Math 6630: Numerical Solutions of Partial Differential Equations

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Course overview

Math 6630: Numerical Solutions of Partial Differential Equations

- In-person (unless otherwise specified)
- Grading based solely on two projects and one end-of-term presentation
- Some core topics covered in class ($\sim$10 weeks), some topics based on student interest ($\sim$2-3 weeks)

Most non-technical information here and in what follows is on the syllabus.

Course topics

A rough outline of what we'll discuss in the "core" topics:

- Background: Basic theory of linear partial differential equations
- Examples, and elliptic, parabolic, hyperbolic problems
- Desiderata for numerical schemes – stability, convergence, efficiency
- Finite-difference methods
- Weighted residuals methods
- Finite element methods
- Global Fourier and spectral methods
- Conservation laws and finite volume methods
- Discontinuous Galerkin methods

Throughout, we'll discuss theory, algorithms, and software considerations.

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Grading: Projects and presentation

Goal: Complete some exercises that are of interest to you.

Projects:

- Essentially homework assignments, with options for written or coding exercises
- Fairly open-ended, **not** explicitly based on practicing concepts discussed in class

Presentation: A $\sim$30-minute presentation at the end of the semester (last few weeks of classes) on a topic of your choice. (I will provide potential topics and guidance.)

Presentation examples:

- Survey numerical methods not discussed in class (e.g., radial basis function, or deep learning-based methods)
- Present a focused research paper on a topic of interest/relevance to you
- Utilize a software package to solve some PDEs, discuss algorithmic backend and coding architecture

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General guidelines

Re: COVID-19, the U encourages

- vaccination + boosting
- routine testing
- masking

We are, unfortunately, still in this pandemic.

- In extenuating circumstances, we'll move class online (Zoom)
- Please isolate appropriately if you contract COVID

General class communication: email is the best way to contact me outside of class. I will send any general announcements through email.

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Theme of this class

Numerical Solutions to Partial Differential Equations, or "Topics in Numerical Methods"

While historically this class might have focused on specific topics, we'll go the generalist route this semester.
In particular, the curriculum is not necessarily fixed and crystallized.

Major goals (and warnings) for this class:

- To provide an overview/survey of numerical methods
- Not to provide substantial in-depth knowledge of any particular method
- To give you general exposure of existing mathematics, tools, and formalisms
- To present you with resources where you could find more detailed information
- Not to provide (important!) mathematical or algorithmic minutiae
- To give an elementary understanding of how PDEs are numerically solved

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Accompanying References

There is no formal textbook for this course.

I'll provide slides (like these) that are a rough outline of topics covered. I apologize in advance for the highly probable typos and mistakes

There are numerous textbooks that I'll draw from for material (see next slide), and I'll identify appropriate texts during class meetings.

References I

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- Langtangen, Hans Petter and Svein Linge (2017). *Finite Difference Computing with PDEs: A Modern Software Approach*. Springer. ISBN: 978-3-319-55456-3.
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