2D Vector Field Visualization

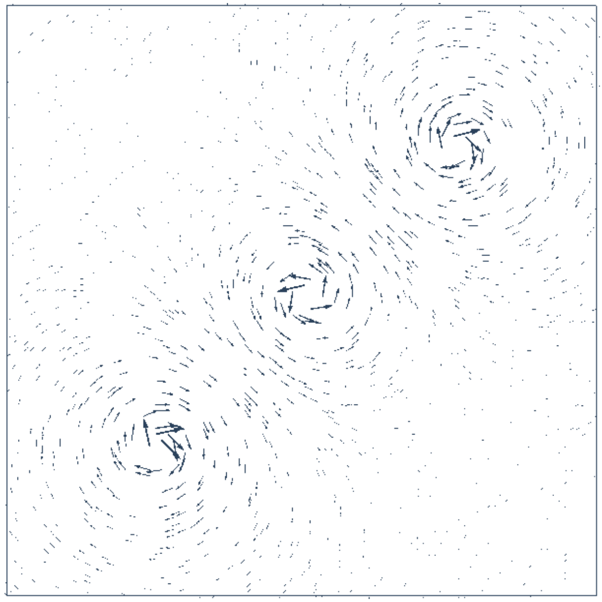
Thanks to Prof. Hansen for integration slides

Vectors

- Directional information
- Wind, mechanical forces (earthquakes)
- **Flows**
- Harder: more than one pixel per vector
  - Clutter

Glyphs

- Place symbols over vector field
  - Regularly spaced
  - Randomly spaced
  - Scale
- Watch out for clutter
- (demo: vector_vis.vt: basic, masking)

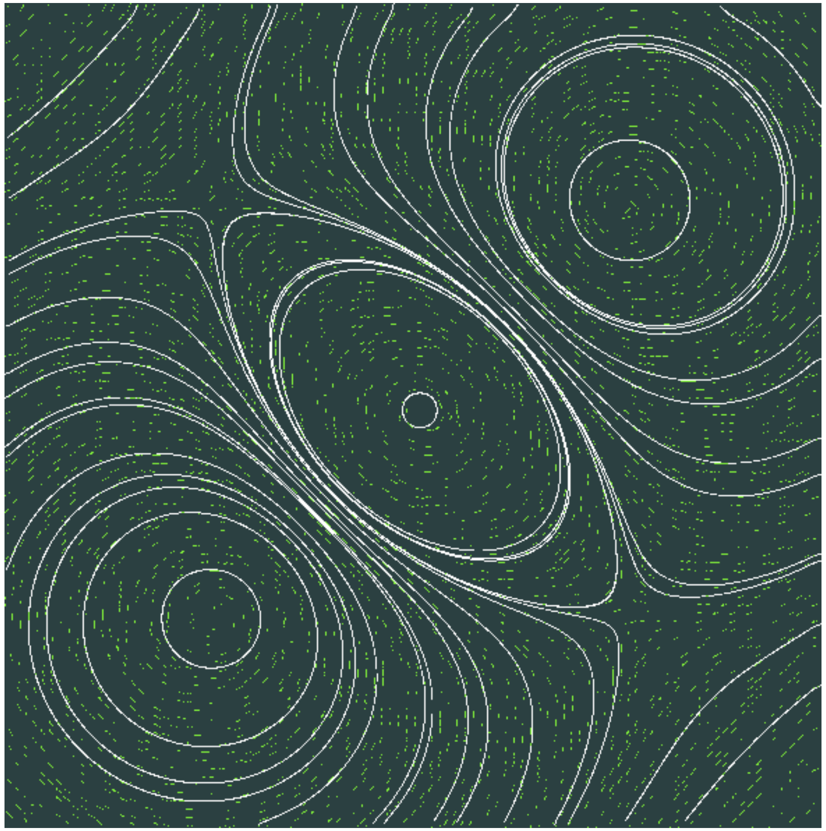


Div, grad, curl and all that

- We've seen grad
- For 2D vector fields, div and curl are scalars
- Use that for additional info
  - "Layers" in the visualization

Streamlines

- Lines that are everywhere tangent to the vector field
  - $f(0) = x_0,\ \dot{f}(x) = u(x)$
- That's a diff. eq.
- Solving for $f(x)$ is an **initial value problem**
- (demo: vector_vis.py, streamlines)



Streamlines are cool

- Streamlines give us a lot of information about the field
  - Partition flow
  - Help portray divergence

Computing Streamlines

- Approximate curve by sequence of line segments
- Naive: compute each segment by jumping in the direction of current vector
  - This is the **Euler integrator, and it is bad**
- Can we do better?

Euler vs. Runge-Kutta

- Euler: accurate if streamlines are lines
  - But error accumulation is typically catastrophic (Why?)
- Runge-Kutta: accurate on higher-order streamlines
  - Family of schemes

Euler's Method

- Let Stock = X
- Let flow = f(t, X)
  [function of time, Stock]
- Compute X(t) from X(t-dt) and time.
  - $\Delta X = dt * f(t-dt, X(t-dt))$
  - $X(t) = X(t-dt) + \Delta X$

Euler's Method

Assume flow = f(t).

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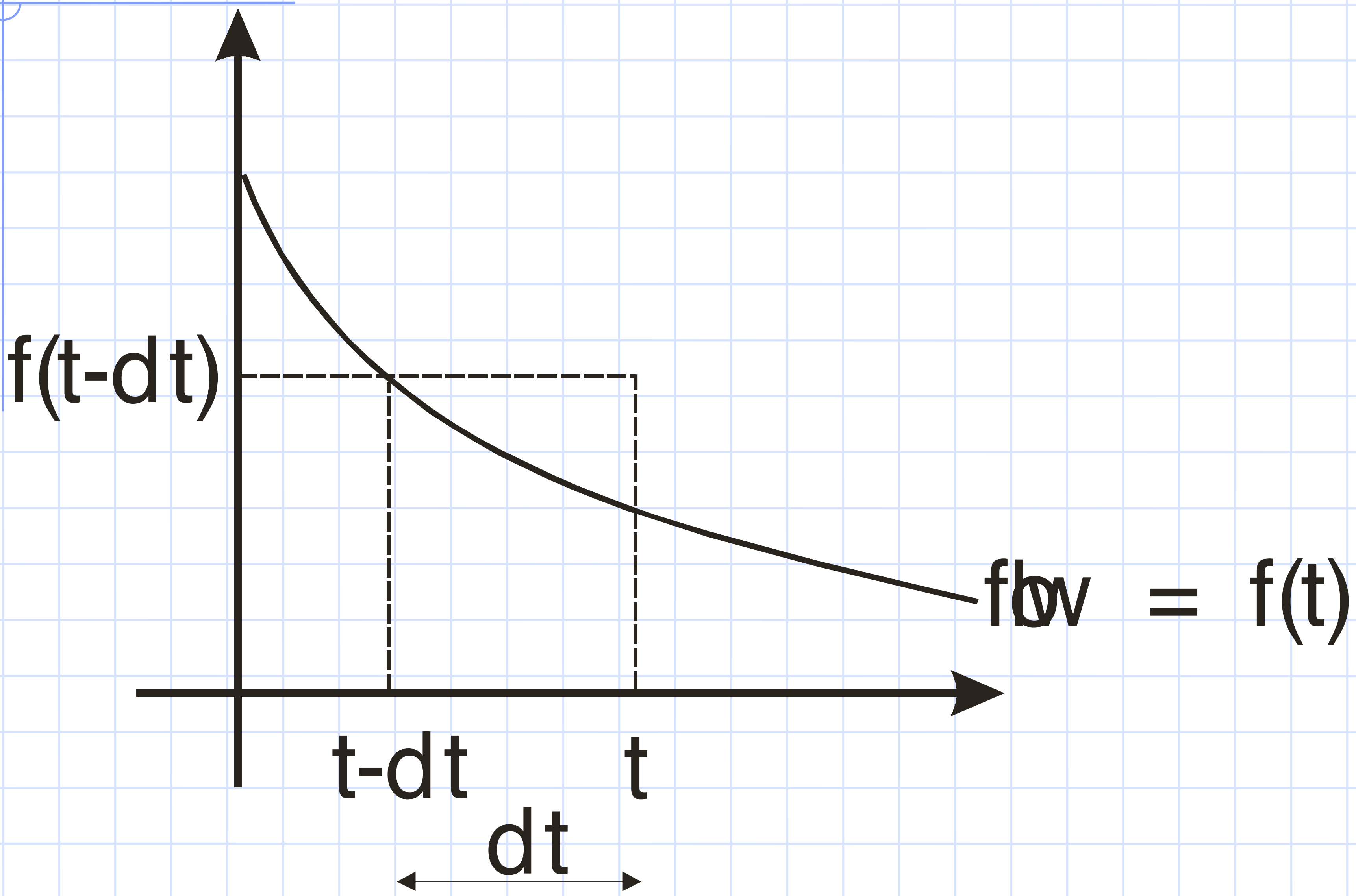

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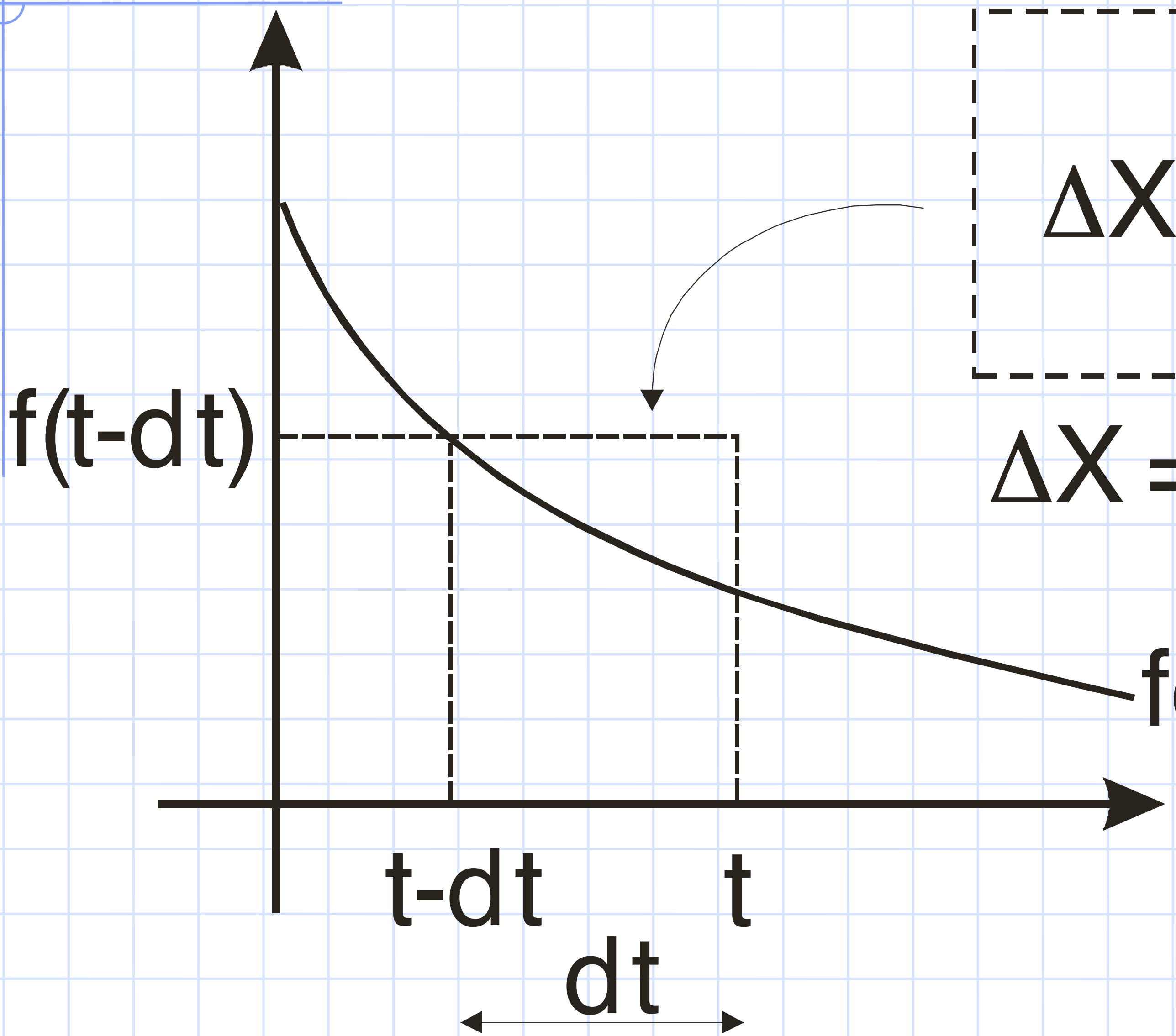

ΔX

$$\Delta X = dt * f(t\text{-}dt)$$

flow = f(t)

Euler's Method

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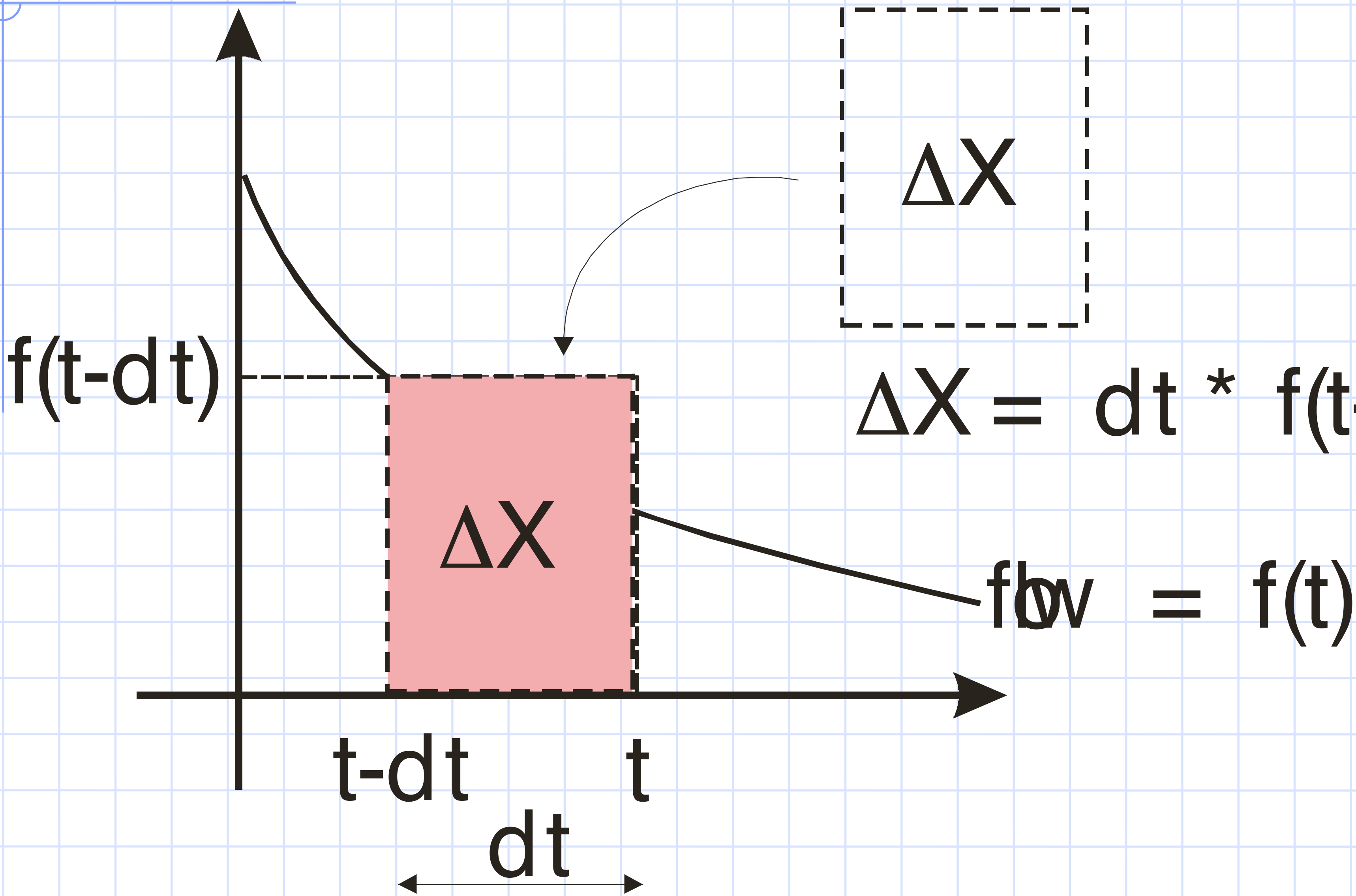

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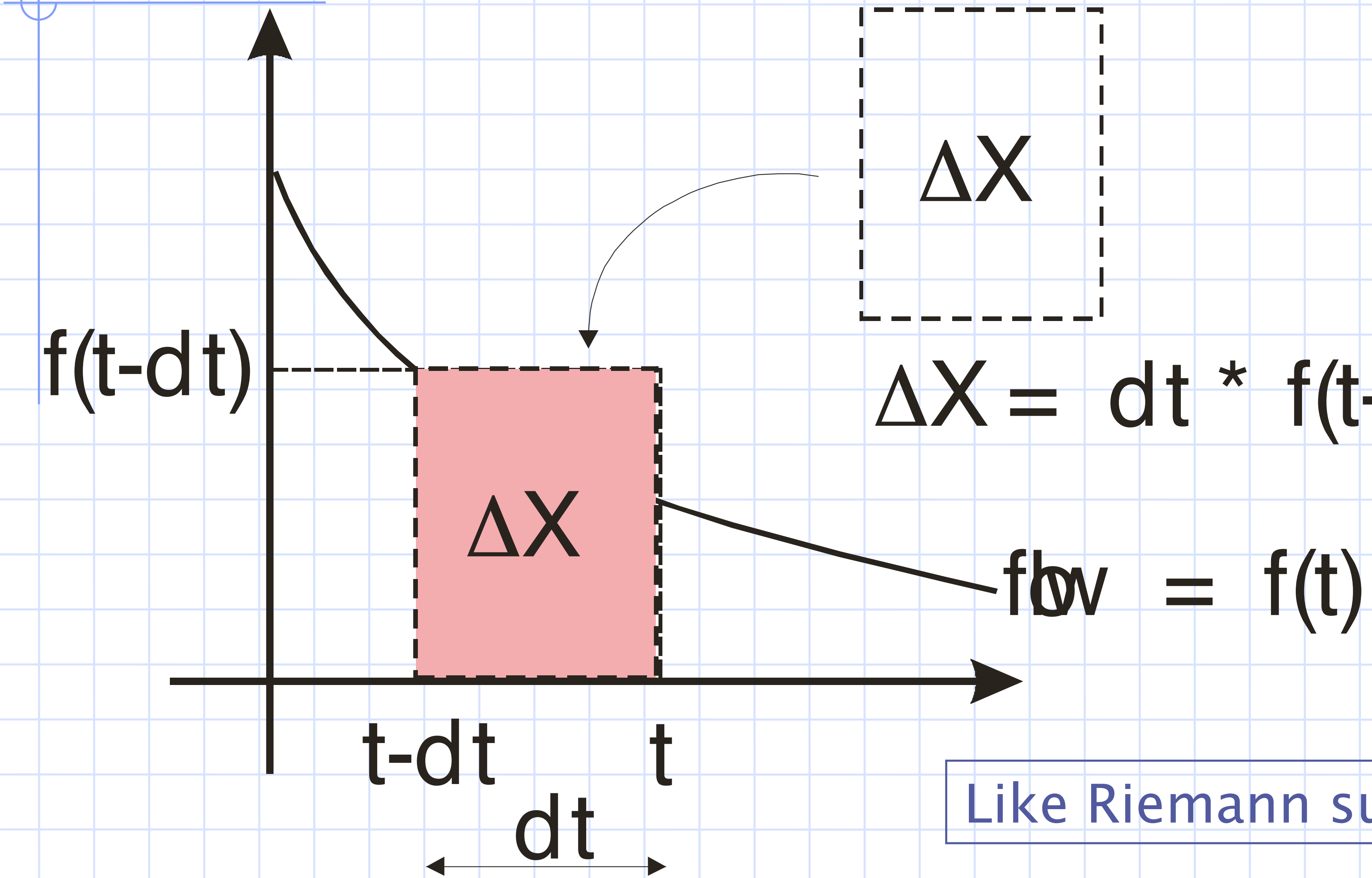

$\Delta X = dt * f(t-dt)$

Like Riemann sum.

Euler Integration Error

- Error = ΔX – area under flow curve

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Euler Integration Error

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Error = $\boxed{\Delta X}$

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Euler Integration Error

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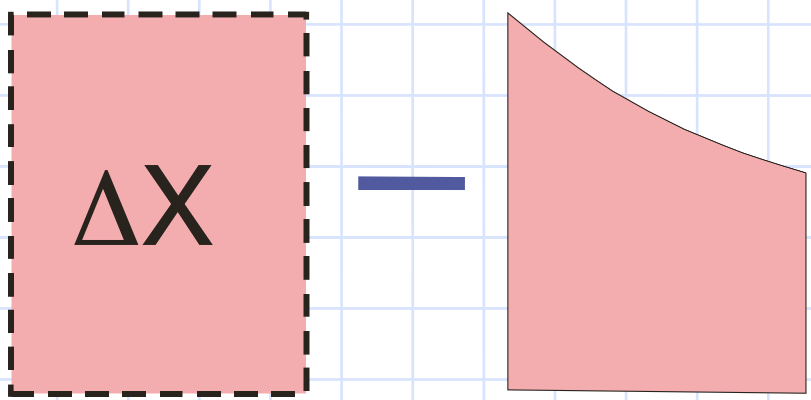

Runge-Kutta 2

- Let Stock = X, flow = f(t,X)
- Estimates for stock updates:
  - $F1 = dt * f(t-dt, X(t-dt))$
  - $F2 = dt * f(t, X(t-dt) + F1)$
- $\Delta X = \frac{1}{2} * (F1 + F2)$
- $X(t) = X(t-dt) + \Delta X$

Runge–Kutta 2

Assume flow = f(t).

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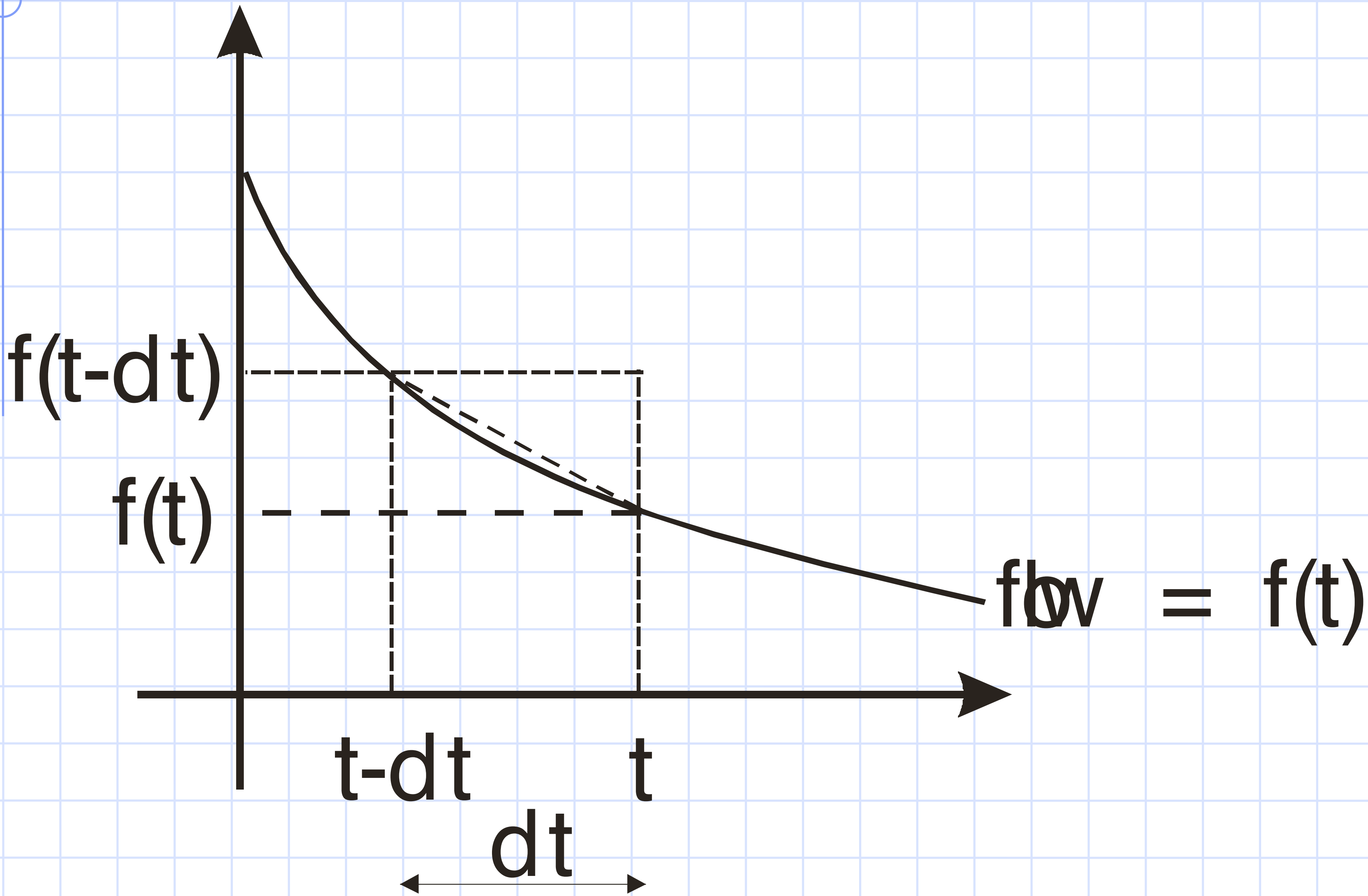

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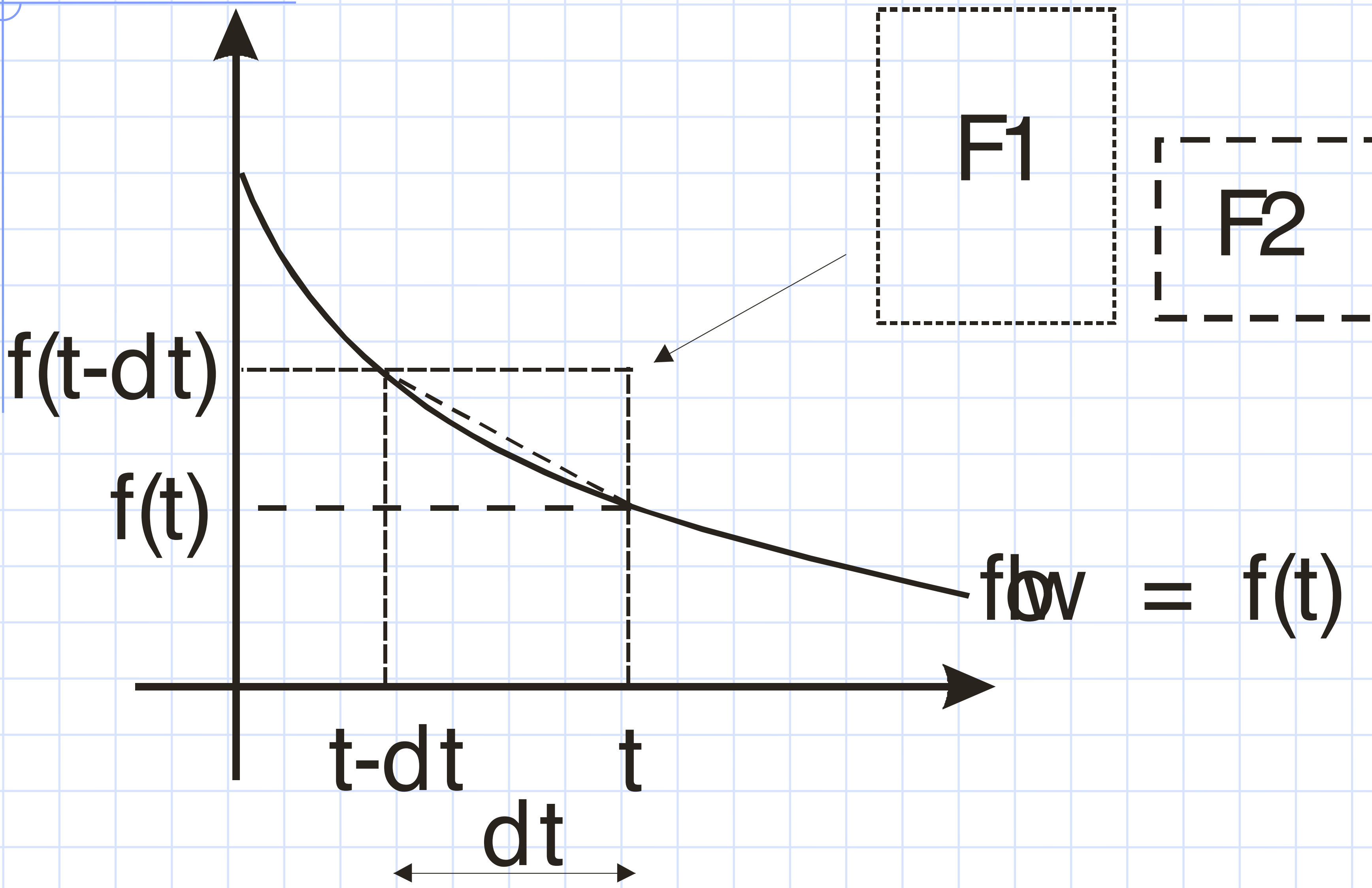

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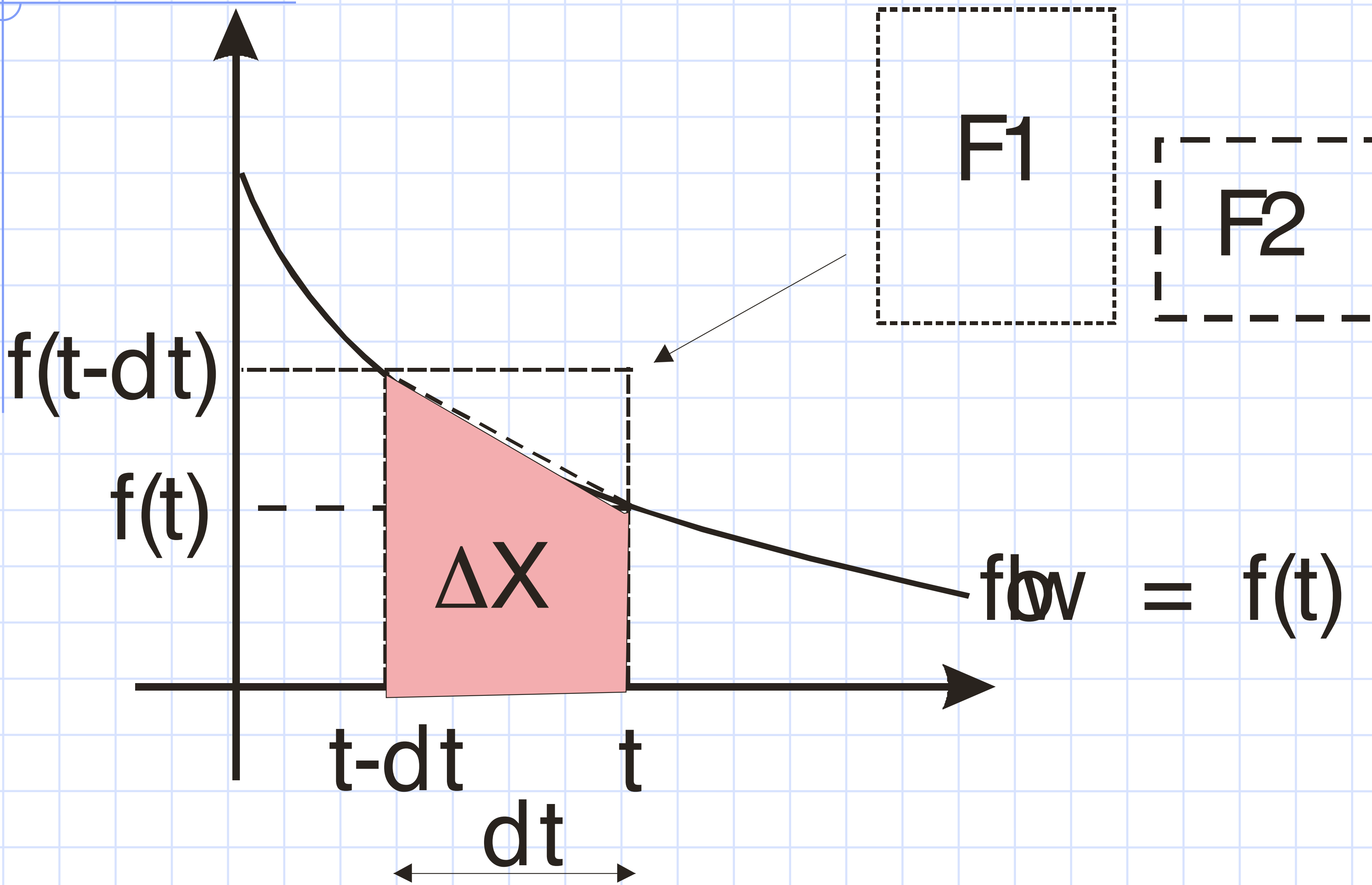

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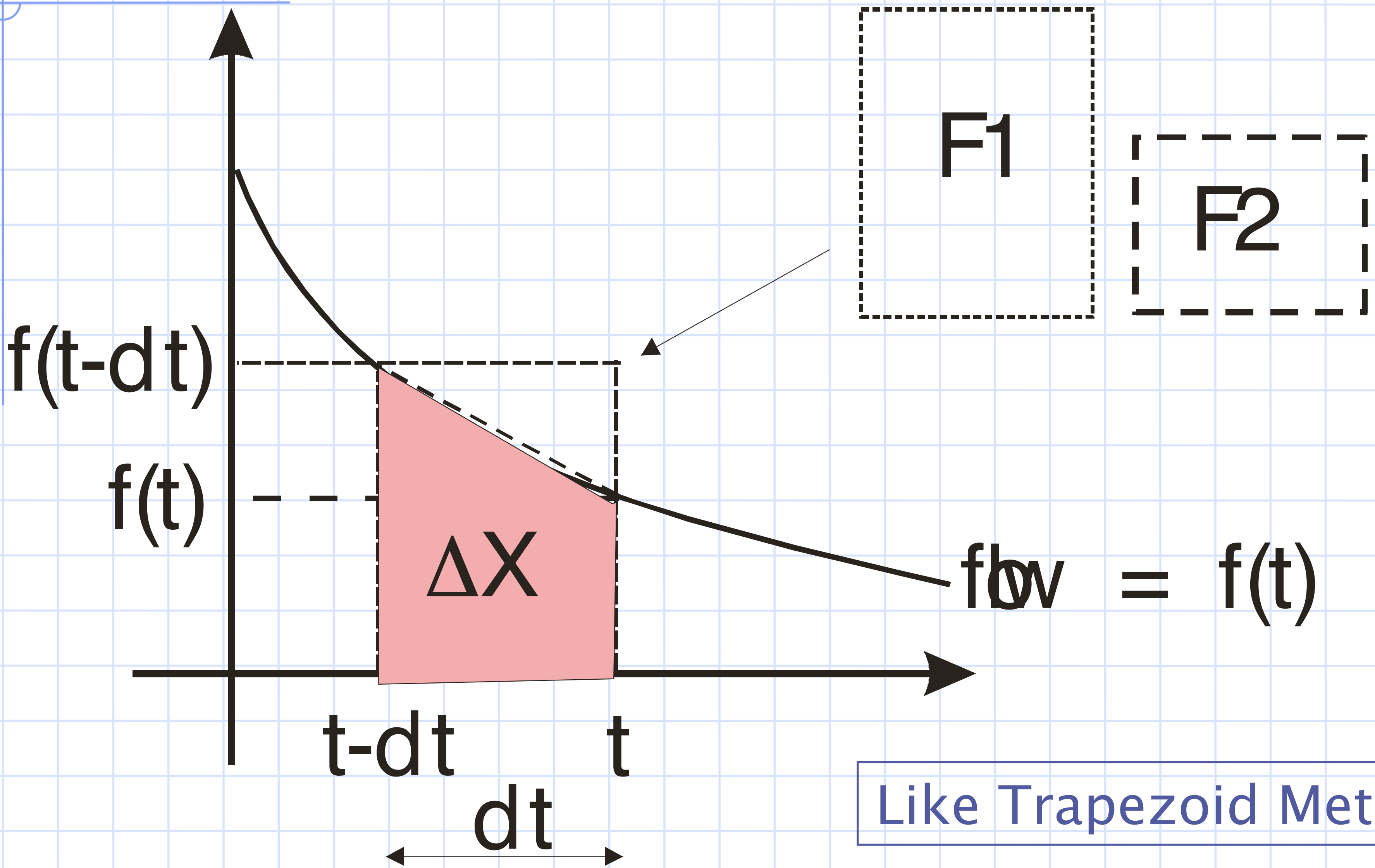

Like Trapezoid Method.

RK2 Integration Error

- Error = ΔX – area under flow curve

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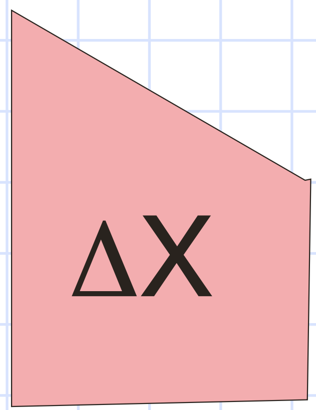
Error = 

RK2 Integration Error

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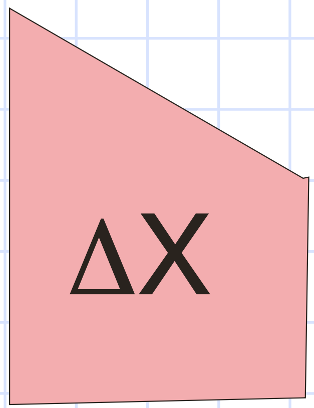

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RK2 Integration Error

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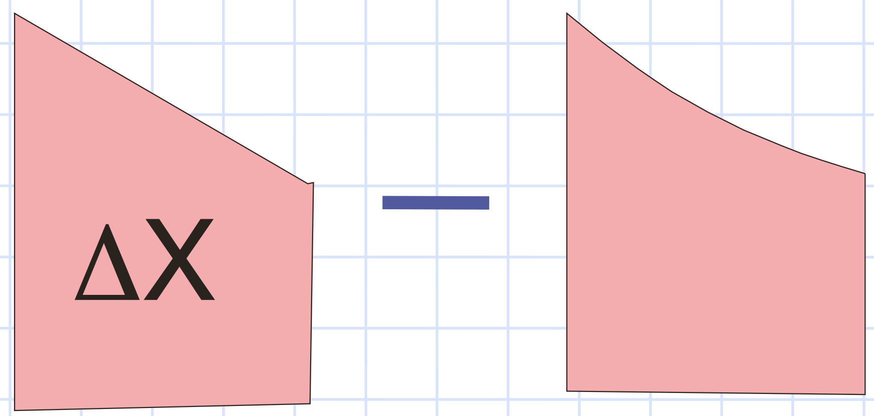


Runge-Kutta 4

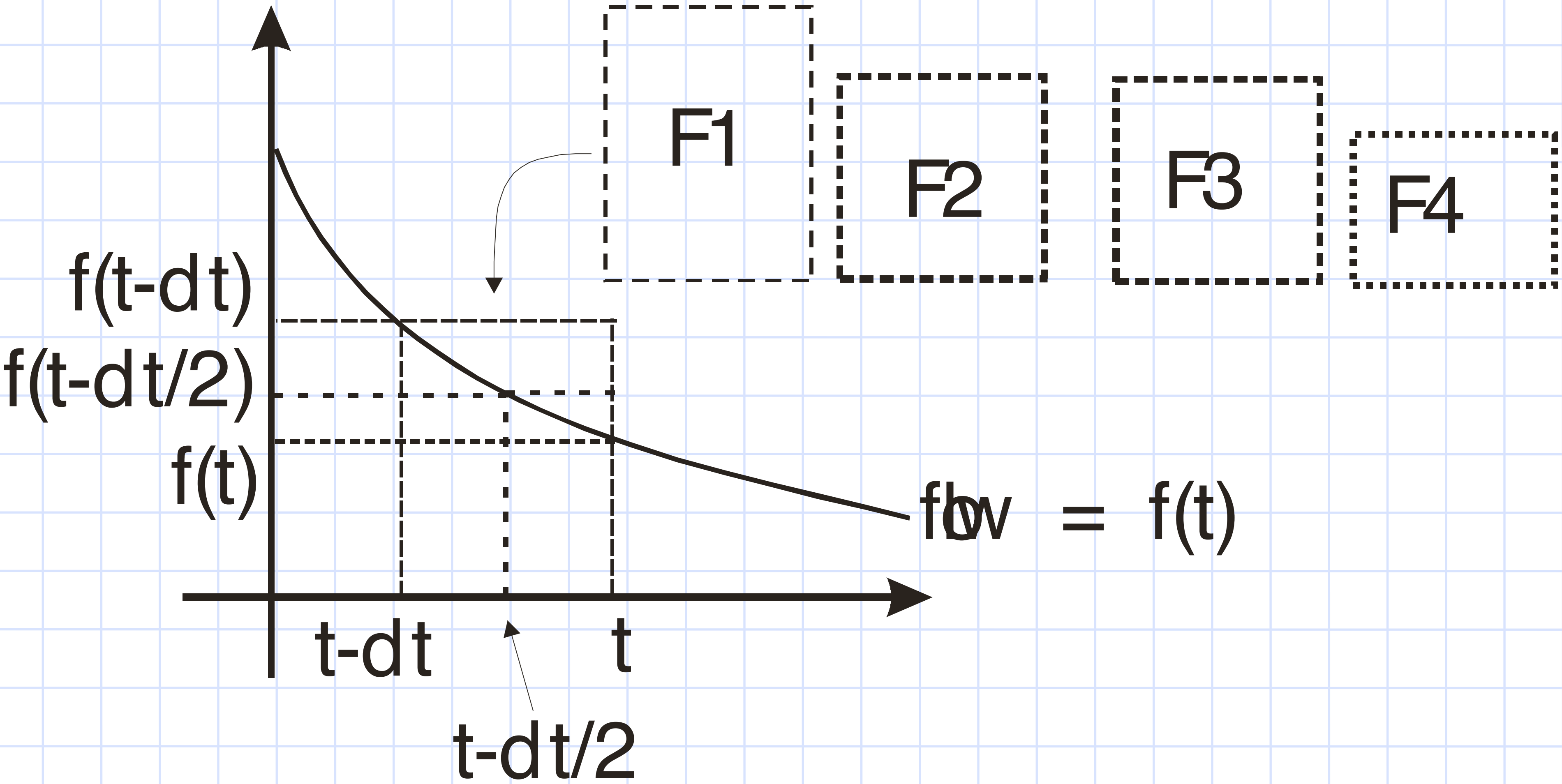
- Let Stock = X, flow = f(t,X)
- Estimates for stock updates:
  - $F1 = dt * f(t-dt, X(t-dt))$
  - $F2 = dt * f(t-\frac{1}{2}dt, X(t-dt) + \frac{1}{2}*F1)$
  - $F3 = dt * f(t-\frac{1}{2}dt, X(t-dt) + \frac{1}{2}*F2)$
  - $F4 = dt * f(t, X(t-dt) + F3)$
- $\Delta X = 1/6 * (F1+2*F2+2*F3+F4)$
- $X(t) = X(t-dt) + \Delta X$

Runge–Kutta 4

$\boxed{\text{Assume flow} = f(t).}$

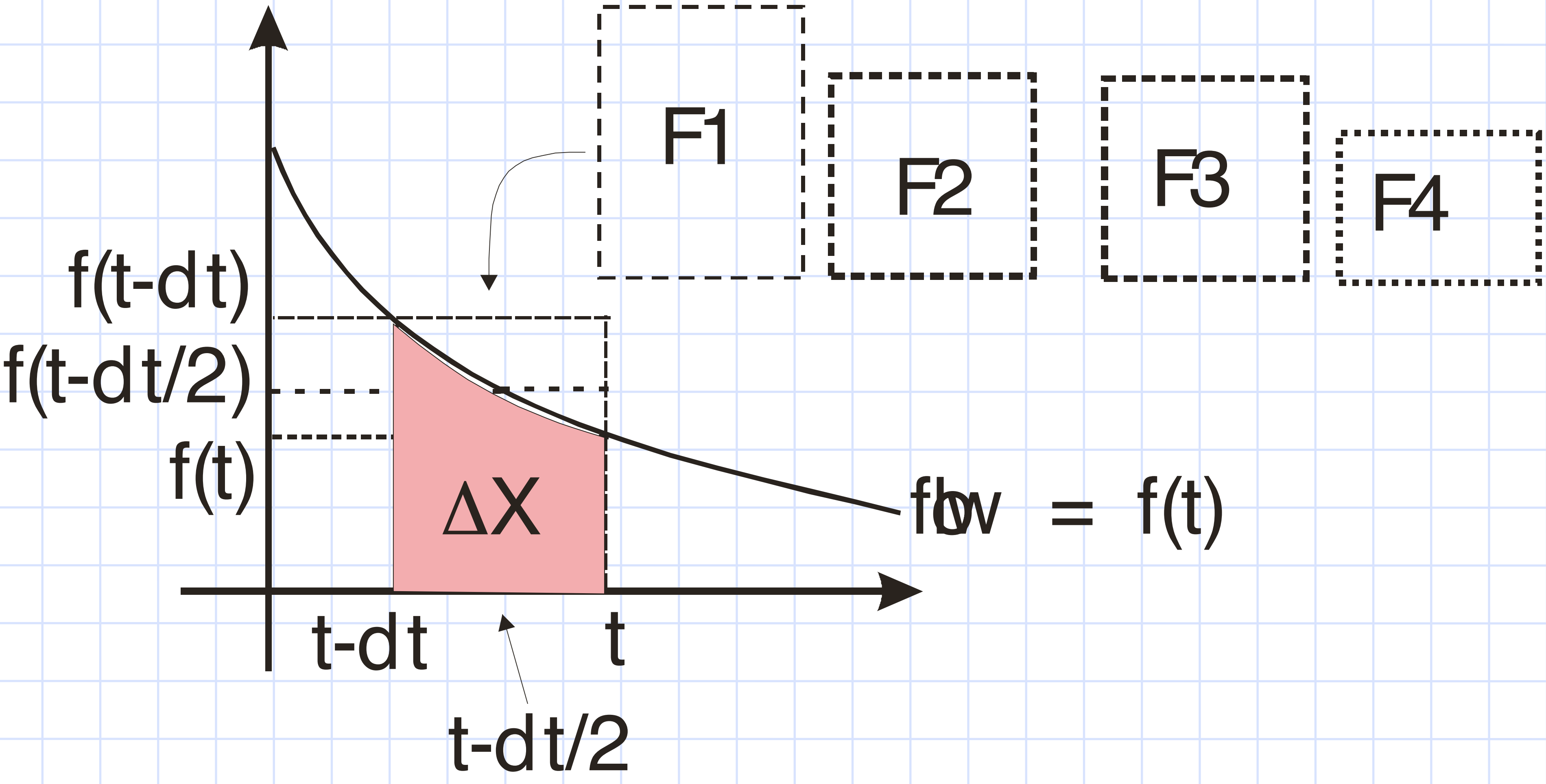
Runge-Kutta 4

Assume flow = f(t).



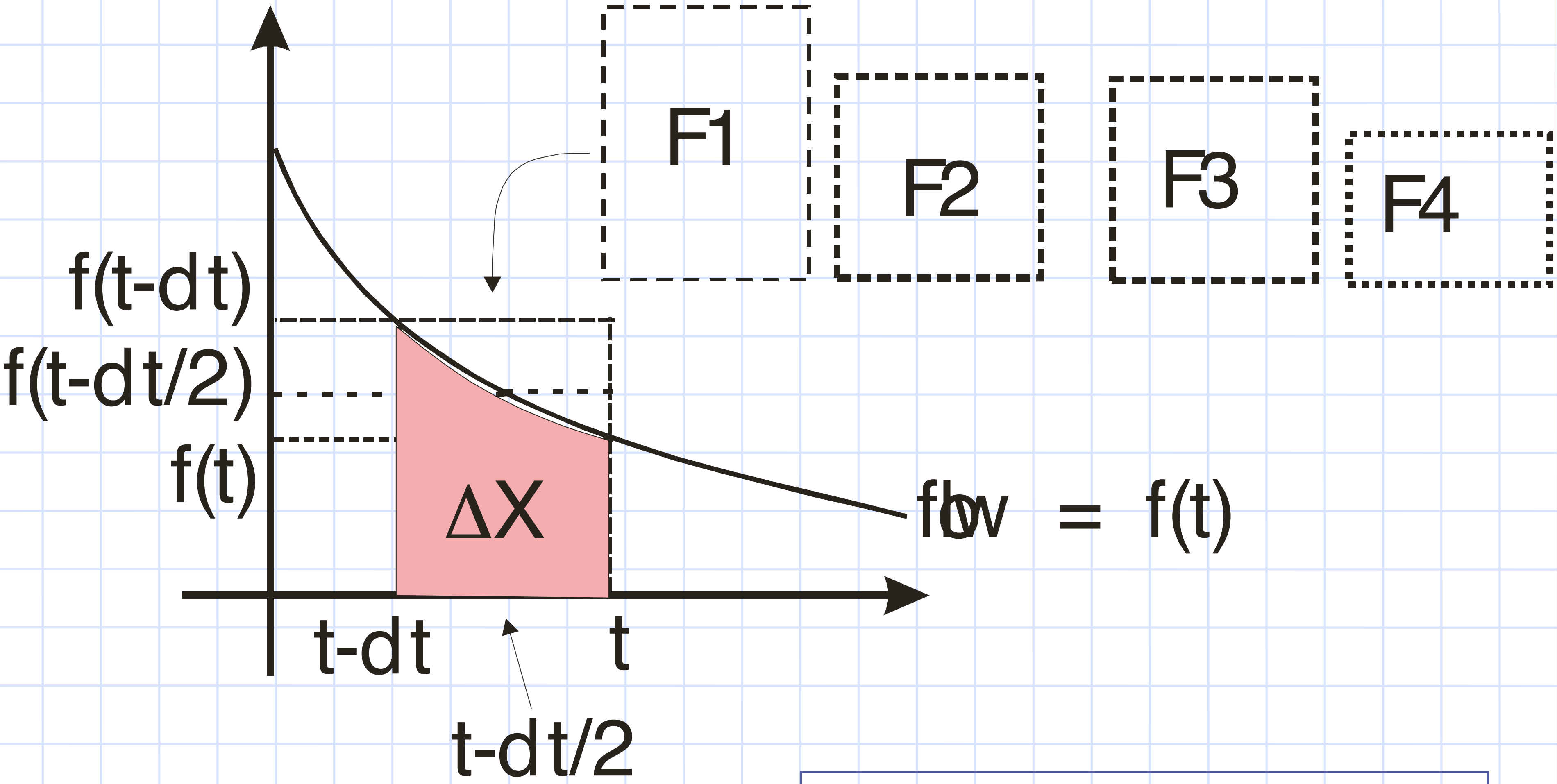
Runge-Kutta 4

Assume flow = f(t).



Runge-Kutta 4

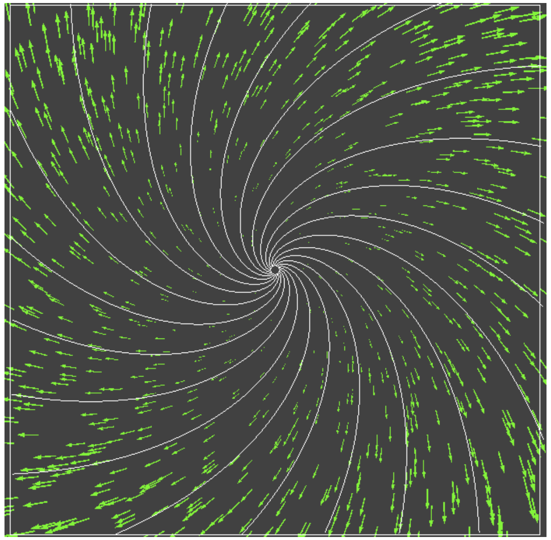
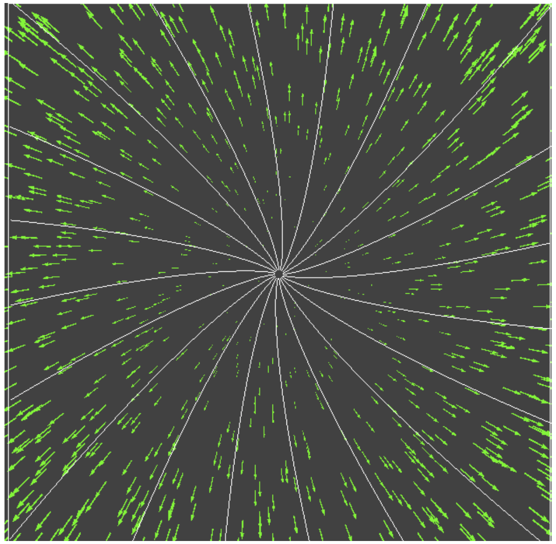
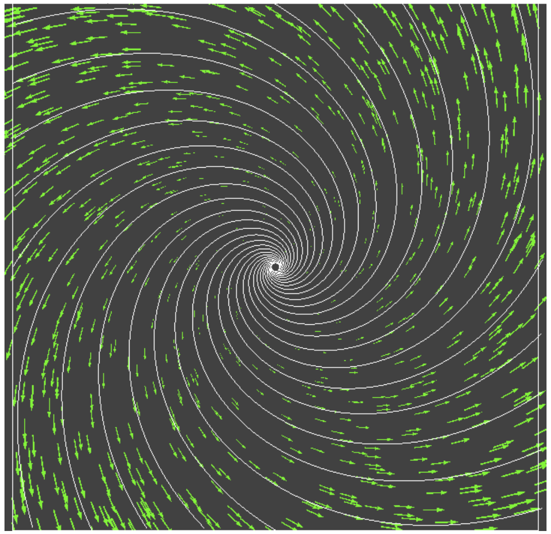
Assume flow = f(t).



Like Simpson's Method.

Steady vs. unsteady

- Flows change with time
- For every timestep, a different vector

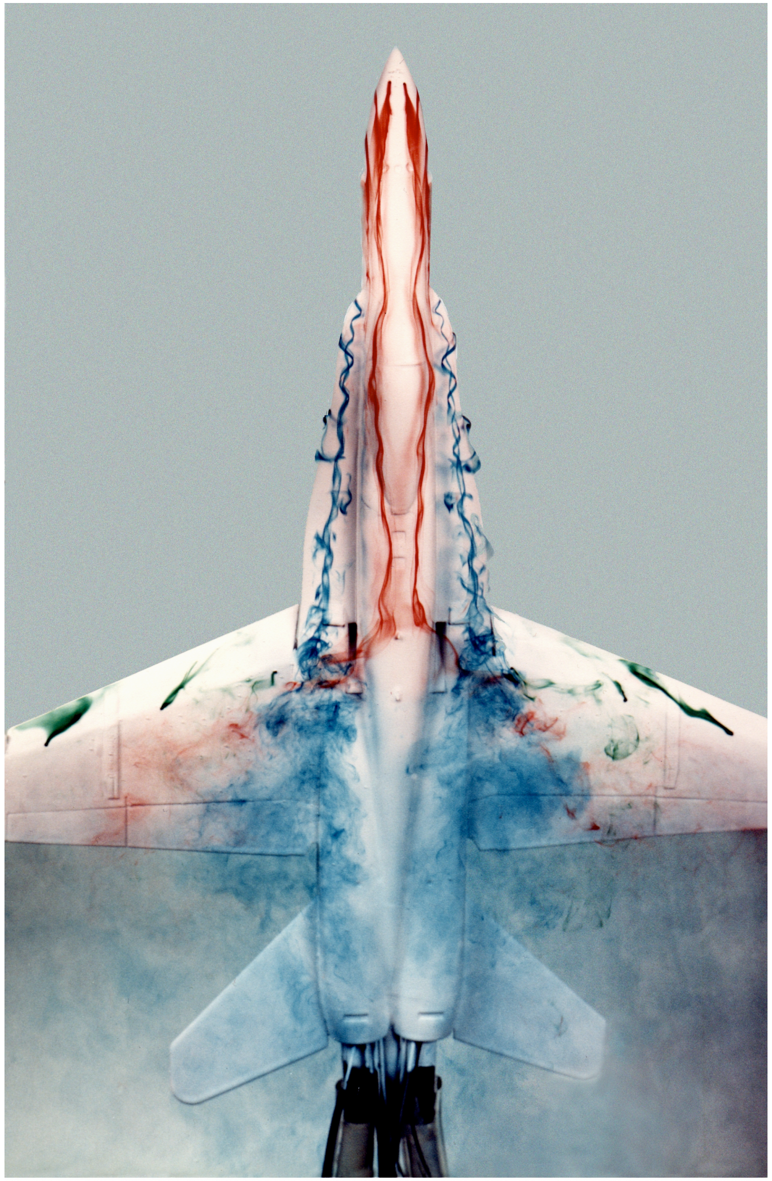




- But, what about streamlines, then?

Pathlines and Streaklines

- Pathlines: look at a single speck of dust as it moves through field
  - demo
- Streaklines: plume of smoke
  - No VTK support for these :(

Streaklines in real life

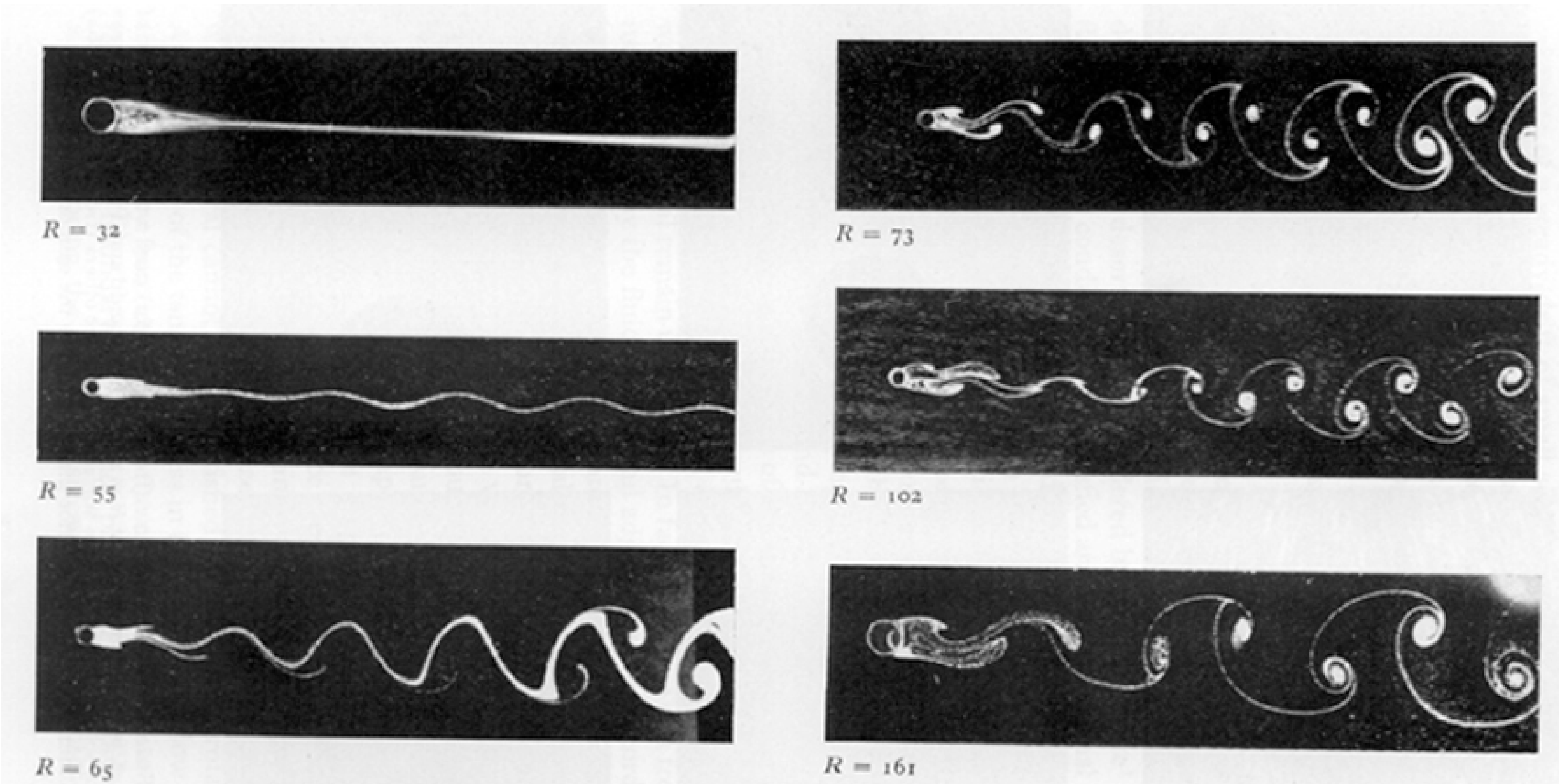


NASA Dryden Flight Research Center Photo Collection
http://www.dfrc.nasa.gov/gallery/photo/index.html
NASA Photo: ECN-33298-03   Date: 1985

1/48-scale model of an F-18 aircraft in Flow Visualization Facility (FVF)

Dryden Flight Research Center ECN 33298-47 Photographed 1985
F-18 water tunnel test in Flow Visualization Facility NASA/Dryden

Streaklines in real life

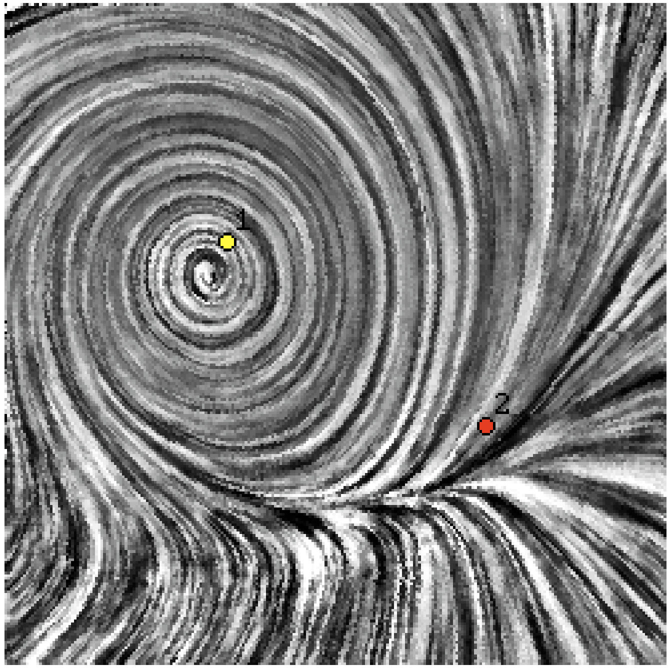


Pathlines and Streaklines

- Streaklines still never cross one another (why?)
- But pathlines, in general, do

Line Integral Convolution

- Basic idea: Integrate noise along streamlines
- demo: http://www.javaview.de/demo/PaLIC.html



IBFV

- LIC gives direction, but not magnitude
- IBFV: “animated LIC, kind of”

  - http://www.win.tue.nl/~vanwijk/ibfv/