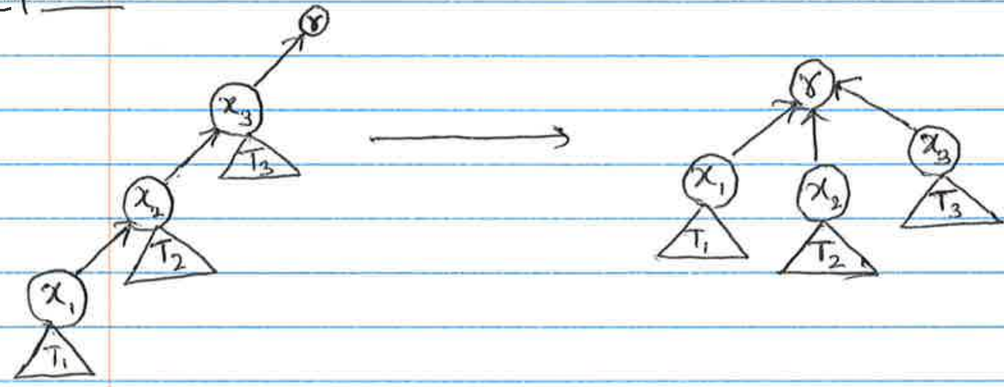
Jan 19

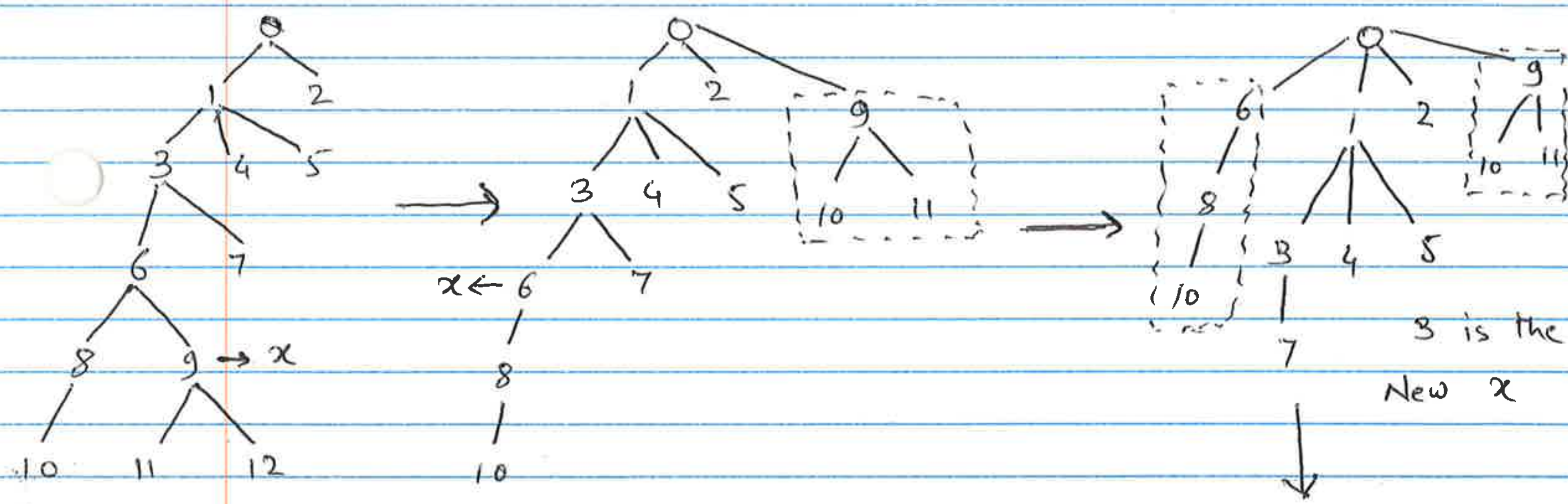
Path Compression: Find (x_1) : all nodes along the path from x_1 to root attached directly to root

Example 1



Traversing the tree up from x_1, apply Find( ) to parent of x_1, and set root to be parent of x_1

Example 2: Find (9)



Pseudocode:

① MakeSet (x)

  parent $(x) \leftarrow x$

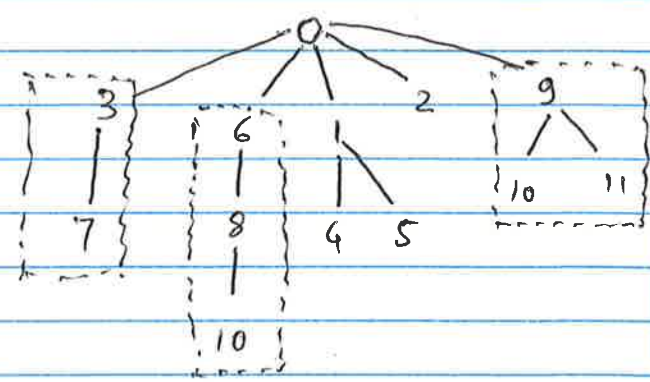
  rank $(x) \leftarrow 0$

② Find (x)

  if $x \neq$ parent (x)

    parent $(x) \leftarrow$ Find (parent (x))

  return (parent (x))



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Union (x, y)
    A ← Find (x)
    B ← Find (y)

    if rank(A) > rank(B) then
            parent (B) ← A

    else
        parent (A) ← B

    if rank (A) = rank (B) then
            rank (B) = rank(B) + 1
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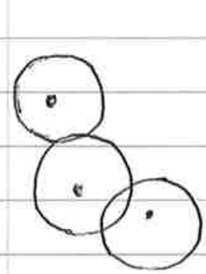
Claim:

the rank here is the upper bound of the true rank.

Data Motivation: Sensor Network Coverage.

→ Assume that sensors are points on the plane and coverage is modelled as disks.

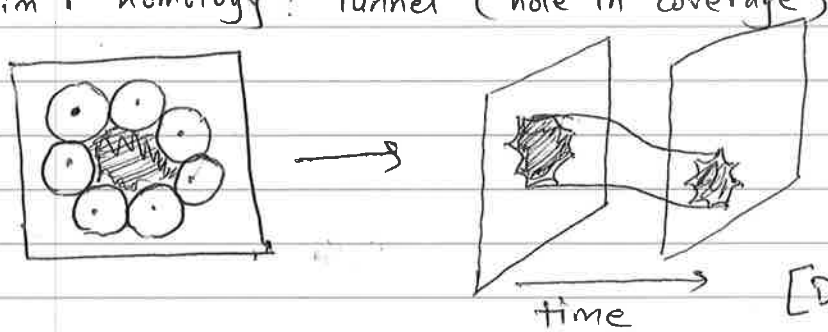


→ Problems: detecting intruders
detecting uncovered area
[holes in coverage]

↓

mobile sensors: time varying positions & coverage.

Dim 1 homology: Tunnel (hole in coverage)



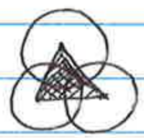
→ Tunnel persists over time then intruder can avoid detection as long as tunnel persists
[Dim 1 persistent homology]

(*) SoC News: Catching a wireless thief
Detecting un-authorized use of cellular spectrum.

→ We can model sensor coverage as simplicial complexes (Čech)
problem transformed into a problem of finding structure.

(#) **Čech Complex** $C(\epsilon)$: Form a d-simplex when there is a common point of intersection of all the $d+1$ $\left(\frac{\epsilon}{2}\right)$-balls

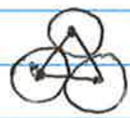
$d=2$: 2-simplex is a triangle



→ 2-simplex
→ No hole in coverage

0-simplex : vertex
1-simplex : edge
2-simplex : triangle
3-simplex : tetrahedron

(#) **Rips Complex** $R(\epsilon)$:
Form a d-simplex if there are pairwise intersections among all the $\left(\frac{\epsilon}{2}\right)$-balls



→ hole in coverage but we still form the 2-simplex.

< www.sci.utah.edu/~tsodergren/network_vis/ >

Def: ~~Simplex~~ Given $k+1$ points $(u_0, u_1, \ldots u_k)$ in $\mathbb{R}^d$ they are affinely independent iff the k vectors $(u_i - u_0)$ for $1 \le i \le k$ are linearly independent

(intuitively : no three points lie on the same line)

Def: A convex hull of a set of points X in $\mathbb{R}^d$ is the smallest convex set containing X
(think of a rubber band stretched around all points)

Def: A point $x = \sum_{i=0}^{k} \lambda_i u_i$ where $\lambda_i \in \mathbb{R}$ is an [affine combination] of u_i if $\sum_{i=0}^{k} \lambda_i = 1$

x is a [convex combination] in $\sum_{i=0}^{k} \lambda_i = 1$ AND $\lambda_i \geq 0$

Def: A K-simplex σ is the convex hull of $K+1$ affinely independent points

$$\sigma = \mathrm{Conv}\{u_0, u_1, \dots, u_K\}$$

dimension of σ : $\dim(\sigma) = K$

Convex hull : all points that can be represented as a convex combination of the $K+1$ points.