Problem A:

An Internet Service Provider (ISP) is assessing its service and decides that the probability of losing a packet p for good service must not be greater than 0.05. Hence, they want to test if $p > 0.05$. They send 20 packets through their network and count the number of lost packets at the other end, denoted X. If $X \geq 3$ the service will be deemed not appropriate.

a) Specify the hypothesis and draw a diagram illustrating the critical region.

b) Compute α.

c) " β for the alternative hypothesis $p = 0.1$.

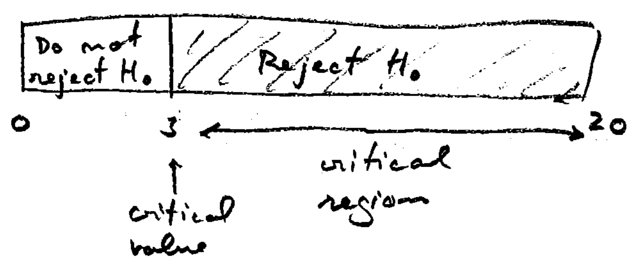
d) Suppose a group of 300 packets is sent. H_0 will be rejected if the number of packets lost $X \geq 22$. Compute α and β, using the standard normal distrib.

$p=0.1$

Solution:

a) $H_0: p = 0.05$

$H_1: p > 0.05$



b) $\alpha = P(\text{type I error}) = P(X \geq 3 \text{ when } p = 0.05)$

$$= \sum_{i=3}^{20} b(i; n=20, p=0.05) = 0.0755$$

note

$$\alpha = 1 - P(X < 3 \text{ when } p = 0.05)$$

$$= 1 - \sum_{i=0}^{2} b(i; n=20, p=0.05) = 1 - 0.9245 = 0.0755$$

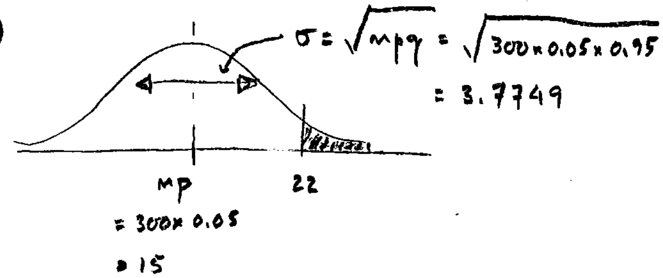
c) $\beta = P(\text{type II error}) = P(X < 3 \text{ when } p = 0.1)$

$$= \sum_{i=0}^{2} b(i; n=20, p=0.1) = 0.6769$$

d) $\alpha = P(X \geq 22 \text{ when } p = 0.05)$

$$= P\left(Z \geq \frac{22 - np}{\sqrt{npq}}\right)\Bigg|_{p=0.05}$$

$$= P(Z \geq 1.8543) = 0.0318$$



$\beta = P(X < 22 \text{ when } p = 0.1)$

$$= P\left(Z < \frac{22 - 300\times 0.1}{\sqrt{300\times 0.1\times 0.9}}\right) = P(Z < -1.5396) = 0.0618$$

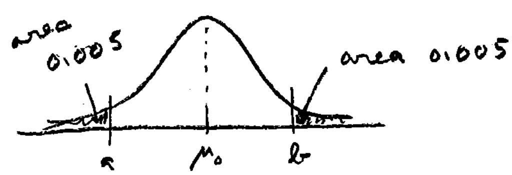
Problem B:

A corporation maintains a large fleet of company cars. To check the average number of miles driven per month per car, a random sample of $n = 40$ cars is examined. The mean of the sample is 2,752 miles, but records for previous years indicate that the average number of miles driven per car per month was 2,600. Test if the current mean differs from 2,600, for a significance level of $\alpha = 0.01$ and standard deviation $\sigma = 350$.

Solution:

$H_0: \mu = 2600$

$H_1: \mu \neq 2600$



$$\alpha = P(z < -z_{0.005}) + P(z > z_{0.005})$$

$$z = \frac{\mu - \mu_0}{\sigma/\sqrt{n}} \Rightarrow \mu < \mu_0 - z_{0.005}\frac{\sigma}{\sqrt{n}} = a$$

$$\mu > \mu_0 + z_{0.005}\frac{\sigma}{\sqrt{n}} = b$$

$$a = 2600 - 2.575\frac{350}{\sqrt{40}} = 2457.5$$

$b = 2742.5$

$\therefore$ Reject H_0.

Q. Could H_0 be rejected at a significance level $\alpha = 0.05$?
Yes. $\alpha = 0.01$ is a stricter criterion anyway.