Wave Equation Based Interpolation on Volumetric Cardiac Electrical Potentials

DJ Swenson[1,2], JG Stinstra[1,2], K Aras[1,2,3], RS MacLeod[1,2,3]

[1]Department of Bioengineering, University of Utah, Salt Lake City, UT, USA
[2]Scientific Computing and Imaging Institute, University of Utah, Salt Lake City, UT, USA
[3]Cardiovascular Research and Training Institute (CVRTI)

Abstract

Like any measurement involving discrete spatial sampling, mapping of the electrical activity in the heart requires interpolation to estimate the values of electric potential between and around measured values. Interpolation is never perfect but can often perform better using application specific approaches than with general purpose methods and cardiac mapping has motivated a number of such customized interpolation techniques. Wave equation based (WEB) interpolation is one such specialized interpolation technique that has been shown to preserve sharp gradients on surface mapping data within electrograms recording from the heart surface, but has not been applied to volumetric data. The special strength of WEB interpolation is the preservation of sharp spatial gradients and we propose that WEB interpolation will also provide accurate unbiased interpolation of volumetric interpolation over sharp gradients in volumetric cardiac mapping data. We applied WEB interpolation in conjunction with linear and volumetric Laplacian interpolation methods and compared them to the same methods in their non-WEB forms. The basis for comparison were extracellular potentials from a high resolution simulation of cardiac activation. The results for the voltage gradients using the WEB methods were more than 2.6 times more accurate than the non-WEB methods at a very slight cost in accuracy in the regions of low spatial gradient. These results demonstrate that WEB approaches capture intrinsic features in the context of cardiac mapping and can augment and improve the accuracy of other interpolation techniques.

1. Introduction

Measurements of multiple spatially distributed time signals (mapping) in cardiac electrophysiology depend heavily on interpolation to reconstruct electrical potential fields across the surface of the heart or intramurally throughout the myocardium. In order to study normal and abnormal propagation due to ischemia, infarction, or other forms of conduction delay, it is necessary for interpolation techniques to preserve the steep potential gradients that define the wavefront location, even in the face of limited spatial sampling by the electrodes. The quality of interpolation is of special importance in studying the features of the border zone between healthy and abnormal tissue, a critical region in the genesis of reentrant arrhythmias[1]. Previous research from our group has demonstrated the effectiveness of wave equation based (WEB) interpolation especially in preserving sharp gradients on cardiac surface mapping, but this approach has not been extended to volumetric data[2]. We propose that WEB interpolation will improve the accuracy of volumetric interpolation of cardiac potentials and remove artifacts in regions with sharp gradients compared to other commonly used approaches. The measurement of such volume potentials typically occurs with multielectrode plunge electrodes inserted from the outside of the exposed ventricles.

2. Methods

For this implementation and evaluation, we used simulated extracellular potentials (kindly provided by Dr. Natalia Trayanova, Johns Hopkins University) computed from a high resolution model of an infarcted canine heart. Virtual intramural needle electrodes were placed in a small region of the simulated heart and the potentials mapped to the nearest electrode. The subset of time signals at the needle electrode sites then provided the input (measured) data for the interpolation methods while the original time signals provided the gold standard test data (Figure 1). We implemented and compared linear, volumetric Laplacian, WEB linear, and WEB volumetric Laplacian (a novel hybrid of WEB and Laplacian methods) interpolation schemes through an entire activation wave and computed the resulting root mean square (rms) value, correlation coefficient, and steepest gradient relative error.

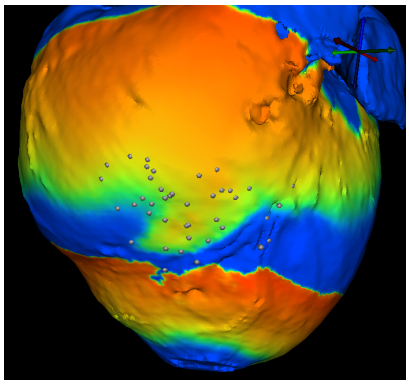

Figure 1. A computational model of activation paced from the apex. The red and yellow indicate the activation wave moving across the myocardium and the round spheres indicate the location of the virtual needles.

2.1. Linear interpolation

A Delaunay[3] tetrahedral mesh was constructed using the electrodes along the transmural needles as input nodes. Elongated or poorly shaped elements resulting from concave regions of the model, were removed from the mesh. The mesh quality was constrained by the number and location of the needle electrodes because no additional nodes could be added for this method. Standard Barycentric coordinates [4] were used to linearly interpolate the data over the mesh and then back onto the fine resolution mesh used in the computational model. The values from the computational model could then be directly compared to the interpolated values because they shared common nodes.

2.2. Laplacian interpolation

The distribution of electrical potentials was assumed to resemble a Laplacian distribution $\Delta\Phi = 0$. A Neumann boundary condition was used on the surface of the heart where no current flows out from the surface of the heart to the body. The data from the electrodes were also added as initial conditions to a small radius of nodes within .5 mm of the electrode location. A discrete approximation of the Laplacian operator was solved for in its weak form using the Galerkin method. The unknown epicardial potentials, or unknown nodes, were adjusted so as to minimize the Laplacian using a conjugate gradient optimization.

2.3. WEB interpolation

Traditional interpolation methods interpolate purely over the spatial distribution of potentials without considering the fact that many morphological differences arise because of time shifts between otherwise very similar waveforms, as suggest by the wave equation 1.

$$V(x,t) = V(x - \theta t) \tag{1}$$

Wave-equation based (WEB) interpolation takes such temporal shift into account by first aligning the wave forms in time, in this case by the activation time, and then performing the spatial interpolation [2].

To implement this idea, it is practical to align time signals by activation time, interpolate them by any of a number of possible schemes, and then shift the interpolated time signals to a new activation time that is also the result of interpolation. The activation time is calculated in the usual fashion by finding the time of largest negative dv/dt for each time signal.

2.4. Evaluation Criteria

Four methods were used to evaluate the quality of the interpolation. The root mean squared error (RMSE), Equation 2 and the correlation coefficient (CC), Equation 3 provided metrics for the quality of fit of the interpolated data over the entire region [5]. The relative error of the maximum gradient, Equation 4, and visual assessments were used to evaluate the ability of each method to preserve important spatial features such as the leading and trailing edges of the activation wave.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(V_i^{in} - V_i^m)^2} \tag{2}$$

$$CC = \frac{\sum_{i=1}^{n}(V_i^{in} - \bar{V}_i^{in})(V_i^m - \bar{V}_i^m)}{\sqrt{\sum_{i=1}^{n}(V_i^{in} - \bar{V}_i^{in})^2}\sqrt{\sum_{i=1}^{n}(V_i^m - \bar{V}_i^m)^2}} \tag{3}$$

$$MaxGradRE = MAX\left(\frac{\nabla V_i^{in} - \nabla V_i^m}{\nabla V_i^m}\right) \tag{4}$$

3. Results

The WEB interpolation methods generated approximately 15% higher RMSE than the non-WEB methods computed over the entire beat with linear interpolation performing slightly better than Laplacian (Table 1). Results of a simular analysis applied to each of the activation (QRS), plateau (ST segment), and repolarization (T wave) phases of the beat showed that the WEB approaches performed equally to their corresponding non-WEB methods during QRS and T waves but were not as effective during the ST segment, a period of low spatial gradients.

Even smaller difference appeared in the correlation coefficient between the WEB and non-WEB methods (Table 2).

Table 1. Root mean square error of the voltage potentials for the different interpolation methods (mean±std) in mV The non-WEB methods vs. the WEB methods had a $p<0.0001$ for a Wilcoxon signed rank test.

Interpolation method	Mean RMSE
Non-WEB Linear	2.47±0.68
Non-WEB Laplace	2.49±0.72
WEB Linear	2.70±0.86
WEB Laplace	2.67±0.89

Table 2. Correlation coefficients comparing the interpolated results to the simulation data (mean±std).

Interpolation method	Mean CC
Non-WEB Linear	0.8675±0.0370
Non-WEB Laplace	0.8641±0.0354
WEB Linear	0.8761±0.0497
WEB Laplace	0.8524±0.0497

Unlike the previous two metrics, the relative error of the maximum voltage gradient showed extreme differences between methods. Relative errors for the non-WEB approaches were approximately three times larger than for their WEB equivalents (Table 3).

Table 3. Relative error of the maximum voltage potential gradients (mean±std). The non-WEB methods were significantly different from the WEB methods ($p<0.0001$ for a Wilcoxon signed rank test.)

Interpolation method	Mean RE
Non-WEB Linear	0.8091±0.0813
Non-WEB Laplace	0.6011±0.1328
WEB Linear	0.2401±0.3629
WEB Laplace	0.2293±0.3429

Visual comparison showed much sharper gradients for the WEB methods (Figure 4) that looked similar in size and location to those seen in the original potentials (Figure 2). Gradients were much more blurred for the non-WEB approaches (Figure 3), making identification of wavefronts less precise.

4. Discussion and conclusions

Based on spatially and temporally global statistical metrics, RMSE and CC, the WEB approaches performed the same or slightly worse than their non-WEB counterparts, a surprising result given the clear improvements when WEB is applied to surface potentials[2, 5]. One possible explanation for this result is the quality of such global metrics when judging performance of a method that seeks to preserve a very focal facet of the wavefront. The WEB results did show a marked improvement in a metric of spatial gra-

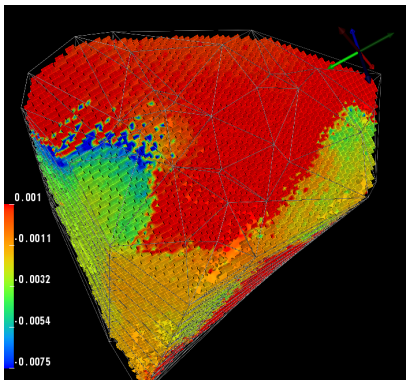

Figure 2. The gold standard electrical potentials from a wedge of the finite element volume.

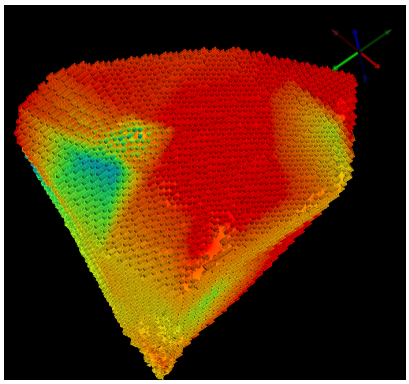

Figure 3. The Non-WEB linear interpolation mapped onto the same nodes as the gold standard potentials.

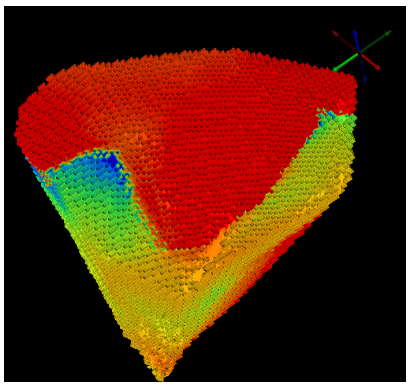

Figure 4. The WEB linear interpolation over the same elements as the gold standard electrical potentials.

dient and during visual examination, indicating that this approach does, indeed, succeed in reconstruction of features that are very valuable when identifying abnormalities in cardiac spread of activation.

A second possibility is that the wave equation assumptions that motivate the WEB approach may not hold as well within the volume of the heart as it does on the surface. Support for this hypothesis comes from the fact that the volume contains much more rapidly changing fiber directions and complex transitions of wave propagation speed and direction than the surface. Under such conditions, the notion of a stable wavefront moving with a locally constant velocity is likely to be more questionable than on the heart surface.

The WEB methods were also at a disadvantage because the model included an infarcted region of tissue to simulate a more realistic comparison for clinically realistic applications. Time signals from the infarcted region were often morphologically different than those in nearby healthy tissue. Such differences made them difficult to time align with any accuracy which resulted in dissimilar segments of the infarcted waveform and normal waveform to be aligned together. The error from miss aligned waveforms would be similar to the error seen in non-WEB interpolation.

The relative error of the maximum gradient in the WEB interpolations were as much as three times more accurate, a finding supported by visual inspection of the results. The WEB interpolations represented the sharp borders of the activation front relatively well in severity and overall shape. In regions of sparsely placed needles, roughly 2 cm or more apart, the activation visibly deviated from the simulation data. The non-WEB methods were much more blurred so that it was difficult to tell exactly where the activation front was located.

In general at least for these test data, linear interpolation performed as well or slightly better than the Laplacian method in terms of the global statistical metrics. However, the Laplacian methods were better at preserving the sharp gradients than the linear methods. The linear interpolation methods also created artifacts in the data at elements that were poorly shaped. Element refinement was not an option because each node in this method needed to be associated to a measurement value. The Laplacian interpolation did not require data at every node location, subsequently it had a much better mesh that did not produce any artifacts. The volumetric Laplacian did not transition as smoothly between electrodes, but rather had a patchy appearance. This was an advantage in regions that actually were patchy, such as the infarct zone, but was a disadvantage in most other places. These advantages and disadvantages lead us to conclude that in general the linear interpolation method is better for cardiac mapping, but in situations where an appropriate mesh cannot be constructed or when the data is very patchy, the Laplacian method may be better.

We conclude that WEB interpolation is much better at representing the spatial gradients within the heart. Much of the time when scientists are creating cardiac maps, they are not looking at absolute potential values, but rather patterns in the spatial distribution of gradients. For studies looking at spatial distributions that potentially have sharp gradients, the WEB interpolation method appears to perform better. However, in cases in which the potentials are very smoothly distributed, the WEB method may actually increase the error, but not significantly.

5. Acknowledgements

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Address for correspondence:

Name: Darrell J. Swenson
Full postal address:
University of Utah
72 S. Central Campus Dr., Room 3750
Salt Lake City, Utah 84112
E-mail address: darrell.swenson@utah.edu