Forward/Inverse toolkit in the SCIRun problem solving environment

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April 1, 2015

Abstract

Here is some abstract text

1 Introduction

Forward and inverse modeling are key components of science, mathematics and engineering. The forward model considers the case in which observed data g is generated from a given source function f and a blurring, or distortion, kernel K ($Kf = g$). Conversely, the inverse model attempts to determine the source function given the observed data and the blurring kernel.

A source is a parameterized function, or set of discrete data points, that defines the initial conditions of the model. The blurring kernel is applied in order to distort the source in some meaningful way, usually being a function of some physical noise term observed in nature. Combining these terms (the forward problem) produces new, or observed, data.

In an inverse sense, one may think that inverting the forward equation is trivial ($f = K^{-1}g$. This is the case for nonsingular kernels; however, in most cases, K is ill-conditioned – translating into large errors upon inversion. Useful data is still contained within the inverse data but must be extracted through regularization. Thus, essential information is heavily weighted, while minimally influential data is given less priority or eliminated completely.

In the following paper, we will illustrate the basic concepts of forward and inverse problems as they relate to electrocardiography, while also introducing a newly developed Forward/Inverse toolkit available in the SCIRun problem solving environment.

2 Methods

SCIRun is an open-source software package supplying researchers with general-purpose, problem-solving tools capable of preparing, executing, solving and visualizing scientific simulations (ref:Macleod 2004). The SCIRun environment works off of a component-based, visual programming model in which researchers are able to link together complex networks of purpose-specific modules that can be interactively modified as the simulation demands (Shepherd 2009). SCIRun is free to downloaded from http://www.sci.utah.edu/download/scirun/4.4.html.

SCIRun also provides generalized toolkits that are specific to individual types of computational problems aimed at modeling specific phenomena. Toolkits are repositories of sample networks and data that can be altered to suit the needs of the user. Within the SCIRun environment, toolkits and tutorials can be accessed from the src/nets/ directory. SCIRun's Forward/Inverse toolkit supplies users with sample networks that illustrate how to define sources, construct blurring kernels, implement mathematical models, apply numerical solvers, manipulate boundary conditions, regularize data and implement custom refinements through interfacing with external packages. Specific networks available within the toolkit relate to clinical problems in electrocardiography.

Electrocardiography is the clinical technique of measuring the electrical signals of the heart as they are observed on the body surface. The cardiac source produces and electrical signal that is attenuated as it passes through the volume conducting body. Resulting electrical signals can then be observed on the body surface.

Forward problem networks in the Forward/Inverse toolkit Setting up the forward problem in SCIRun environment treat the cardiac source conditions are known values that are connected to a passive volume conductor, representing the body tissue. Torso geometries are comprised of inhomogenous conducting regions that passively attenuate the electrical signal passing through them. Body surface potentials (BSPs) are generated as the resulting, observed data. Case Study 3.1 supplies an example of this forward modeling method as it is applied to a clinical defibrillation model with finite element approximation.

Case Study 3.2 is a boundary element approximation of the activation based inverse solution. That is, the torso potentials are provided as the known values. Given these values, as well as certain properties of the volume conducting torso, the activation times of the cardiac tissue are extracted.

3 Case Studies

3.1 Defibrillation Surface Potentials: A Forward Study

Implantable Cardioverter Defibrillators (ICDs) are relatively common devices, implanted into cardiac patients, that provide an electric shock to treat fatal arrhythmias. Though ICDs are common, they have not been rigorously optimized for pediatric patients. Children, due to their smaller size and often abnormal anatomy, require more specialized ICD configuration than adult patients. Forward problems can be used to determine optimal energy discharge and placement of the ICD. The following case study illustrates how these forward solutions can, and have, aided clinicians implanting ICDs for maximum efficiency.

Determining the optimal energy required for defibrillating a patient, or defibrillation threshold (DFT), involves generating ICD and patient-specific geometries, solving the forward problem for BSPs, and calculating the DFT from the solution. Segmentation of patient MRI or CT data provides the torso geometry into which CD geometry is interactively placed. Local mesh refinement around the ICD reduces the overall number of elements while maintaining crucial details.

All tissues are modeled as passive with ICD geometry locations acting as sources and sinks. This allows the solution to be approximated via FEM by satisfying Poisson's equation. The DFT is found by scaling the calculated potential field so that the electric field throughout the myocardial tissue satisfies the critical mass hypothesis, that is 95% of the myocardium has an electric field strength greater than 5 V/cm. The scaled potential field magnitude is then to find the DFT.

Explain figures that I'll put in.

3.2 Cardiac Activation Times From Body Surface Potentials: An Inverse Study

During a cardiac cycle, tissues throughout the heart activate and differing times. Coordination of activation throughout the myocardium produce synchronous heart beats. Alterations in cellular activation times (and therefore contraction times) may lead to serious, even fatal, consequences. BSPs over the course of a cardiac cycle contain information about myocyte activation time that may be clinically useful in diagnosis of patients with abnormal cardiac activity.

Generation a computational inverse model to find cardiac activation times requires knowledge of both the BSPs during the QRS segment of the cardiac cycle as well as the volume conducting torso. The epicardial wavefront is treated as a state model where cells transition for the 'off' (resting) to 'on' (depolarized) states. An initial guess of the activation times (time when cells switch from off to on) is supplied by the user. From this initial guess a Gauss-Newton optimization routine is used to find the local minima of the Tikhonov objective function.

$$min(\|AX(\tau) - Y\|^2 + \lambda\|LX(\tau)\|^2) \tag{1}$$

where L is the regularization matrix, AX is the forward solution matrix (*i.e.* the blurring kernel, Y is the observation matrix, and τ are the activation times.

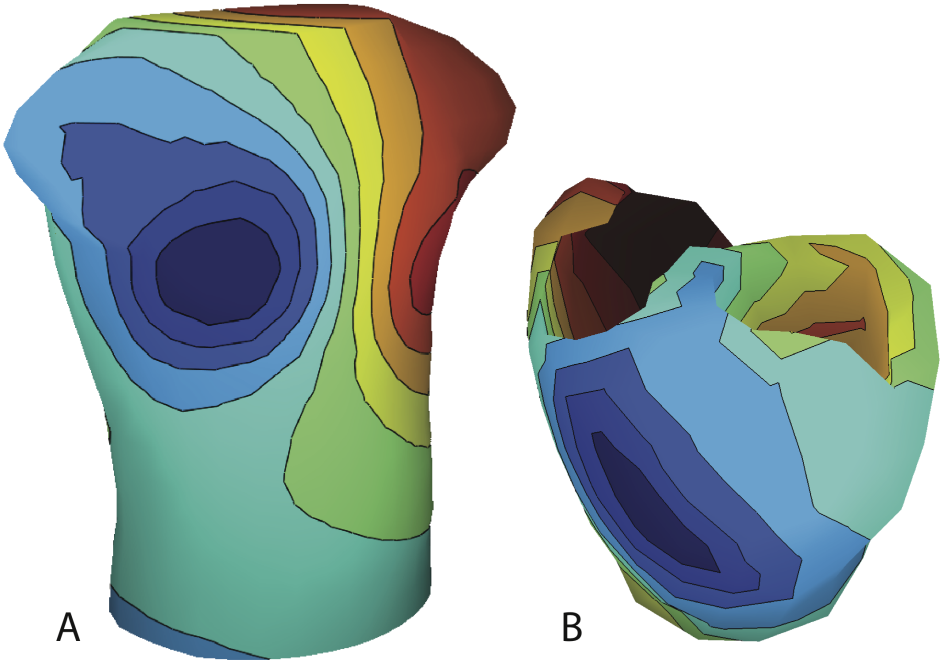

Figure 1: Activation-time Inverse Solution. From the body surface potentials (A) observed on the surface of the torso during a QRS segment, Isochronal values on the epicardium (B) can be extracted.

In essence, the inverse solution requires the user to perform multiple forward simulations that are incrementally minimized, with respect to the original torso potentials, until activation times are obtained. Figure 1 shows an isochronal, cardiac activation map that was derived from an isopotential torso map. Such results may offer clinicians substantially more information than current ECG techniques, making diagnosis of activation time sensitive maladies more efficient and accurate.

4 Discussion

SCIRun's Forward/Inverse toolkit provides users with adequate resources to define, compute and visualize components of forward and inverse problems. Through clinically relevant sample networks, users obtain a basic sense of these problems as well as an introduction to the interactivity and performance of the SCIRun environment. External interface options allow for custom data manipulation within the SCIRun environment, while collaborative efforts with major educational and industrial packages (such as ECGSIM, CHASTE and CARP) aim to generate output files that are compatible between programs.

Interfacing with these more specialized, external packages allows SCIRun to maintain its generality as a problem solving environment while taking advantage of the more accessible user interfaces of development-specific tools. Indeed, general purpose tools, such as SCIRun, generate a large amount of complexity. Complexity that is usually apparent in user interfacing and software developmental expansion. External package integration and compatibility as well as intuitive visualization tools are ways that SCIRun is able to reduce apparent complexity while extending a broad range of usable tools to users. Furthermore, recent updates in the SCIRun source code structure has added fast, parallel computing feature and allowed for more accessibility to users and developers in updating and extending its capabilities.

5 Conclusion

The Forward/Inverse toolkit, available in the SCIRun problem solving environment, has been able to address many of the software needs of researchers performing forward and inverse simulations, while also supplying them with illustrative networks in the fiend of electrocardiography. Though some limitations still exist, the Forward/Inverse toolkit has addressed many of the needs of researchers and continues to develop and change as new needs arise among an expanding body of worldwide researchers aiming improve scientific knowledge in medicine, science, and engineering.