Path Compression: Find($x_i$): all nodes along the path from $x_i$ to root attached directly to root

Example 1:

Traversing the tree up from $x_i$, apply Find(.) to parent of $x_i$, and set root to be parent of $x_i$.

Example 2: Find(9)

Pseudocode:

1. MakeSet($x$)
   - parent($x$) ← $x$
   - rank($x$) ← 0

2. Find($x$)
   - if $x \neq$ parent($x$)
     - parent($x$) ← Find(parent($x$))
   - return (parent($x$))
Union \((x, y)\)

\[
A \leftarrow \text{Find } (x) \\
B \leftarrow \text{Find } (y) \\
\text{if } \text{rank}(A) > \text{rank}(B) \text{ then} \\
\text{parent}(B) \leftarrow A \\
\text{else} \\
\text{parent}(A) \leftarrow B \\
\text{if } \# \text{rank}(A) = \# \text{rank}(B) \text{ then} \\
\text{rank}(B) = \text{rank}(B) + 1
\]

**Claim:**

the rank here is the upper bound of the true rank.

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**Data Motivation:** Sensor Network Coverage.

Assume that sensors are points on the plane and coverage is modelled as disks.

Problems:
- detecting intruders
- detecting uncovered area [holes in coverage]
- mobile sensors: time varying positions & coverage

Dim 1 homology: Tunnel (hole in coverage)

… Tunnel persists over time then intruders can avoid detection as long as tunnel persists

[Dim 1 persistent homology]

SoC News: Catching a wireless thief
Detecting unauthorized use of cellular spectrum

We can model sensor coverage as simplicial complexes ([Čech] problem transformed into a problem of finding structure).
Čech Complex $C(\varepsilon)$: Form a $d$-simplex when there is a common point of intersection of all the $d+1 \left(\frac{\varepsilon}{2}\right)$-balls.

$d=2$: 2-simplex is a triangle
- 0-simplex: vertex
- 1-simplex: edge
- 2-simplex: triangle
- 3-simplex: tetrahedron

Rips Complex $R(\varepsilon)$: Form a $d$-simplex if there are pairwise intersections among all the $\left(\frac{\varepsilon}{2}\right)$-balls.
- hole in coverage but we still form the 2-simplex.

[www.sci.utah.edu/mtsodergren/network_vis/]

**Def.** Given $K+1$ points $(u_0, u_1, \ldots, u_K)$ in $\mathbb{R}^d$, they are affinely independent if the $K$ vectors $(u_i - u_0)$ for $1 \leq i \leq K$ are linearly independent.

(intuitively: no three points lie on the same line)

**Def.** A convex hull of a set of points $X$ in $\mathbb{R}^d$ is the smallest convex set containing $X$.

(think of a rubber band stretched around all points)

**Def.** A point $x = \sum_{i=0}^{K} \lambda_i u_i$ where $\lambda_i \in \mathbb{R}$ is an [affine combination] of $u_i$ if $\sum_{i=0}^{K} \lambda_i = 1$.

$x$ is a [convex combination] in $\sum_{i=0}^{K} \lambda_i = 1$ AND $\lambda_i \geq 0$. 
Def: A $k$-simplex $\delta$ is the convex hull of $k+1$ affinely independent points.

$$\delta = \text{conv} \left\{ u_0, u_1, \ldots, u_k \right\}$$

dimension of $\delta$: $\dim(\delta) = k$

Convex hull: all points that can be represented as a convex combination of the $k+1$ points.