# Partial Differential Equations

MATH 6610 Lecture 00

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Types of equations:

- *Algebraic* equations: Solve for $x$:

$$x^2 - 4 = 0$$

- (Ordinary) *differential* equations: Solve for $y(x)$:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3y$$

- *Partial differential* equations: Solve for $u(x,t)$:

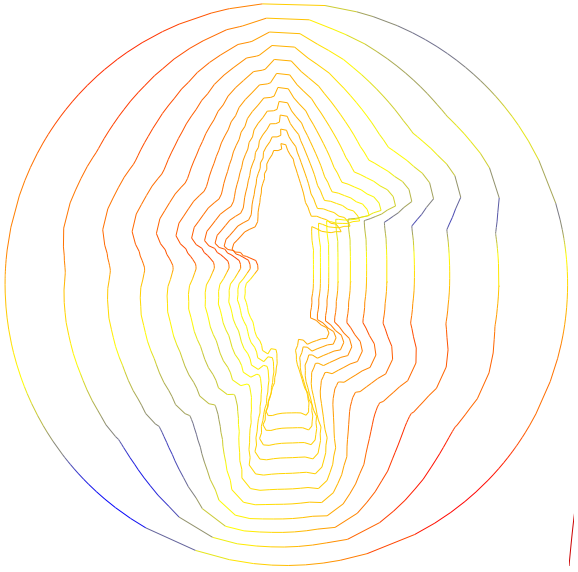
$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}.$$

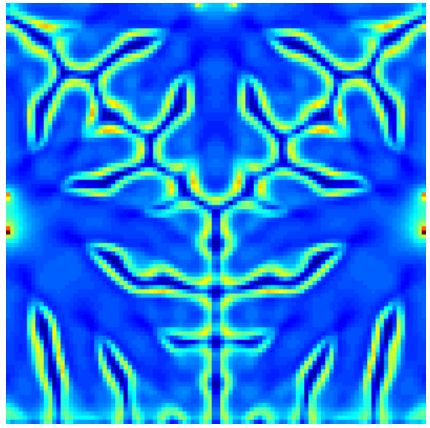
# Partial Differential Equations (PDEs)

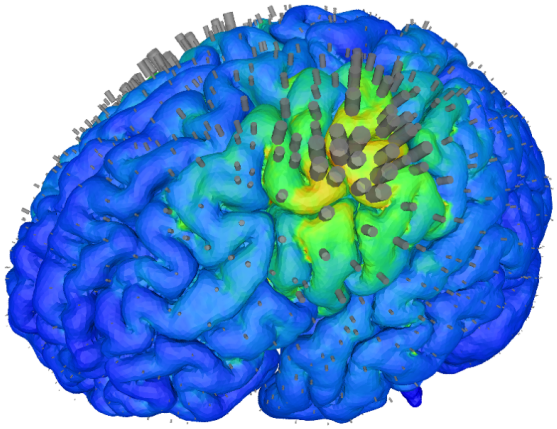
PDEs are, essentially, *mathematical models*.

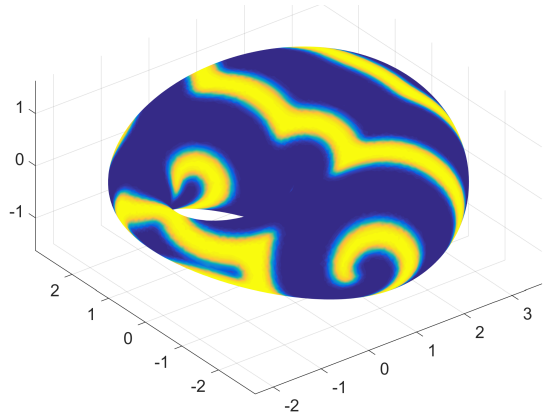
- Astronomical/cosmological models
- Biophysical models
- Chemical flows and reactions
- Data analysis and clustering
- Fluid dynamics
- Imaging
- Neurological models
- Optimization and design
- Population dynamics, swarm behavior
- Structural mechanics/dynamics

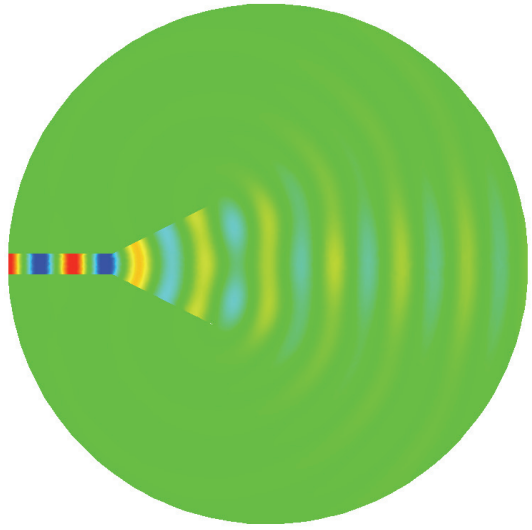
# PDE applications









# Scope of this class

This class is a first look into PDEs.

Specifically, we consider the following linear PDEs:

- The heat equation: $u_t = u_{xx}$
- Laplace's equation: $u_{xx} + u_{yy} = 0$
- The wave equation: $u_{tt} = u_{xx}$

We are interested in (a) solving these PDEs, (b) understanding what kind of behavior these PDEs model.

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There are many more PDEs that we don't cover in this class.

# Prerequisites

We assume some background:

- Fluency in calculus (derivatives+integrals of common functions, $u$-substitution, integration by parts, ...)
- Familiarity with ordinary differential equations (simple harmonic oscillators)

This class will be difficult without knowledge of the above.