Filtering in the spatial domain

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Outline

Introduction

Concepts of image filtering

Smoothing filters

Sharpening filters

Outline

Introduction

- Filtering is a fundamental signal processing operation, and often a pre-processing operation before further processing.
- Applications:
    - Image denoising
    - Image enhancement (i.e., "make the image more vivid")
    - Edge detection
    - ...

Outline

Basic idea

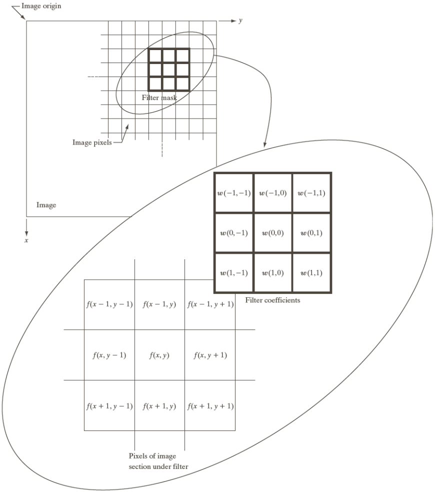


FIGURE 3.28 The mechanics of linear spatial filtering using a 3×3 filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.

Convolution

- The filtered image is the convolution of the original image with the filter impulse response (or “mask”).
- So, if $f(x, y)$ denotes the original image and $w(x, y)$ the filter impulse response, then their convolution is

$$\begin{aligned} f'(x, y) &= (w * f)(x, y) \\ &= \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x - s, y - t). \end{aligned}$$

Note that, with respect to the indices (s, t), this means that either the impulse response or the original image (original image in the equation) is mirrored vertically and horizontally before the pixel-wise product and sum.

Correlation

- The book mention the concept of correlation, which is somewhat similar to convolution.
- Mathematically is

$$(w \star f)(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t).$$

  (No mirroring involved!)
- Differences to statistics:
  - This is called cross-correlation in statistics.
  - Here, the normalizing factor can be neglected.
- This concept is important for applications such as matched filtering.

Convolution vs. Correlation

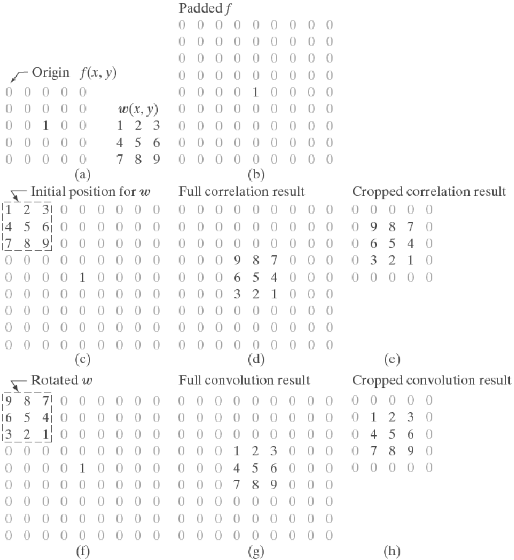


FIGURE 3.30 Correlation (middle row) and convolution (last row) of a 2-D filter with a 2-D discrete, unit impulse. The 0s are shown in gray to simplify visual analysis.

Separable filters I

- Separable filters represent a subset of all possible linear filters.
- An image filter is separable if it can be expressed as the outer product of two vectors.
- Example,

$$h = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 2 \end{bmatrix} = \mathbf{u}\mathbf{v}^T$$

Separable filters II

- Thus, separable filters can be applied in two steps:
  1. Filter along rows,
  2. Filter along columns

  (or vice-versa).
- The advantage is reduced computation.
  For an $M \times N$ image and an $P \times Q$ mask,
  - Direct approach is $\mathcal{O}(MNPQ)$,
  - Separable approach is $\mathcal{O}(MN(P+Q))$.

Outline

Averaging filters I

- Basic idea: replace each pixel by the average of the pixels in a square window surrounding the pixel.
- Example: for a 3×3 averaging filter,

$$f'(x, y) = \frac{1}{9} \sum_{s=-1}^{1} \sum_{t=-1}^{1} f(x - s, y - t).$$

- Extends the idea of "moving average" for images.
- This means that the mask is constant,
  with all values equal to 1/9 in this case.

Averaging filters II

- <u>General case:</u> For an $n \times n$ averaging filter,

$$w(x,y) = \frac{1}{n^2} \underbrace{\left.\begin{bmatrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 1 \end{bmatrix}\right\} n \text{ rows.}}_{n \text{ columns}}$$

  where, typically, n is an odd number.

- The elements of the mask must sum to one!
  This applies to any filter that we see. (why?)

Averaging filters: examples I

Averaging with a 3×3 averaging filter:

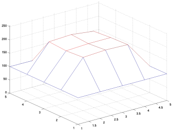
Original image

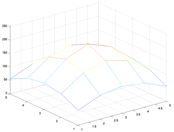
100	100	100	100	100
100	200	205	203	100
100	195	200	200	100
100	200	205	195	100
100	100	100	100	100

$\longrightarrow$

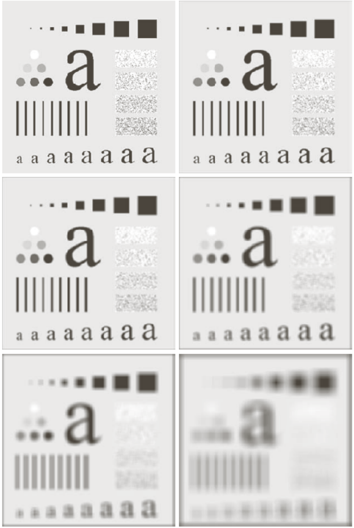
Filtered image

56	89	101	90	56
88	144	167	145	89
99	167	200	168	100
88	144	166	144	88
56	89	100	89	55





Averaging filters: examples II



a b
c d
e f

FIGURE 3.33 (a) Original image, of size 500×500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50×120 pixels.

Averaging filters: summary of properties

- Averaging filters can be applied for image denoising since the image pixel values change slowly but noise is a wide band signal (see previous figure).
- This filters blur image edges and other details.
  - This means that for image denoising there is a trade-off between noise remove capability and blurring of image detail.
  - Larger windows remove more noise but introduce more blur.
- Fundamentally, an averaging filter is a low-pass filter.

Weighted averaging filters

- Instead of averaging all the pixels in a window equally, give the pixels a weight inversely proportional to the distance to the center of the window.
- Example of a 3×3 weighted mask,

$$w(x, y) = \frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}.$$

  (Again, notice that weights sum to one.)
- Still, a low-pass filter. However, better behaved. (Think of rectangular and Hamming window in 1-D signal processing, for example.)

Generating smoothing filters I

- Examples of smoothing filters:

$$\frac{1}{10}\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 1\end{bmatrix}, \quad \frac{1}{16}\begin{bmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{bmatrix}, \quad \frac{1}{(b+2)^2}\begin{bmatrix}1 & b & 1\\ b & b^2 & b\\ 1 & b & 1\end{bmatrix}.$$

- Criteria for designing a smoothing filter:
  - $h(s,t) \geq 0$, so that it functions as averaging,
  - $\displaystyle\sum_{s=-s_0}^{s_1}\sum_{t=-t_0}^{t_1} h(s,t) = 1$, to preserve the dynamic range.

Generating smoothing filters II

- Designing a Gaussian smoothing mask:
  1. Create a distance matrix to the center of the mask:

$$d(x,y) = \begin{bmatrix} \ddots & & \vdots & & \\ & \sqrt{2} & 1 & \sqrt{2} & \\ \cdots & 1 & 0 & 1 & \cdots \\ & \sqrt{2} & 1 & \sqrt{2} & \\ & & \vdots & & \ddots \end{bmatrix}.$$

  2. Apply the Gaussian function to the matrix,

$$h'(x,y) = \exp\left[-d(x,y)^2/(2\sigma^2)\right].$$

  3. Normalize: $h(x,y) = h'(x,y) \Big/ \left(\sum_s \sum_t h'(s,t) \right).$

Nonlinear filters: introduction

- For image denoising (and other applications), the blurring associated with linear filters is undesired.
- Moreover, linear filters are ineffective to remove some types of noise; e.g., impulsive noise.

- Solution: use nonlinear filters.

Median filter I

- The simplest and most widely used filter to deal with impulsive noise is the median filter.
- Idea: The filtered pixel value is the median of the values in a window centered on the pixel.
- Example: Median filter with a 3×3 filter,

Original image

100	100	100	0	100
100	100	200	200	200
100	255	200	200	200
100	100	200	0	200
100	100	200	200	200

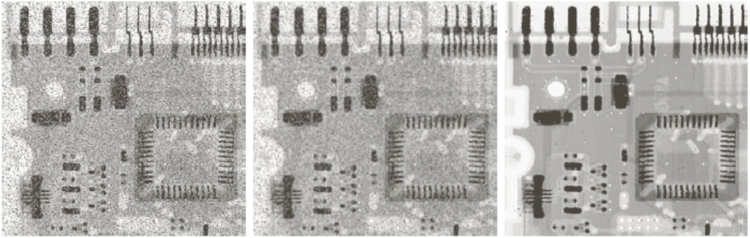
$\longrightarrow$

Filtered image

100	100	100	100	100
100	100	200	200	200
100	100	200	200	200
100	100	200	200	200
100	100	200	200	200

Median filter II

- Comparison:



a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Outline

Image sharpening I

- The main idea of sharpening is the enhance line structures of other details in an image.
- Thus, the enhanced image contains the original image with the line structures and edges in the image emphasized.

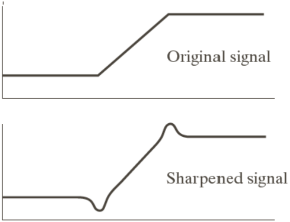


Image sharpening II

- Line structures and edges can be obtained by applying a difference operator (an high-pass filter) on the image.
- Resulting operation is a weighted averaging operation, in which some weights will be negative (why?).

Designing an isotropic high-pass filter

- Criterion for designing an high-pass filter:

$$\sum_{s=-s_0}^{s_1} \sum_{t=-t_0}^{t_1} h(s,t) = 0.$$

- A filter is isotropic if it is rotation invariant.
- Examples of isotropic high-pass filters:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Laplacian operator

- The Laplacian operator is an approximation to the second-order derivative of an image,

$$
\begin{aligned}
\nabla_f^2 &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \\
&\approx [f(x+1,y) + f(x-1,y) + f(x,y-1) + f(x,y+1)] \\
&\quad - 4f(x,y).
\end{aligned}
$$

- Typical implementations:

$$
\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad
\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}, \quad
\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}.
$$

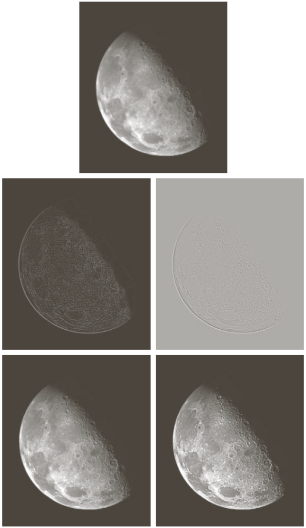
Image enhancement with Laplacian operator I

- “Add” result of filtering with Laplacian operator to image. In general,

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{if center of Laplacian mark is negative} \\ f(x,y) + \nabla^2 f(x,y) & \text{if center of Laplacian mark is positive} \end{cases}$$

- Why? It reinforces line and edges detail.

Image enhancement with Laplacian operator II



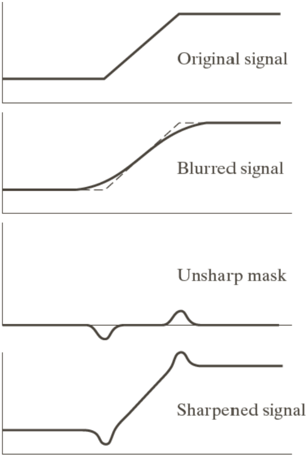
a
b c
d e

FIGURE 3.38 (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

Unsharp masking I

- Idea: Sharpen images using low-pass filters!
- How?
    1. Blur the original image
    2. Subtract the blurred image from the original
    3. Add the previous result to the original.

Unsharp masking II



a
b
c
d

FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

Is this a new concept?

- What is the difference to the approach using the Laplacian operator?

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- Why?

Is this a new concept?

- What is the difference to the approach using the Laplacian operator? Conceptually, they are the same!
- Why?
  Because the first two steps, blur the image and subtract the blurred image from the original, implement a high-pass filtering operation.

Reading

- Sections 3.4 through 3.6 of the textbook.