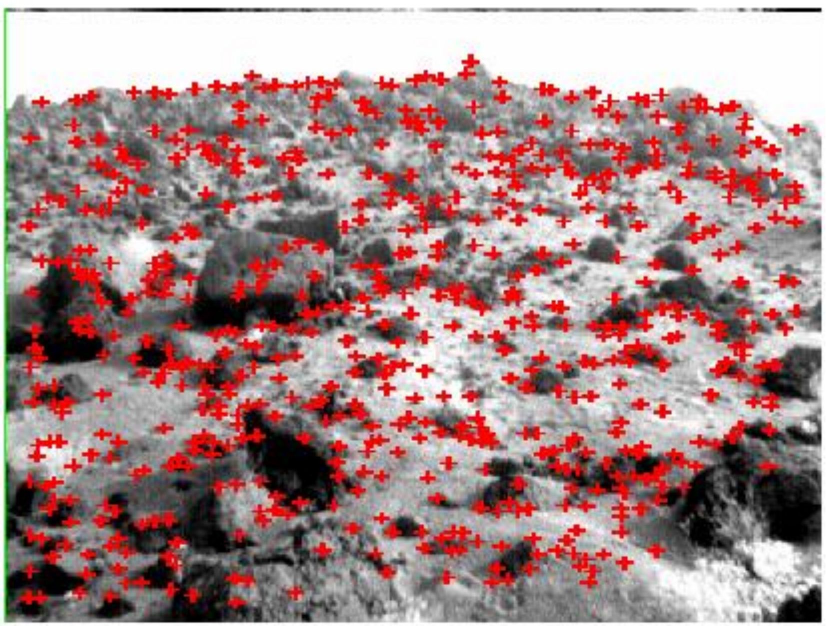




Fundamental matrix between the two cameras:

5.2049303e-07	-2.9159992e-05	-2.5405448e-04
2.9090123e-05	6.7508671e-08	-5.6408271e-03
-5.0032429e-03	5.6687599e-03	1.0e-0

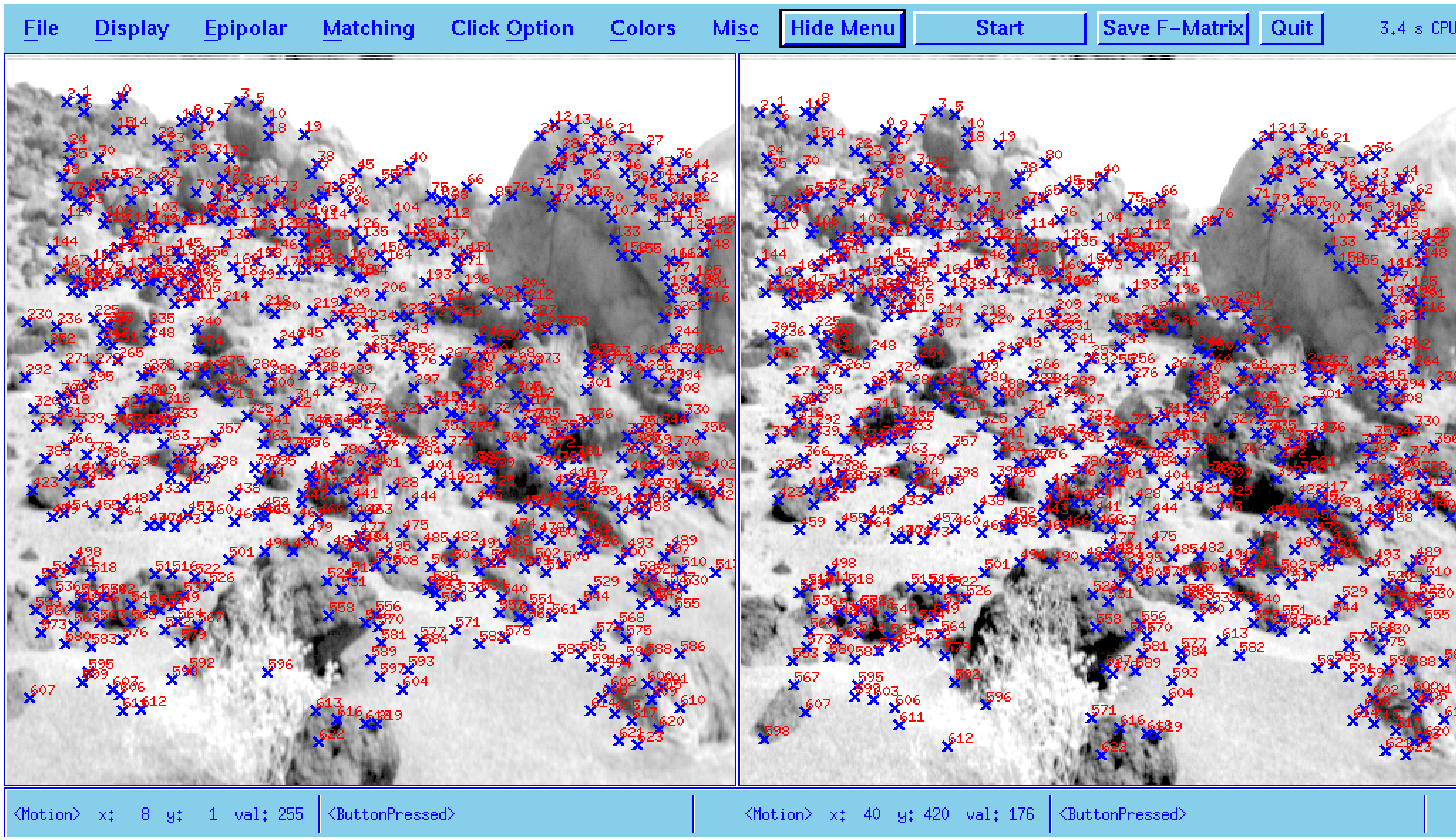




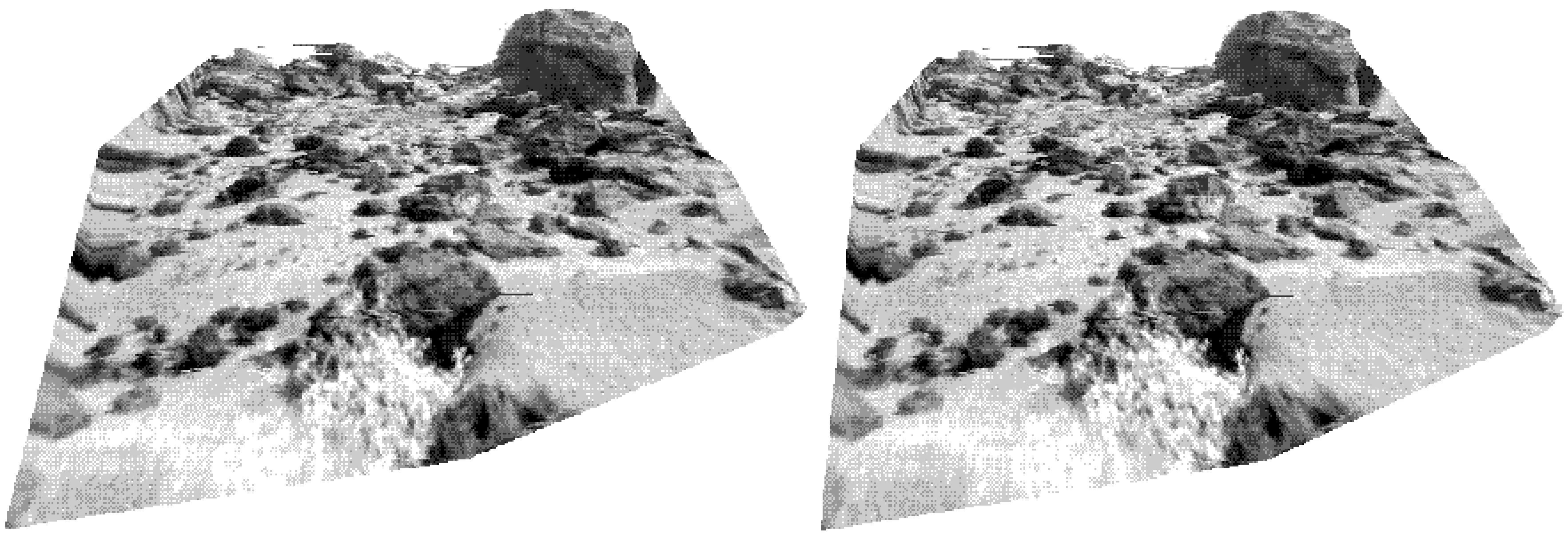
Points have been extracted using Harris corner detector, point matches via fundamental matrix F and search along epipolar lines.

2.019618e-06	1.887944e-04	-3.978027e-01
-1.702288e-04	-2.145494e-07	6.907326e-03
3.978027e-01	-7.553172e-03	8.266808e-01

Point matches found by a correlation technique



3D reconstruction represented by a pseudo stereogram



http://research.microsoft.com/en-us/um/people/zhang/INRIA/SFM-Ex/SFM-Ex.html

Additional Materials: Forward Translating Cameras

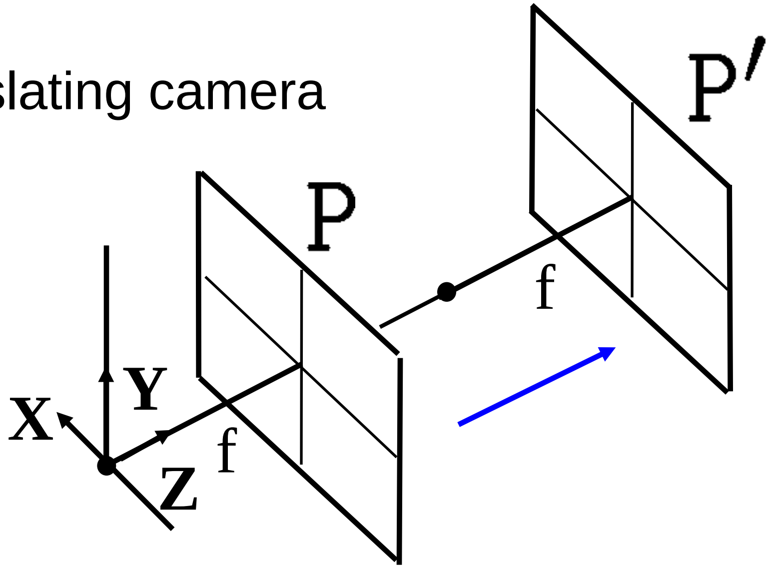

OXFORD


OXFORD

Example II: compute F for a forward translating camera

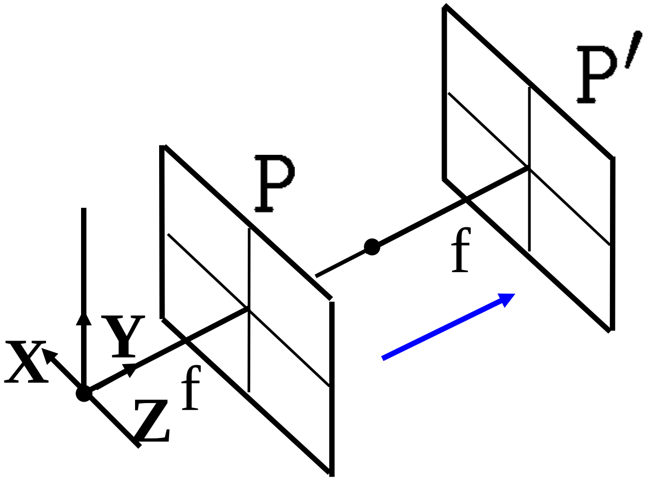
$$\mathtt{P} = \mathtt{K}[\mathtt{I} \mid \mathbf{0}] \qquad \mathtt{P}' = \mathtt{K}'[\mathtt{R} \mid \mathbf{t}]$$

$$\mathtt{K} = \mathtt{K}' = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathtt{R} = \mathtt{I} \quad \mathbf{t} = \begin{pmatrix} 0 \\ 0 \\ t_z \end{pmatrix}$$



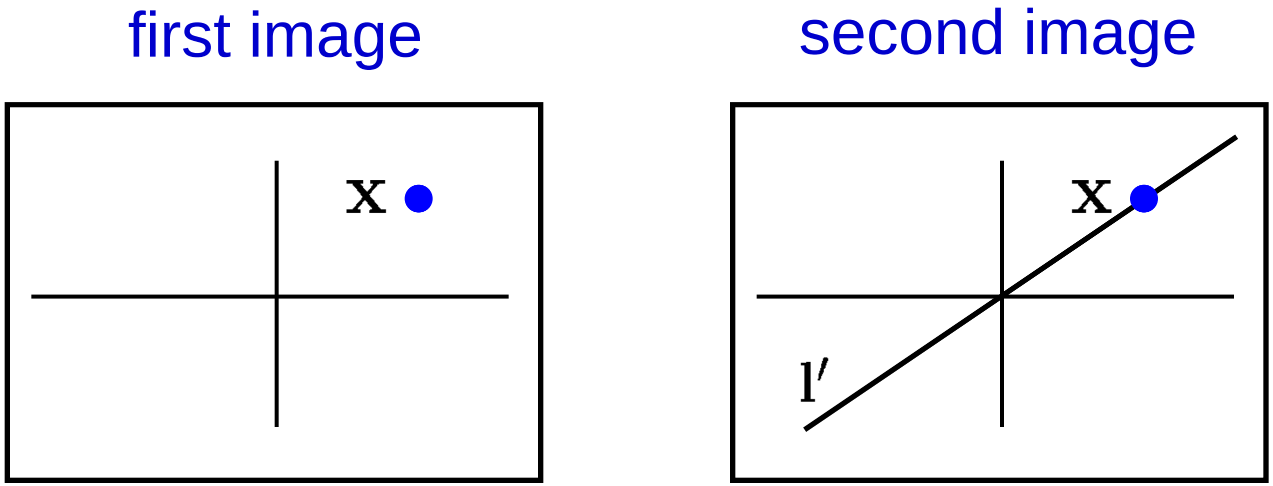
$$\begin{aligned} \mathtt{F} &= \mathtt{K}'^{-\top}[\mathbf{t}]_\times \mathtt{R}\mathtt{K}^{-1} \\ &= \begin{bmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -t_z & 0 \\ t_z & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

From $\mathbf{l}' = \mathtt{F}\mathbf{x}$ the epipolar line for the point $\mathbf{x} = (x, y, 1)^\top$ is



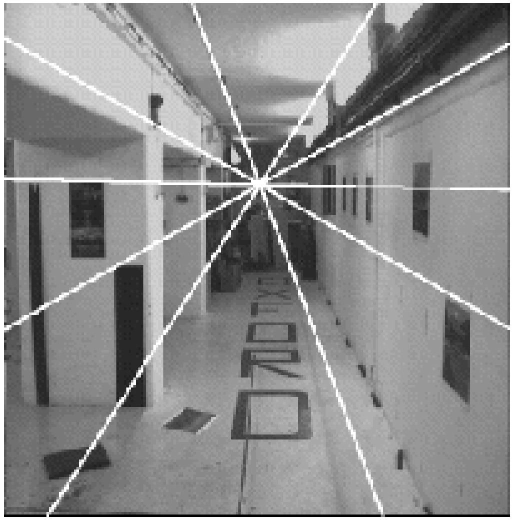
$$\mathbf{l}' = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$$

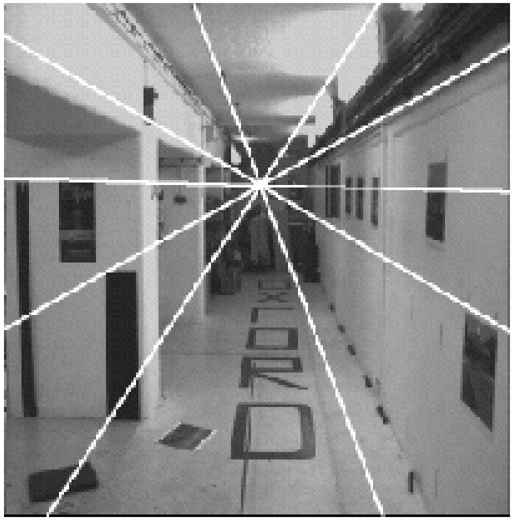
The points $(x, y, 1)^\top$ and $(0, 0, 1)^\top$ lie on this line

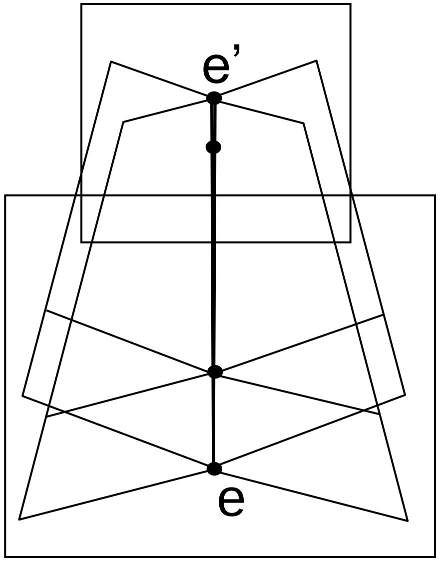



OXFORD


OXFORD






e’
e

Summary: Properties of the Fundamental matrix

- $\mathtt{F}$ is a rank 2 homogeneous matrix with 7 degrees of freedom.

- Point correspondence:

  if $\mathbf{x}$ and $\mathbf{x}'$ are corresponding image points, then $\mathbf{x}'^{\top}\mathtt{F}\mathbf{x} = 0$.

- Epipolar lines:

  ◇ $\mathbf{l}' = \mathtt{F}\mathbf{x}$ is the epipolar line corresponding to $\mathbf{x}$.

  ◇ $\mathbf{l} = \mathtt{F}^{\top}\mathbf{x}'$ is the epipolar line corresponding to $\mathbf{x}'$.

- Epipoles:

  ◇ $\mathtt{F}\mathbf{e} = \mathbf{0}$.

  ◇ $\mathtt{F}^{\top}\mathbf{e}' = \mathbf{0}$.

- Computation from camera matrices $\mathtt{P}, \mathtt{P}'$:

  $\mathtt{P} = \mathtt{K}[\mathtt{I} \mid \mathbf{0}],\ \mathtt{P}' = \mathtt{K}'[\mathtt{R} \mid \mathbf{t}],\ \mathtt{F} = \mathtt{K}'^{-\top}[\mathbf{t}]_{\times}\mathtt{R}\mathtt{K}^{-1}$

Goal: 3D from Stereo via Disparity Map

image I(x,y)

Disparity map D(x,y)

image I´(x´,y´)

$$(x', y') = (x + D(x,y), y)$$

F&P
Chapter 11

Example: Stereo to Depth Map

Example: Stereo to Depth Map