CS 6170: Computational Topology, Spring 2019 Lecture 20

Topological Data Analysis for Data Scientists

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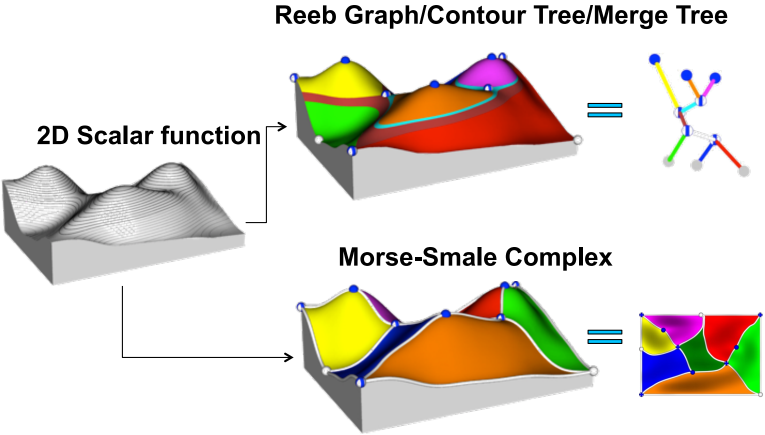
Key development in topological data analysis

1. Abstraction of the data: topological structures
2. Separate features from noise: persistent homology

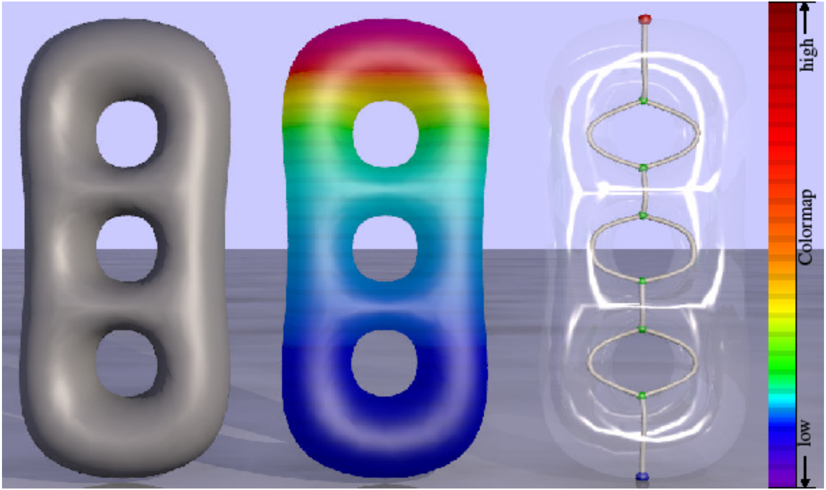


van Kreveld et al. (1997); Carr et al. (2003); Edelsbrunner et al. (2003a,b)

Reeb Graphs

Reeb graph

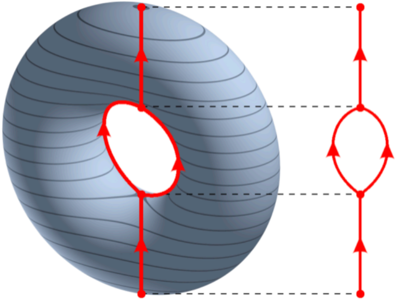
Graph obtained by continuous contraction of all the contours in a scalar field, where each contour is collapsed to a distinct point.



Cole-McLaughlin et al. (2003)

Reeb Graph

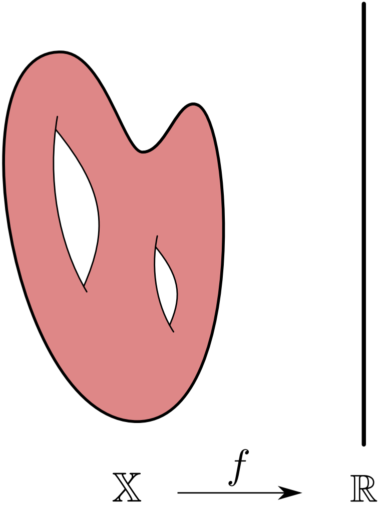
- Let $f : \mathbb{X} \to \mathbb{R}$ be a generic, continuous mapping
- Two points $x, y \in \mathbb{X}$ are *equivalent*, demoted by $x \sim y$, if $f(x) = f(y)$ and x and y belong to the same path-connected component of the pre-image of f, $f^{-1}(f(x)) = f^{-1}(f(y))$.
- The *Reeb graph*, $\mathcal{R}(X, f) = \mathbb{X}/\sim$, is the quotient space contained by identifying equivalent points together with the quotient topology inherited from $\mathbb{X}$.



https://en.wikipedia.org/wiki/Reeb_graph

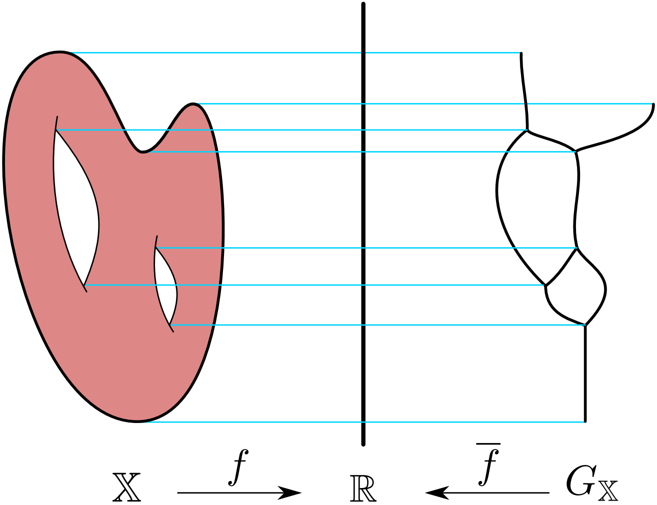
Reeb Graph in Shape Analysis

Reeb Graph



- Input: $(\mathbb{X}, f)$
- $f : \mathbb{X} \to \mathbb{R}$

Reeb Graph

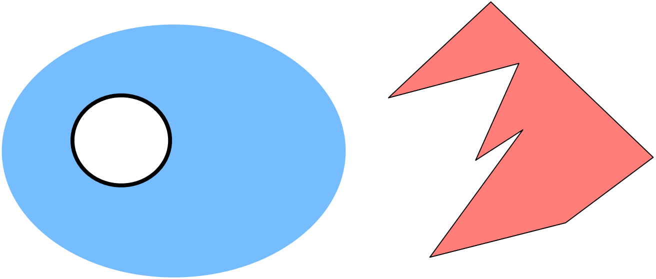


- Output: $(G_{\mathbb{X}}, \bar{f})$
- $G_{\mathbb{X}} := \mathcal{R}(\mathbb{X}, f)$, $\bar{f} : G_{\mathbb{X}} \to \mathbb{R}$

Contour Trees

Contour tree

- A contour tree is a special type of Reeb graph when the domain is simply connected.
- A topological space is simply connected if it is path-connected and every path between two points can be continuously transformed into any other such path while preserving the endpoints.



Contour tree

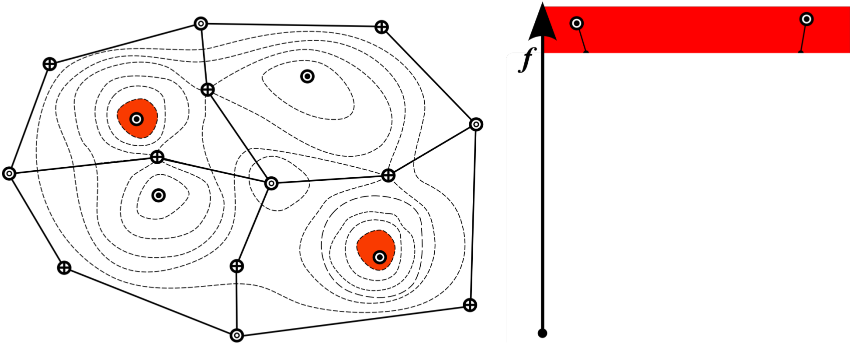


Image courtesy: V. Pascucci

Contour tree

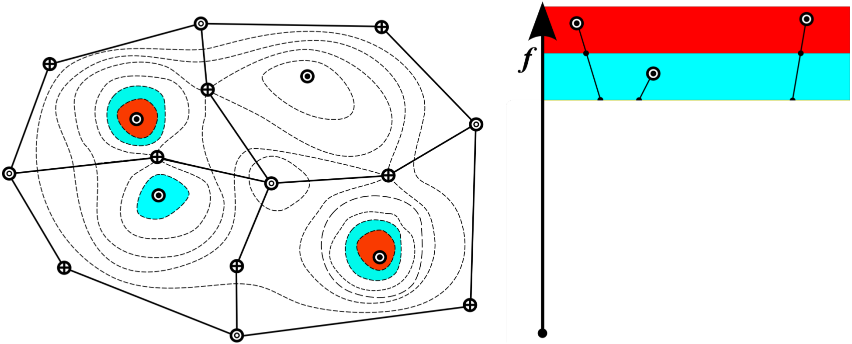


Image courtesy: V. Pascucci

Contour tree

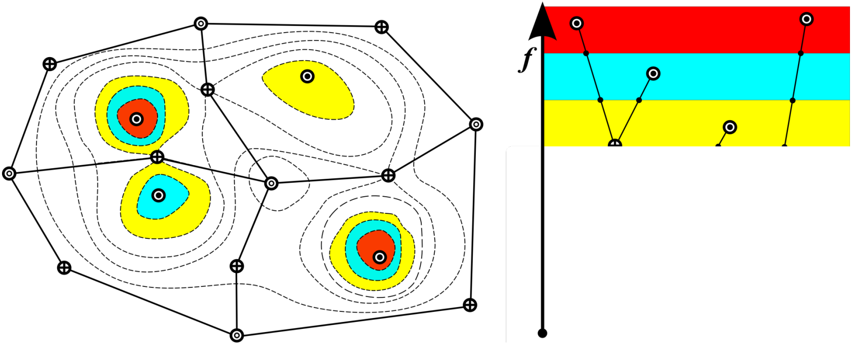


Image courtesy: V. Pascucci

Contour tree

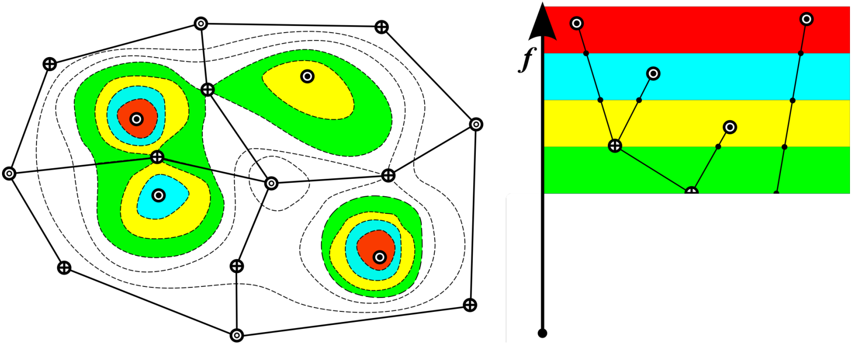


Image courtesy: V. Pascucci

Contour tree

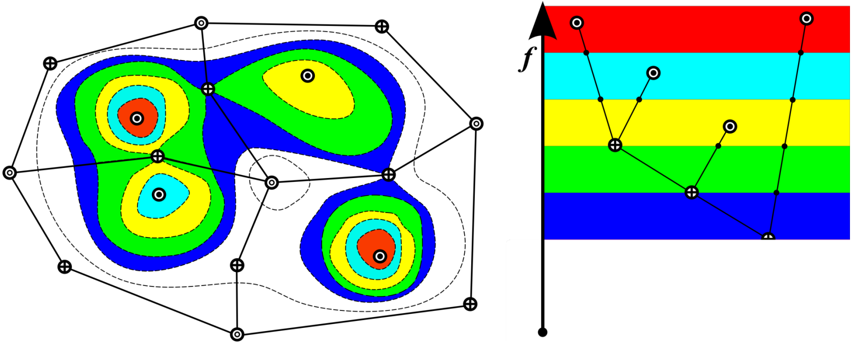


Image courtesy: V. Pascucci

Contour tree

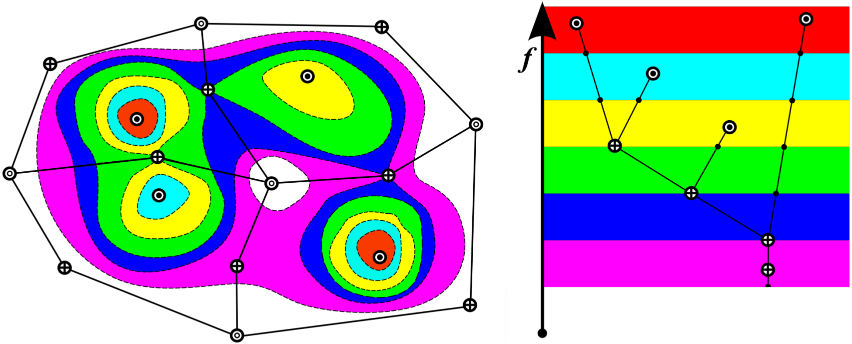


Image courtesy: V. Pascucci

Contour tree

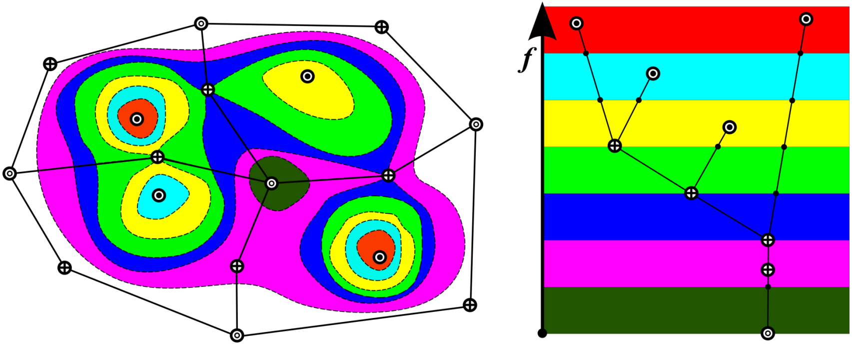
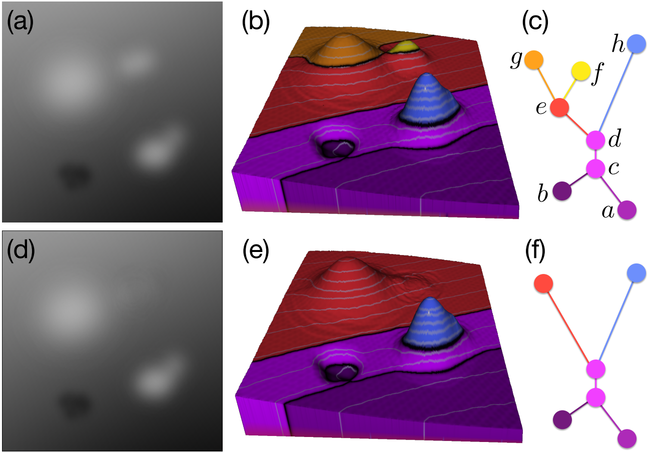


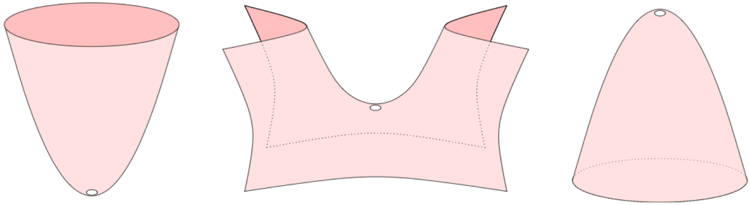
Image courtesy: V. Pascucci

Contour tree based simplification

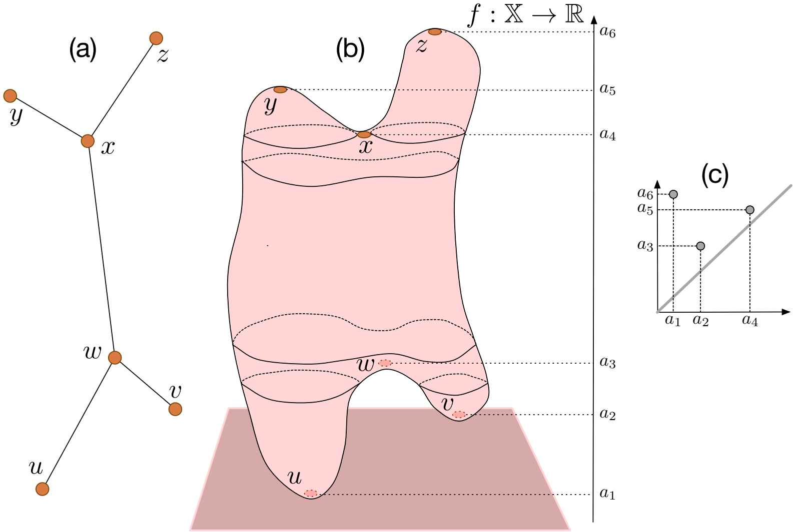


Rosen et al. (2017)

Critical points of Morse function on a 2-manifold

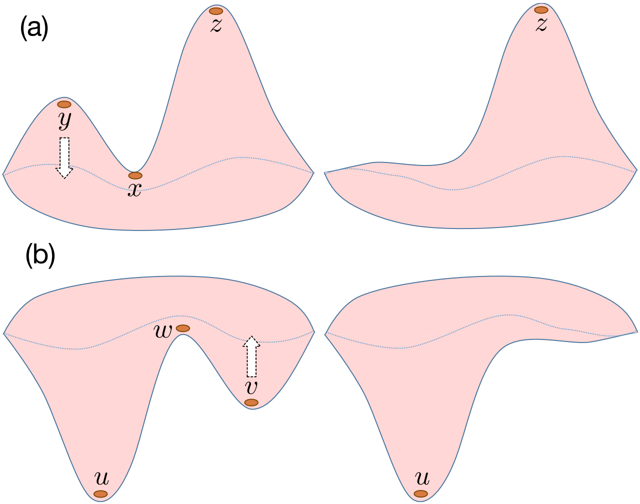


Contour tree for functions on 2-manifolds



Rosen et al. (2017)

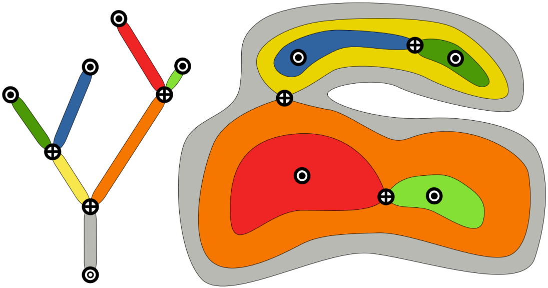
Contour tree based simplification



Rosen et al. (2017)

Merge Trees

Merge tree

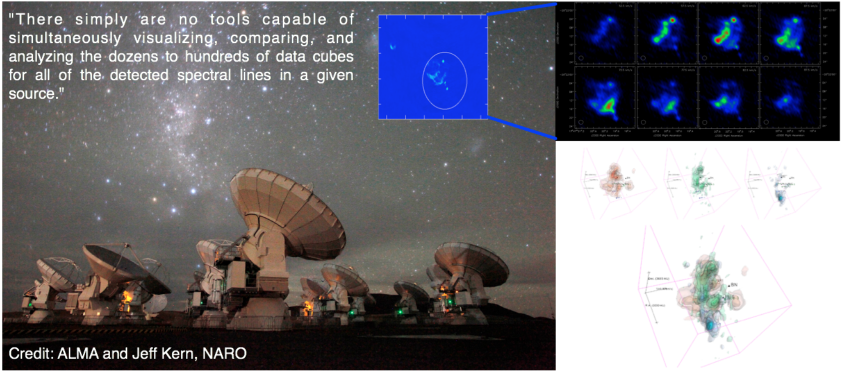


Bennett et al. (2012) A merge tree tracks the connected components of *sublevel sets* of a function (instead of level sets).

Applications in Astronomy
Rosen et al. (2017)

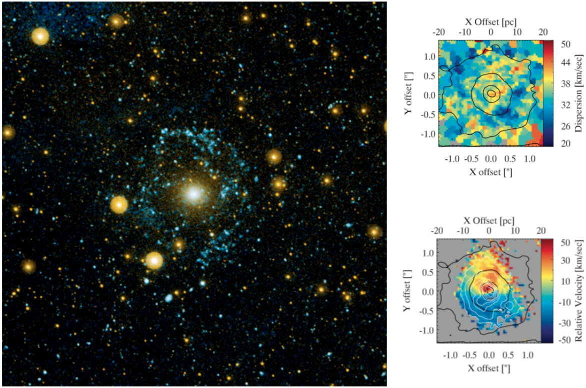
Analysis and visualization of ALMA data cubes

- Atacama Large Millimeter/submillimeter Array (ALMA)
- One of the world's most powerful telescopes, located in Chile
- Collaborate with NRAO scientists: analysis and vis of spatial and kinematic structures within ALMA data cubes, e.g. black holes
- Develop techniques and software tools for data transformation, feature extraction, feature exploration and feature comparison



Study black hole within the Ghost of Mirach Galaxy

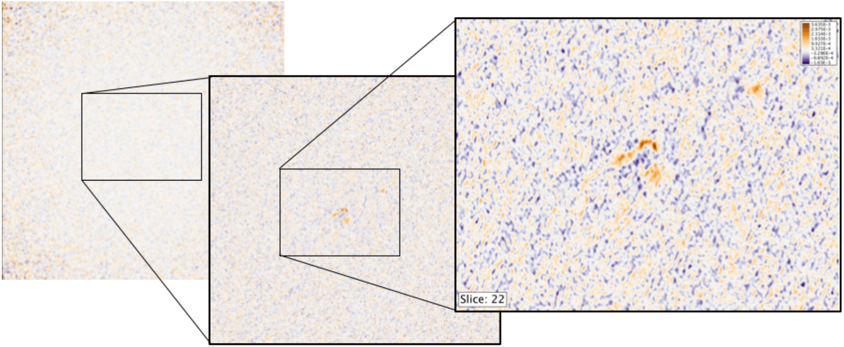
- Stellar and gas kinematics (movement of stars/gas w/o needing to understand how they acquired their motion)
- Contours: light distribution of the galaxy, or the luminosity in the gas emission lines.



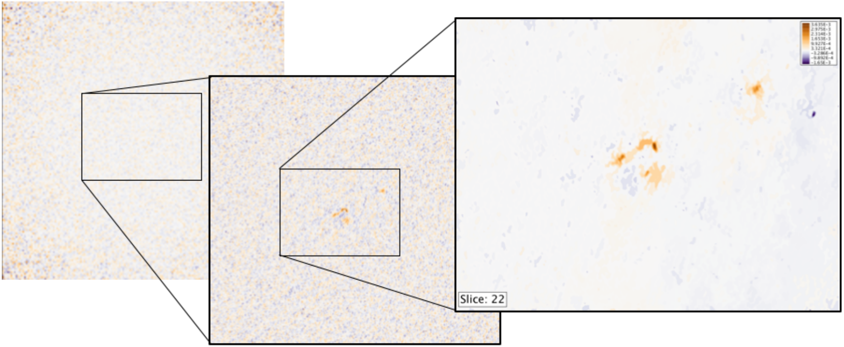
Stellar and gas kinematics from shorter wavelength near infrared data

Collaborators: Bei Wang (Utah SCI), Anil Seth (Utah Astronomy), Jeff Kern (NRAO), Betsy Mills (NRAO), Chris Johnson (Utah SCI), Paul Rosen (USF)

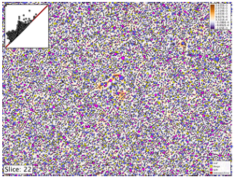
Feature de-noising and source finding

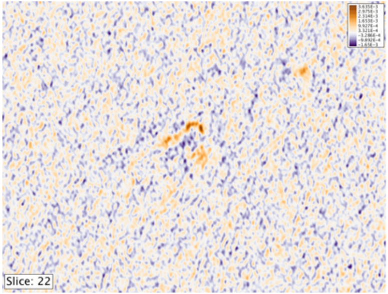


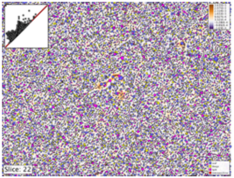
Feature de-noising and source finding

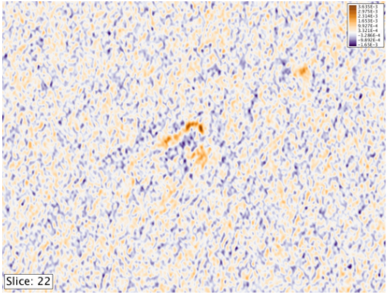


De-noising at multiple scale and source finding

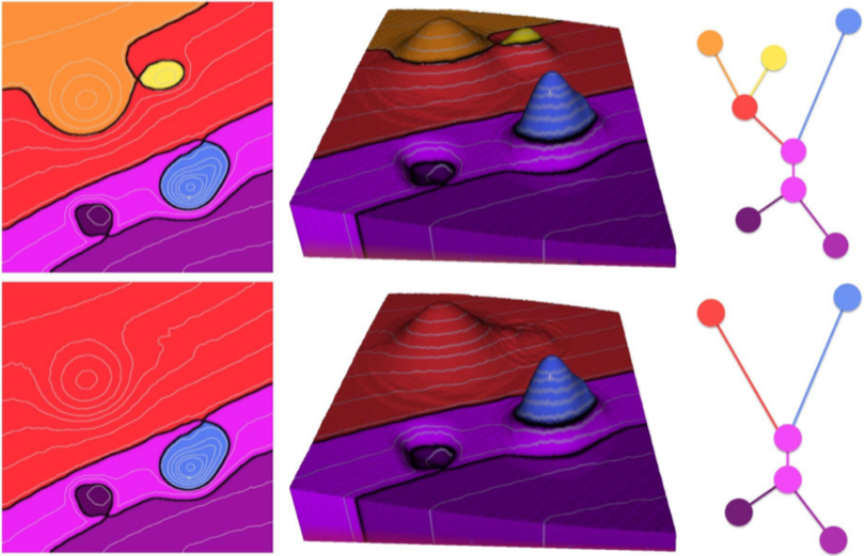




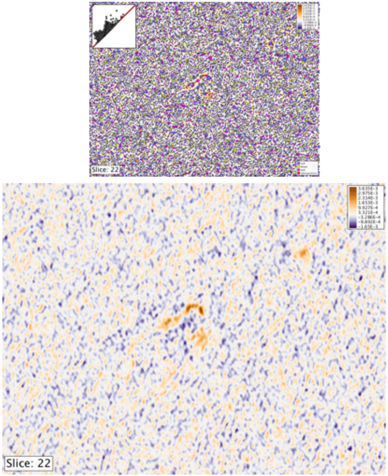


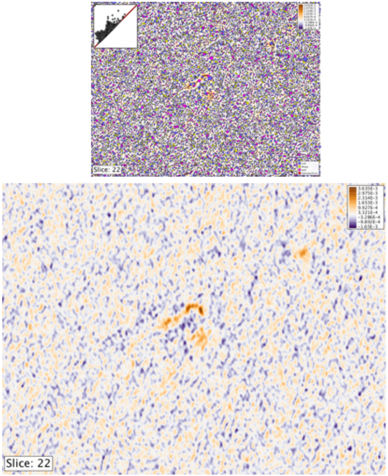


Idea: Using Contour Tree



De-noising at multiple scale and source finding





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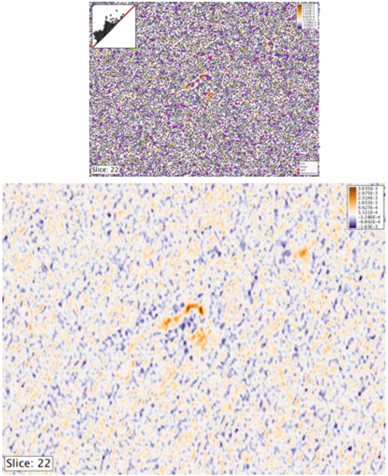

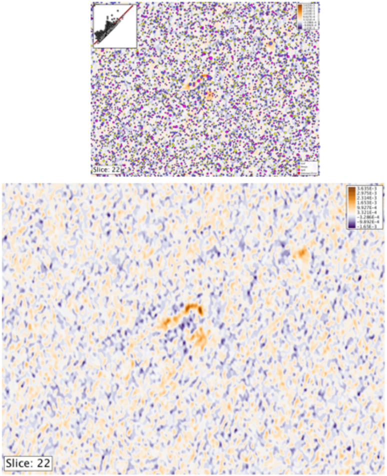

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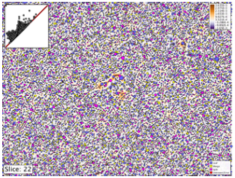

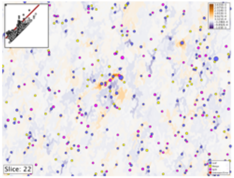

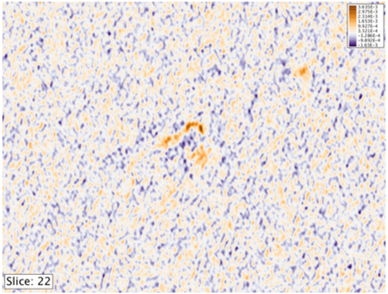

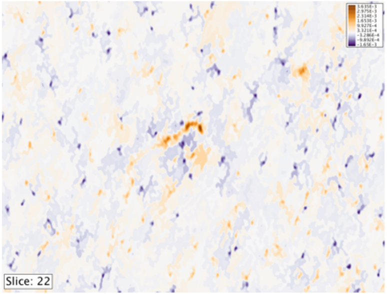

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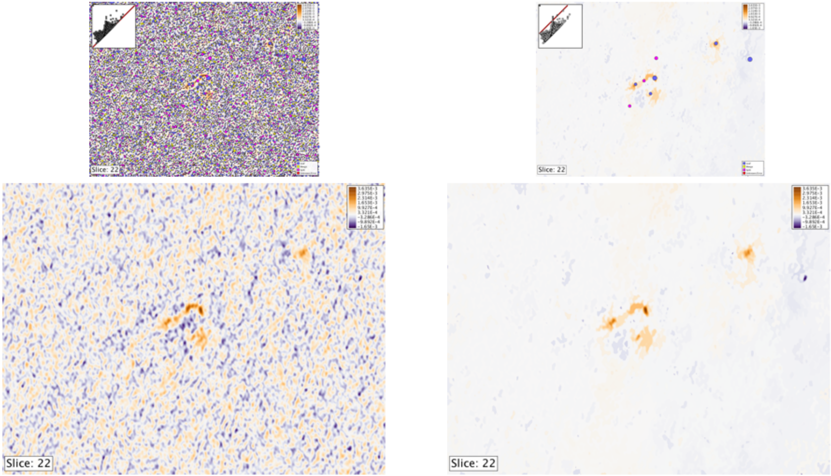

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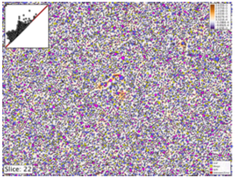
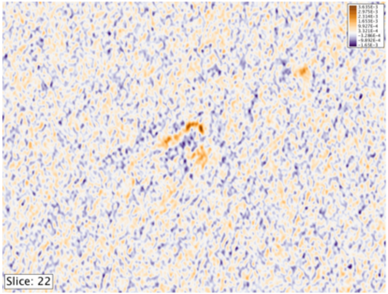
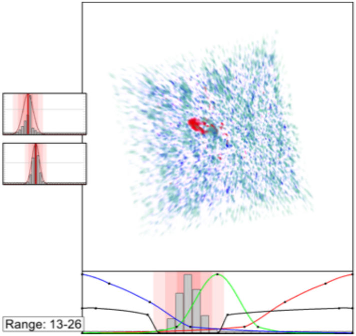
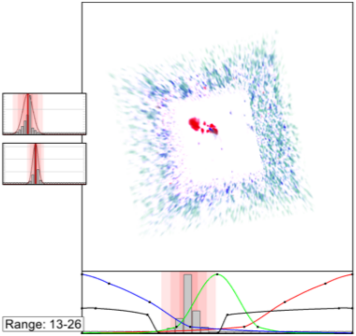




Volume rendering





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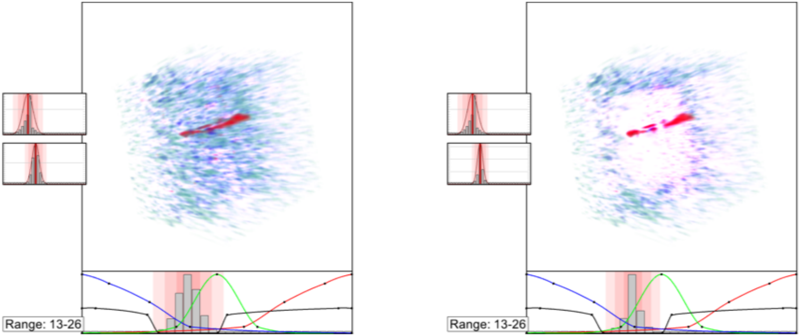

Other Types of Astrophysical Data Cubes

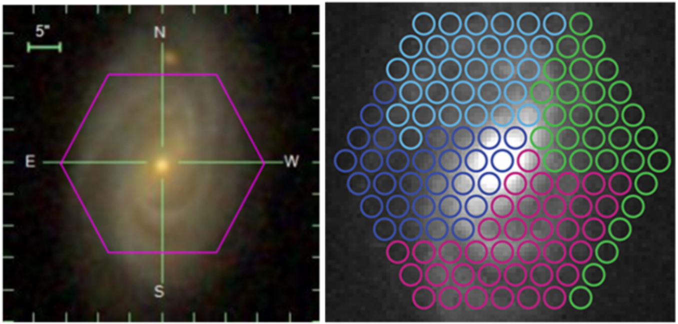

FIGURE 1: *Left*: A face-on spiral galaxy seen by MaNGA – the red hexagon shows the coverage of the MaNGA IFU instrument. *Right*: The same spiral galaxy, now showing circles for the individual IFU optical fibers (Images courtesy www.sdss.org)

A Collaboration with Carnegie institution for science

References I

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References II

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