A Comparison of Delaunay-Based Meshing Algorithms for Electrophysical Cardiac Simulation

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Mesh generation for finite element simulation of biomedical domains has emerged as a key open problem to be addressed by the scientific community. Building representative models of organ systems that can provide accurate simulations is a cross-cutting issue requiring domain expertise from both biologists and computational scientists. Often these two groups have attacked the problem from independent viewpoints. In particular, a number of software packages for automatic mesh generation have been developed from the computational fields that provide various levels of control for geometric "quality" of the meshes they create. However, there is a divide between these geometric quality measures and the desired properties a mesh should have to achieved robust biomedical simulation results.

Our focus for this work is to help bridge the gap between these two communities by investigating mesh generation within the pipeline from acquiring physical data to analysis of simulation results. The main goal is to better understand which properties of meshes have the most impact and how varying them translates to effects on the simulation. We take an empirical point of view. Many simulations require domain-dependent meshes that are catered to the particular simulation type. While we narrowed the focus for this work to the study of electrical simulations of the heart, in particular modeling ischemia using bidomain simulations, we hope to learn lessons that can be applied to biomedical simulations on multi-material domains in a general sense.

Delaunay Meshing The computational geometry community has put forth a number of automatic algorithms for mesh generation. Of particular interest are Delaunay-based algorithms. These algorithms produce superior (geometric) quality elements while providing a number of essential features for a meshing algorithm, i.e. robustness and numerical stability, simplicity of implementation, domain conformation, and topological correctness. In general, *geometric quality* refers to building elements (triangles or tetrahedra) that have *aspect ratios* within some tolerance. Aspect ratio is measured using the shape of elements [9], such as the ratio of circumscribing sphere to shortest edge length (naturally optimized by the Delaunay triangulation) or the ratio of circumscribing sphere to inscribing sphere. In addition, the same algorithms can be used to control element size, typically by uniformly scaling elements or building size which scales with respect to the features of the domain geometry.

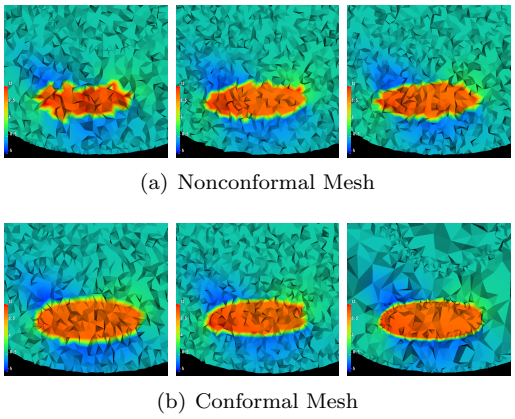
What is unclear, and a major focus of this work, is if the collection of output mesh properties exposed to a user of these algorithms provides an appropriate interface to build meshes suitable for their downstream application. In particular, many of the parameters provide only high-level controls. For example, as quality increases, the number of elements the mesh requires often increases as well. For many simulations (including the ones used here) this results in an explosion of elements where often only high quality features are needed in specific places. Moreover, some of the available set of controls do not appear to impact simulation quality significantly. Finally, significant research effort is often spent trying to ensure that both topological and geometric features of the domain are preserved, whereas it is arguable that sometimes these features are unnecessary. By designing these algorithms without the application in mind, inferring when these meshing constraints can be relaxed is a major challenge. On the other hand, an end user of these algorithms is left with the significant burden of trying to translate these geometric notions into their intuition on the properties a mesh needs for their simulation.

Experimental Setup We have identified three publicly available algorithms using Delaunay-based techniques to compare. We give a brief overview of them here. The first algorithm [2, 8], released in the CGAL [1] library, uses a Delaunay refinement approach, but does not build meshes that conform to smooth surface boundaries. The second algorithm [3] is based on the DelPSC [4] meshing algorithm for Delaunay refinement of piecewise-smooth complexes. Finally, the third algorithm, released as BioMesh3D [7], uses particle systems [6] to distribute a set of mesh vertices along all surfaces, and then constructs a 3D Delaunay triangulation using TetGen [10].

Using these algorithms we generated a number of various meshes to perform simulations of the electrical activity on the heart. In particular, we varied a number of the typical parameters exposed by the

algorithms to build meshes of various size and with elements of different quality.

In our first experiment, these meshes were used in a two part process to simulate acute cardiac ischemia. First, experimentally recorded electrical potentials were place on the mesh using a finite element Laplacian interpolation [13] and an appropriate voltage was selected to threshold a volume representing the ischemic region. For each mesh, the volume and center of the ischemic regions was recorded and compared to the results from the other meshes. The second step in the experiment was to run a bidomain simulation [5, 11] modeling electrical conduction of an ischemic heart during one time instance of the cardiac cycle. The geometry and conduction anisotropy were taken from imaging data of an actual canine heart and the ischemic region was modeled as a voltage source as described in previous simulations [12]. The minimum and maximum epicardial potentials were recorded along with the volume of depressed cardiac tissue.



(a) Nonconformal Mesh

(b) Conformal Mesh

Figure 1: From left to right are the output meshes of BioMesh3D, CGAL, and DelPSC.

Our second experiment evaluated the shape of the ischemic region. Of particular interest was the simulation behavior when the mesh conformed to the boundary of the ischemic zone. Figure 1 shows visualizations of the meshes for all three algorithms.

Results The ischemic regions based on the finite element Laplacian interpolation had volumes seen in Table 1. The lowest resolution of mesh, 50k vertices, produced results that varied by 37% from algorithm to algorithm. While both the medium and high resolution meshes produced consistent results from algorithm to algorithm. The total volume for the medium and high resolution meshes were within 1% of each other and the DelPSC low resolution also produced a result within a few percent of the medium and high resolution meshes.

Table 1: Summary of Experiment 1. Shown are the mesh resolution (number of vertices) and the mean, scaled inscribed/circumscribed ratio of the mesh tetrahedra. Simulation results of the volume off ischemic regions (mm^3), maximum and minimum epicardial potentials (mV), and volume of the voltage depression (mm^3).

Input Summary			Output Summary			
Algorithm	Mesh Res.	I/C Ratio	Isc. Vol	Max Pot.	Min Pot.	Dep. Vol
BioMesh3D		0.597	46044	11.39	-10.26	3334.9
CGAL	50k	0.790	32389	9.47	-10.93	2872.2
DelPSC		0.676	51651	10.06	-11.19	3226.3
BioMesh3D		0.678	54002	9.87	-11.45	3089.8
CGAL	250k	0.806	54314	9.85	-11.52	3130.7
DelPSC		0.751	54388	9.71	-11.65	3232.8
BioMesh3D		0.767	54283	9.68	-11.68	3314.8
CGAL	1000k	0.806	54248	9.71	-11.59	3136.9
DelPSC		0.692	54195	9.67	-11.66	3260.0

The same trend held for the bidomain solution where the low resolution meshes did not produce consistent solutions varying by 12% and 14% as indicated by Table 1, showing results for the extrema of the epicardial potentials and the total volume of the voltage depressions.

The second experiment measured the effects of conformal meshing on the voltage distribution. Figure 2 shows the difference between the conformal and non-conformal ischemic regions as a function of distance from the ischemic boundary. The maximum difference was for BioMesh3D, differing by 44%, while DelPSC had a difference of 39% and CGAL only had a max difference of 20%. It is important to note that CGAL had the least conformal mesh which could have contributed to the significantly better correspondence to the non-conformal mesh. The large differences between solutions was primarily localized near the ischemic boundary, within less than 3 mm, regardless of the mesh resolution. Further from the ischemic boundary, greater than 5 mm, the differences became increasingly smaller, but so did

the amplitude of the voltage potential such that the percentage did not significantly change. The reported values for the non-conformal meshes were taken from the 250k vertices meshes of each algorithm. The 1000k vertices meshes were also compared, but results did not significantly change (less than 1% improvement).

Qualitatively, the non-conformal meshes produced a less heterogeneous transition from the ischemic region to the healthy tissue with the concave and convex boundary representation produced local minimums and maximums that were not as prominent in the conformal meshes. However, when the voltage profiles along the entire ischemic zone were averaged together for both the conformal and non-conformal meshes, the resulting profiles were very similar and had a maximum deviation of only 6%. The biggest difference between the two sets of profiles was that the non-conformal meshes had voltage profile that were shifted by .45 mm when compared to the voltage profiles of the conformal mesh. In this instance the non-conformal meshes underrepresented the ischemic border and the ischemic region as a whole.

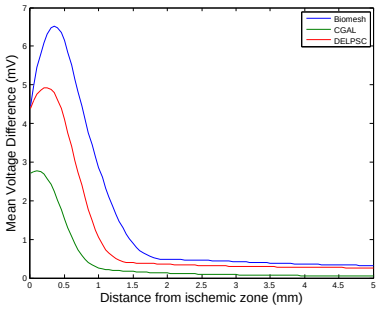


Figure 2: Mean difference between voltages of non-conformal and conformal meshes as a function of distance from ischemic border.

Conclusions In this work we have illuminated some of the challenges for meshing physiological domains and noted that a joint effort from both the biological and computational communities is required to properly address them. By studying electrophysical cardiac simulations, we have given a case study of some of the difficulties inherent to the meshing process and its impact on the simulation pipeline as a whole. Future efforts will be devoted to studying the dependency of simulation results on geometric quality measures.

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