# Wave Equation Based Interpolation 

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## Cardiac Mapping



Discrete Electrical Measurements


Interpolation


Isocontours

## Cardiac Mapping



Needle Electrodes


Volumetric Interpolation

## Potential Maps

Relative min and max
Activation times
Gradients

- Direction
- Border zones



## Interpolation



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## Wave Equation Based






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## Wave Equation Based





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## Wave Equation Based





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## Wave Equation Based



## Wave Equation Based



## Surface WEB Interp.



## Implementation

Create test data set
Linear Interpolation
Laplacian Interpolation
WEB Linear and Laplacian Interpolation

- Time align data
- Interpolate potentials
- Interpolate activation times
- Re-align potentials by activation time


## Evaluate results

## Simulated Data

Dr. Natalia Trayanova


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## Linear Interpolation



Tetrahedral Mesh
Barycentric coordinate linear interpolation

## Laplacian Interpolation

Laplacian interpolation

- Minimize the Laplacian of the mesh
- Electrode data as boundary conditions
- FE solution of the discrete Laplacian operator


## Evaluation Criteria

0. 

## Evaluation Criteria

$$
\begin{aligned}
& \text { Root Mean Squared Error } \\
& R M S E=\sqrt{\frac{1}{n} \sum_{i=1}^{n}\left(V_{i}^{i n}-V_{i}^{m}\right)^{2}}
\end{aligned}
$$

## Evaluation Criteria

## Root Mean Squared Error <br> $$
R M S E=\sqrt{\left.\frac{1}{n} \sum_{i=1}^{n}\left(\sqrt{V_{i}^{2 n}}\right)-V_{i}^{m}\right)^{2}}
$$ <br> Interpolated Voltage <br> Potential

## Evaluation Criteria

Root Mean Squared Error<br><br>Interpolated Voltage<br>Potential<br>Measured Voltage<br>Potential

## Evaluation Criteria

## Root Mean Squared Error

$$
R M S E=\sqrt{\frac{1}{n} \sum_{i=1}^{n}\left(\sqrt{\left(V_{i}^{i n}\right)-\left(V_{i}^{m}\right)^{2}}\right.}
$$

Interpolated Voltage
Potential

Measured Voltage
Potential

## Correlation Coefficient

$$
C C=\frac{\sum_{i=1}^{n}\left(V_{i}^{i n}-\bar{V}_{i}^{i n}\right)\left(V_{i}^{m}-\bar{V}_{i}^{m}\right)}{\sqrt{\sum_{i=1}^{n}\left(V_{i}^{i n}-\bar{V}_{i}^{i n}\right)^{2}} \sqrt{\sum_{i=1}^{n}\left(V_{i}^{m}-\bar{V}_{i}^{m}\right)^{2}}}
$$

## Evaluation Criteria

## Root Mean Squared Error

$$
R M S E=\sqrt{\frac{1}{n} \sum_{i=1}^{n}\left(\sqrt{\left(V_{i}^{i n}\right)-\left(V_{i}^{m}\right)^{2}}\right.}
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Interpolated Voltage
Potential

Measured Voltage
Potential

## Correlation Coefficient

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C C=\frac{\sum_{i=1}^{n}\left(V_{i}^{i n}-\bar{V}_{i}^{i n}\right)\left(V_{i}^{m}-\bar{V}_{i}^{m}\right)}{\sqrt{\sum_{i=1}^{n}\left(V_{i}^{i n}-\bar{V}_{i}^{i n}\right)^{2}} \sqrt{\sum_{i=1}^{n}\left(V_{i}^{m}-\bar{V}_{i}^{m}\right)^{2}}}
$$

Max Gradient Relative Error

$$
\text { MaxGradRE }=M A X\left(\frac{\nabla V_{i}^{i n}-\nabla V_{i}^{m}}{\nabla V_{i}^{m}}\right)
$$

## Results

## RMSE

Mean CC
Max Grad RE


## Visual Inspection

Gold Standard
Non-WEB Linear


## Discussion

WEB methods do not improve global statistics

- Preserves focal facet of the activation wave
- WEB assumptions not as accurate transmuraly as they are across the epicardium


## Discussion

WEB methods preserve gradients

- More than 3 times more accurate gradients
- Wave front delineated better


## Conclusion

What are we looking for

- Gradients
- Activation front
- Relative minimum and maximum potentials


## Questions

