CS7640: Advanced Image Processing **Due Tu. 2/7**

Project 1: Statistics in Shape Spaces

In this project you will implement Kendall's shape space and simple shape statistics, namely, the Fréchet mean. You first must decide on a data structure for representing a 2D point set (called an "object" throughout this assignment). For an object containing n points, you may decide to use either an $2 \times n$ real matrix or a list of n complex numbers. Also, you will want to have a way to plot your resulting shapes. Matlab, Python, or R (recommended) make such plots easy, but other languages are possible. Along with a written report, you should turn in all source code that you write.

Methods: You will need to implement functions that perform the following operations:

1. Project an object onto the preshape sphere by removing its centroid and scaling it to unit norm. This is a preprocessing step that all objects will go through before applying any of the following routines.
2. Perform Ordinary (a.k.a. Orthogonal) Procrustes Analysis (OPA). This should input a target preshape and moving preshape and output the aligned version of the moving preshape.
3. Compute the exponential map on Kendall shape space, i.e., given a preshape and tangent vector, compute the preshape at the end of the corresponding geodesic.
4. Compute the log map on Kendall shape space, i.e., given two preshapes, compute the tangent vector at the first preshape that is the initial condition of the geodesic segment between the two.
5. Estimate the Fréchet mean shape using gradient descent as described in class.
6. Compute the approximate principal geodesic analysis (PGA), that is, perform principal component analysis (PCA) in the tangent space to the Fréchet mean.

Experiments: You will test your shape analyis routines using two datasets: one synthetic and one from real images.

1. Random triangles. Generate a set of 100 triangles with the following three points:

   $$p_0 = (-1, 0), \quad p_1 = (1, 0), \quad p_2 = (0, s),$$

   where s is a Gaussian random variable with mean 1 and standard deviation 0.5. Next, add zero-mean Gaussian noise with s.d. 0.1 to all six coordinates of each of the 100 triangles.
2. Corpus callosum shapes: `http://www.sci.utah.edu/~fletcher/CS7640/cc-shapes.zip`

   Each corpus callosum object consists of 64 points listed in an ASCII text file. Each line of the file contains a single point (x and y coordinates).

Report: You should submit a report (either as html or pdf) describing your work. Be sure to include the following:

- Describe your implementation. You do not need to recount the theory and equations behind all of the methods, just a brief description of how you implemented them and the issues you faced.
- Describe both the triangle and corpus callosum experiments. For each experiment include:
  - Pick two objects from the data set and plot a geodesic between them (this should be a sequence of objects deforming from one to the other).
  - Plots of 1) the raw object data (i.e., all points from all objects overlaid in one plot), 2) the preshapes aligned to the Fréchet mean, and 3) the Fréchet mean shape (in a different color). Can you see an obvious difference in the aligned shapes?
  - A plot of the eigenvalues for each mode (scree plot). Looking at this plot, how many modes would you use to describe the data?
  - Plot the modes of variation at $0, \pm 1, \pm 2$ standard deviations. You only need to plot up to 3 different modes (maybe fewer, if you decided in the previous question that fewer were needed to represent the data).
  - Describe what you are seeing in the modes of variation. For the triangle case, is this the variation you expected?
- Discuss: What is the maximal number of modes that are possible for triangle shapes? In general, what are the maximal number of modes that are possible for a set of objects with n points?