The quiz is open-book, open-notes, but close-internet. In particular, no laptops, calculators, cell phones, or other electronic devices are allowed.

The point value of each question is clearly marked, so allocate your time wisely. The quiz is worth a total of 10 points; with a bonus question worth 5 points.

You must complete all work in 10 minutes, there are no exceptions.

This quiz constitutes 10% of your final grade (if you complete the bonus question, you can earn potentially another 5% towards your final grade).

Total __________________________ (out of 10 points)
Question 1 (Compute a Reeb Graph, 7 points).

Given the following scalar function $f$ on a 2-dimensional manifold $X$, $f : X \to \mathbb{R}$, complete its corresponding Reeb graph by connecting the marked critical points in Figure 1. The critical points are labeled in increasing height order. (You lose 1 point for every 1 edge you get wrong in the Reeb graph).

Figure 1: Left: 2-manifold with a height function $f$ defined on it. Right: the corresponding Reeb graph (to be completed).

Question 2 (Persistence Pairing, 3 points).

As shown in Figure 1, suppose each critical point $i$ has a height value of $a_i$, the (extended) persistence diagram of the sublevel set filtration of $f$ contains two points $(a_5, a_6)$ and $(a_1, a_{10})$.

Please list the other 3 pairs in the persistence diagrams in the form of $(a_i, a_j)$. 
(Bonus) Question 3 (Compute a Reeb Graph, 5 points). Given the following scalar function $f$ on a 2-dimensional manifold $X$, $f : X \rightarrow \mathbb{R}$, complete its corresponding Reeb graph by connecting the marked critical points in Figure 2. The critical points are labeled in increasing height order. (You lose 1 point for every 1 edge you get wrong in the Reeb graph). The figure is adaptive from Figure 2 in paper [Extreme Elevation on a 2-Manifold by Agarwal, Edelsbrunner, Harer and Wang].

Figure 2: Left: 2-manifold with a height function $f$ defined on it. Right: the corresponding Reeb graph (to be completed).