Project 3: Diffeomorphic Image Registration II

In this project you will implement diffeomorphic image registration in 2D, building on the previous project. Here you will implement a gradient descent methods for doing image matching. Along with a written report, you should turn in all source code that you write.

Methods: You will need to implement functions that perform the following operations:

1. **Image Match Gradient.** Compute the gradient of the sum-of-squared image match with respect to the final velocity at the end time point.

2. **Jacobi Field Equations.** Compute the forward Jacobi field equation in the Lie algebra.

3. **Adjoint Jacobi Field Equations.** Compute the backward adjoint Jacobi field equation in the Lie algebra.

4. **Greedy Image Registration.** Given a source and target image, use your image match gradient to do a greedy gradient descent image registration.

5. **Geodesic Shooting Image Registration.** Given a source and target image, use your adjoint Jacobi field function to do a gradient descent on the initial velocity of a geodesic for diffeomorphic image registration.

Experiments: For these experiments, you will use source and target images provided here:

http://www.sci.utah.edu/~fletcher/CS7640/hw3/

1. **Jacobi Fields.** Test that your Jacobi fields and adjoint Jacobi fields are truly adjoints of each other. Use random velocities u, v, w to test the relationship:

$$\langle d\mathrm{Exp}_{\mathrm{id}}(v)u, w\rangle_V = \langle u, d\mathrm{Exp}_{\mathrm{id}}^{\dagger}(v)w\rangle_V.$$

2. **Provided Images.** Download the provided source and target images and perform diffeomorphic image registration between them using both your greedy algorithm and geodesic shooting.

Report: You should submit a report (either as html or pdf) describing your work. For both greedy and geodesic shooting algorithms, be sure to include the following:

- Display your deformed source images, as well as difference images between deformed source and target.

- Show a graph of the objective function value versus iteration (note: the objective function for greedy and geodesic shooting are different!)

- Display your final image transformation as either (a) a deformed grid, or (b) an image of the log determinant of the Jacobian. (You should feel free to try visualizing both!)