Project 5: Markov Random Fields

In this project you will be experimenting with Markov Random Fields (MRFs). You will be working with simple segmentation problems with two labels, in other words, the Ising model. Along with a written report, you should turn in all source code that you write.

Methods:

1. Write a function to perform Gibbs sampling with the Ising model. This function should take an $\alpha$ and $\beta$ parameter, and generate a random binary image (labels in the set $\{-1, 1\}$) according to the Gibbs distribution. The Ising model you should use is a slight modification of Example 1 in the Perez paper. The energy should look like this:

   $$U(x) = -\alpha \sum_i x_i - \beta \sum_{(i,j)} x_i x_j$$

   The $\alpha$ parameter controls the proportion of labels that are $-1$ vs. $+1$. Negative values of $\alpha$ will favor more $-1$ pixels, and positive values of $\alpha$ will favor more $+1$ pixels.

2. Write a function that will perform two-label segmentation of an image using Iterated Conditional Modes (ICM). Use the same Ising model as above and assume a Gaussian noise model for the likelihood.

Experiments:  All of the randomly generated images below should be of size $128 \times 128$.

1. Generate several random binary images using your Gibbs sampler. Try different values for the $\alpha$ and $\beta$ parameters. Be sure to show several examples in your report, and discuss what effect the parameters had. Next, set $\alpha = 0$ and let $\beta$ get really large. What happens? Also, how long of a burn-in period do you need before the image “settles in”?

2. Generate a random image with $\alpha = 0$ and $\beta = 1$. (Again with sufficient burn-in). Save this label image. Now, add Gaussian noise with mean zero and $\sigma = 2$. Run your ICM routine on this noisy label image (with known values for $\alpha, \beta, \sigma$). How close do you get to the original “clean” image? You may want to try this experiment a few times to get a feel for how well it works (it is random after all).

3. Download the image www.sci.utah.edu/~fletcher/CS7960/NoisyMessage.png Run your ICM algorithm to segment the image. Note, the image intensities are in the range $[0, 255]$. To make them correspond to the $\{-1, +1\}$ labels, you will want to transform them by the function $f(x) = (x - 128)/64$. You will have to try several values of $\alpha, \beta, \sigma$ in your ICM algorithm until you find the best parameters that work. Try initializing the ICM label image with two different random label images. Do the results change? Make sure to describe your results (and include the pictures) in your report.
Extra Stuff: If you have extra time to kill (not required):

- Write a Gibbs sampler that does anisotropic neighborhood priors like in the Perez paper.
- Implement one of the parameter estimation procedures (least-squares, pseudo-likelihood) so that you don’t have to specify parameters at the beginning.
- Implement a Gaussian MRF model (see Perez, Example 2) and Gibbs sampling procedure for it.