

# KERNEL PARTIAL LEAST SQUARES REGRESSION FOR RELATING FUNCTIONAL BRAIN NETWORK TOPOLOGY TO CLINICAL MEASURES OF BEHAVIOR

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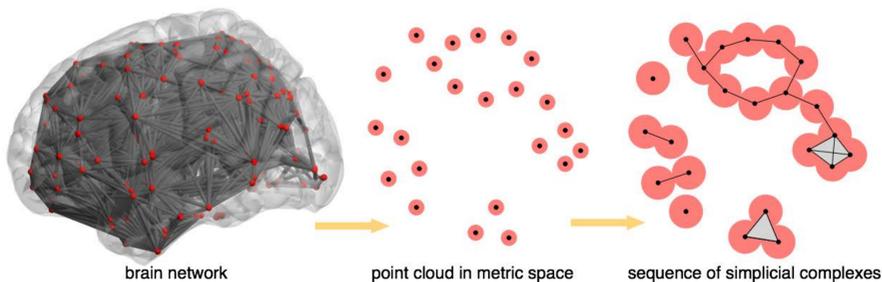
## Overview

We propose a method for analyzing relationships between functional brain networks and behavioral phenotypes.

We map **topological data features** extracted from resting-state fMRI networks to a kernel space and use **kernel partial least squares** (kPLS) regression to quantify their relationship to autism. The advantages are:

- Typical correlation-based methods use thresholds to determine the existence of an edge in the network. Topological features capture network organization across all threshold values.
- Multiple image features can be easily combined by a summation of kernels.

## Topology of Brain Networks

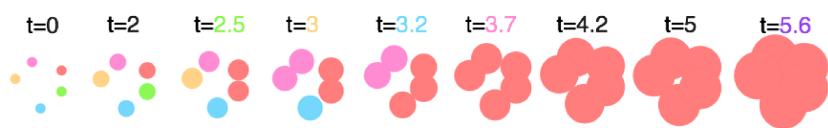


Procedure:

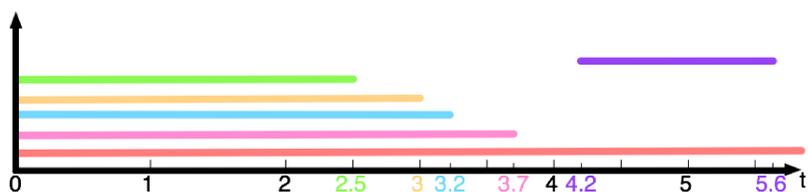
- For each subject, map the nodes from a brain network to points in a metric space using the distance measure,

$$d(u, v) = \sqrt{(1 - \text{corr}(u, v))}.$$

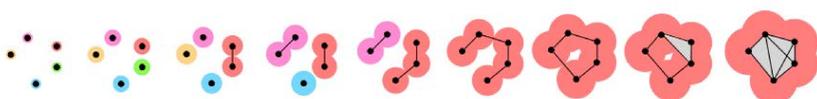
- Grow the radius of a neighborhood around each point. Neighborhoods begin to merge.
- Track the 0-, 1-, 2-dim topological features (connected components, holes, and voids) that appear and disappear over time.



**Persistent homology** follows the birth and death times of these features, which can be visualized as a barcode.



It is equivalent to capturing the topological changes of the *Rips complexes*, formed by connecting the points in intersecting neighborhoods with pairwise edges.



## References

- [1] Rosipal, Roman, and Leonard J. Trejo. "Kernel partial least squares regression in reproducing kernel hilbert space." *The Journal of Machine Learning Research* 2 (2002): 97-123.  
 [2] Reininghaus, Jan, et al. "A stable multi-scale kernel for topological machine learning." *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*. 2015.

## Kernel Partial Least Squares

PLS projects two datasets,  $X$  and  $Y$ , of sizes  $n \times N$  and  $n \times M$  down to  $p$  latent dimensions that maximally covary. kPLS [1] extends this, assuming that  $X$  is mapped by  $\Phi$  to a higher dimensional inner product space  $\mathcal{F}$ .

### kPLS Algorithm

$$t = \Phi(x)\Phi(x)^T u = Ku$$

$$\|t\| \rightarrow 1 \quad K : \text{Gram matrix of } X$$

$$c = Y^T t \quad K(x, x') : \langle \Phi(x), \Phi(x') \rangle_{\mathcal{F}}$$

$$u = Yc \quad t, u : \text{covarying latent variables}$$

$$\|u\| \rightarrow 1 \quad c : \text{loadings for } Y$$

Once all the latent dimensions are found, the regression from kernel to  $Y$  is  $\hat{Y} = \Phi(x)B = KU(T^T KU)^{-1}T^T Y$ .

### TDA Kernel

We take all subject persistence barcodes to construct a multi-scale topological kernel [2] with stability properties,

$$K_{\sigma}^{TDA}(A, B) = \frac{1}{8\pi\sigma} \sum_{p \in A, q \in B} e^{-\frac{\|p-q\|^2}{8\sigma}} - e^{-\frac{\|p-\bar{q}\|^2}{8\sigma}}$$

where for every  $q = (a, b) \in B$ ,  $\bar{q} := (b, a)$ .

## Experiments

**Goal** Predict ADOS score from resting state fMRI using data from one site in the ABIDE dataset (30 control, 57 ASD subjects)

**Results** Compared with using just raw correlations from 264 ROIs ( $d=34716$ ), augmentation with topological features improves predictive power.

We did LOOCV and evaluated the RMSE of the predictions. We used the following comparisons:

- $K^{\text{cor}}$ : Linear kernel from correlation matrices by taking Euclidean dot products between subjects
- $K^{\text{TDA}_0}$  and  $K^{\text{TDA}_1}$ : 0-, 1-dim TDA feature kernels
- $K^{\text{TDA+cor}} = w_0 K^{\text{TDA}_0} + w_1 K^{\text{TDA}_1} + (1 - w_0 - w_1) K^{\text{cor}}$ : the combined kernel with 4 parameters to cross validate over. Weights  $w_0, w_1$  in the range 0 to 1 by 0.05, and log kernel sizes  $\log_{10}(\sigma_0), \log_{10}(\sigma_1)$  from -8 to 6 by 0.2
- ADOS mean baseline: mean from n-1 subjects to predict leave-out subject. We also looked at random noise which performed worse than this baseline.

	RMSE	ADOS mean	$K^{\text{TDA}}$	$K^{\text{cor}}$
ADOS mean	6.4302	-	-	-
$K^{\text{TDA}}$	6.3553	0.316	-	-
$K^{\text{cor}}$	6.0371	0.055	0.095	-
$K^{\text{TDA+cor}}$	<b>6.0156</b>	<b>0.048</b>	0.075	0.288

We used permutation tests to get a p-value for the significance of the results.

### Acknowledgements

This work was supported by NSF grants IIS-1513616 and IIS-1251049.