Department of Mathematics, University of Utah
Numerical Solutions of PDE
MATH6630 – 001 – Spring 2023

Project guidelines and potential papers

A required component of this course is the in-class presentation of either a paper, or a summary of a project that you are investigating. Potential types of presentations are:

- Overview of a research paper (see below for suggestions). This includes a high-level summary of the problem the paper is attempting to solve, the main ideas of the approach, a description of the numerical results (if appropriate), and a discussion of the advantages/disadvantages and open problems or future directions from this work. You need not be an expert in the content of the paper, and you need not fully understand all technical details, but you absolutely should be well-versed enough to present the paper and answer some basic questions. You are encouraged to read related references as well to gain a fuller picture of the landscape of the paper's focus.
- Presentation of a project you have pursued during this semester. "Project" is loosely defined – it can be a numerical implementation of an algorithm to solve a problem, an empirical investigation of several algorithms, a presentation of theory that you have learned, etc. The major goal is to have a well-crafted message and presentation.

In terms of topics, I am quite flexible – anything related to numerical methods for PDEs is fine. See below for some paper suggestions. (They are only suggestions, you may choose other options.) **In general I am happy to meet to discuss anything related to these projects**, including deciding on a topic, settling on what to present, how much detail to present, etc. Please do reach out if you'd like to discuss at any point.

Timeline

Here is a proposed timeline for these presentations:

- Now until April 14 – Decide on a topic/paper and finalize a presentation date.
- April 18 - April 24 – Presentations. I plan on 2-3 per class meeting.

You can interpret the above to mean that **April 14 is the deadline for deciding on a project topic + presentation date**. I am aiming to have two or three presentations per class meeting, and I will fill people into slots on a first-come first-served basis.

Presentation "Guidelines" and Preparation

I am leery of providing rules on presentations since I don't want to impose too many constraints. But here is a general outline of what I expect:

- Presentation length: 20-25 minutes, as this is probably a reasonable amount of time to adequately give background on the project. As a general rule of thumb, most folks generally underestimate the amount of time they need to speak.

- Presentation format: you are welcome to use electronic slides, or present on the whiteboard. (We do not have a chalkboard in our classroom.) I would recommend slides if you're going to show numerical results/pictures. The class has an HDMI input, and I will always have a USB-C adapter + HDMI cable. If you need other adapters, please let me know as soon as you can so I can arrange to have them available. If you prefer to present virtually over Zoom, please let me know well ahead of time so we can iron out logistics for that.
- I encourage you to share with me either a draft of your slides or an outline of your presentation a few days before your talk. If you choose to do this then I'll give some feedback on your slides.

Potential papers/topics

The ultimate goal of this is for you to present on a topic that you are excited about. Below I list some topics that either I am familiar with, or would be interesting for me, but you should prioritize your own interest in choosing your topics. In particular, do not consider the list below as a comprehensive collection of materials: There exist appropriate references that I have not listed in relevant sub-topics below, and there exist entire sub-topics that I have omitted.

- Radial basis function methods
  - [5], *Preconditioning for Radial Basis Function Partition of Unity Methods*
  - [9], *A Radial Basis Function (RBF)-Finite Difference (FD) Method for Diffusion and Reaction–Diffusion Equations on Surfaces*
- Physics-informed neural networks
  - [4], *Physics-Informed Graph Neural Galerkin Networks: A Unified Framework for Solving PDE-governed Forward and Inverse Problems*
  - [8], *Physics-Informed Neural Networks: A Deep Learning Framework for Solving Forward and Inverse Problems Involving Nonlinear Partial Differential Equations*
- Discontinuous Galerkin methods
  - [6], *Nodal High-Order Methods on Unstructured Grids: I. Time-Domain Solution of Maxwell's Equations*
  - [12], *An Optimization Based Discontinuous Galerkin Approach for High-Order Accurate Shock Tracking*
  - [2], *The Development of Discontinuous Galerkin Methods*
- Particle-in-cell methods
  - [11], *The Particle-In-Cell Method*
- Lagrangian/semi-Lagrangian methods
  - [10], *Semi-Lagrangian Integration Schemes for Atmospheric Models—A Review*
  - [3], *Conservative Semi-Lagrangian Schemes for Vlasov Equations*
- Learning PDEs from data
  - [7], *PDE-Net: Learning PDEs from Data*
  - [1], *Data-Driven Discovery of Coordinates and Governing Equations*

References

[1] Kathleen Champion et al. “Data-Driven Discovery of Coordinates and Governing Equations”. In: *Proceedings of the National Academy of Sciences* 116.45 (Nov. 5, 2019), pp. 22445–22451. ISSN: 0027-8424, 1091-6490. DOI: 10.1073/pnas.1906995116. pmid: 31636218. URL: https://www.pnas.org/content/116/45/22445.

[2] Bernardo Cockburn, George E. Karniadakis, and Chi-Wang Shu. “The Development of Discontinuous Galerkin Methods”. In: *Discontinuous Galerkin Methods*. Ed. by Bernardo Cockburn, George E. Karniadakis, and Chi-Wang Shu. Lecture Notes in Computational Science and Engineering. Berlin, Heidelberg: Springer, 2000, pp. 3–50. ISBN: 978-3-642-59721-3. DOI: 10.1007/978-3-642-59721-3_1.

[3] Nicolas Crouseilles, Michel Mehrenberger, and Eric Sonnendrücker. “Conservative Semi-Lagrangian Schemes for Vlasov Equations”. In: *Journal of Computational Physics* 229.6 (Mar. 20, 2010), pp. 1927–1953. ISSN: 0021-9991. DOI: 10.1016/j.jcp.2009.11.007. URL: https://www.sciencedirect.com/science/article/pii/S0021999109006263.

[4] Han Gao, Matthew J. Zahr, and Jian-Xun Wang. “Physics-Informed Graph Neural Galerkin Networks: A Unified Framework for Solving PDE-governed Forward and Inverse Problems”. In: *Computer Methods in Applied Mechanics and Engineering* 390 (Feb. 2022), p. 114502. ISSN: 00457825. DOI: 10.1016/j.cma.2021.114502. arXiv: 2107.12146. URL: http://arxiv.org/abs/2107.12146.

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[9] Varun Shankar et al. “A Radial Basis Function (RBF)-Finite Difference (FD) Method for Diffusion and Reaction–Diffusion Equations on Surfaces”. In: *Journal of Scientific Computing* 63.3 (Sept. 6, 2014), pp. 745–768. ISSN: 0885-7474, 1573-7691. DOI: 10.1007/s10915-014-9914-1. URL: http://link.springer.com/article/10.1007/s10915-014-9914-1.

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