Department of Mathematics, University of Utah
Anaylsis of Numerical Methods I
MTH6610 – Section 001 – Spring 2019

Homework 0
Basics of submission

Write a program that uses a Monte Carlo technique to estimate π.

Let $\rho : \mathbb{R}^d \to \mathbb{R}$ be a given probability density and let $f : \mathbb{R}^d \to \mathbb{R}$ be a given function. If X is a random variable distributed according to ρ, then a Monte Carlo estimate of $\mathbb{E}[f(X)]$ using a size-M ensemble is the approximation,

$$\mathbb{E}[f(X)] \coloneqq \int_{\mathbb{R}^d} f(x)\rho(x)\,\mathrm{d}x \approx F_M =: \frac{1}{M}\sum_{m=1}^{M} f(X_m), \tag{1}$$

where $(X_m)_{m=1}^M$ are independent random variables distributed according to the density ρ. Note that F_M itself is a random variable.

Let $R = \left\{(x_1, x_2) \in \mathbb{R}^2 \,\middle|\, x_1^2 + x_2^2 \le 1\right\}$ be the unit ball in $\mathbb{R}^2$. Define ρ as the uniform (probability) density on the square $[-1,1]^2 \subset \mathbb{R}^2$, i.e., $\rho(x_1, x_2) = 1/4$ if both $|x_1|$ and $|x_2|$ are at most 1, and $\rho(x_1, x_2) = 0$ otherwise. Define $f(x)$ as the characteristic function of R, i.e.,

$$f(x) = \mathbb{1}_R(x) = \begin{cases} 1, & x \in R, \\ 0, & x \notin R \end{cases}$$

Since

$$\mathbb{E}[f(X)] = \int_{[-1,1]^2} f(x)\frac{1}{4}\,\mathrm{d}x\,\mathrm{d}y = \frac{1}{4}\int_R \mathrm{d}x\,\mathrm{d}y = \frac{\pi}{4},$$

then F_M defined in (1) is an estimator for $\pi/4$.

Using your programming language of choice, write a program that computes $4F_M$ and hence estimates. (Note that from (1), to generate one instance of F_M you need only the ability to generate instances of the uniform random variable X, and to implement the function f.) Report the convergence of $4F_M$ to π as a function of M. Since F_M is random, you should report ensemble statistics of F_M (e.g., the deviation of a computed mean from π, an ensemble-computed standard deviation, etc.) Use your code to verify the central limit theorem for a sufficiently large M.

Submission instructions

These instructions are a template for all future homework assignments. You are required to submit all assignments via the version control system Git. You will do so by "pushing" updates to a remote repository. You will need to

1 Complete the assignment, including writing any code necessary to solve the problems.

2 Type up your solutions in LaTeX. Your solutions should contain as much detail as you would normally include in a handwritten assignment. Any problems requiring computer simulation should have documented results (e.g., a table or plots) in the solution document.
3 Create a git repository on your computer (the "local" repository) that contains both a LaTeX report with tables/figures as appropriate, and any source code that reproduces figures/tabular values as they appear in the solution.
4 Create an account on github (`github.com`). Do not submit your work through another remote system (such as bitbucket).
5 On Github, create a (private) repository named `math6610-homework-0` on your account (the "remote" repository). In your local repository, create a pointer to the remote repository with the `git remote` command. (Github will have instructions for how to do this.)
6 Submit your project to the remote repository with a `git push` command from your local repository. (You will need to create an SSH public/private keypair and upload your public key to bitbucket to accomplish this.)
7 Give me (username `akilnarayan`) write access to your `math6610-homework-0` github repository.

Note that you will not be submitting this assignment, but the instructions above are the same for future assignments **except** that the homework assignment number will not be 0. So, e.g., for the first assignment, you will create a repository named `math6610-homework-1`.

Submission expectations

A LaTeX solution report is *always* a component of the submission. Proofs and/or arguments are expected to be cogent, well-articulated, and well-formatted. If the assignment requires coding, then submitting the source code is also mandatory.

General repository etiquette

- If there are unusual things about the way you've organized your files, provide a `README` text file that documents the general structure of the repository.
- Provide a `makefile` that allows me to easily compile your LaTeX report. Your `makefile` should be in the top-level directory, and I should be able to compile your solution report immediately upon cloning your repository. (You can check this yourself by performing a fresh clone on your local machine and trying it yourself.)
- Your repository should in general only track ASCII-type text files (images are an exception). It should in general not track compiled binaries, pdf output from tex compilation, or datasets generated by your code. In general your repositories should be *small* in disk space, with the largest files probably being any images used in your LaTeX reports.
- General rule of thumb: if I cannot figure out how to generate your LaTeX report and/or use your computer code within 10 seconds of looking at your repository, I will return it to you for you to document things properly.

Submission structure

The base folder of your repository should contain the LaTeX file (`.tex` file) for your report. You may structure all dependent files (images, bib files, secondary tex files, etc.) in any way you choose. The base folder should contain a subfolder called `code` that contains *all* code that reproduces figures in

your LaTeX document.

LaTeX submission expectations

- Write your solutions so that someone can understand the assignment without having seen the assignment sheet. You need not be meticulous, but you should ideally write your document as if it presents solutions/proofs as found in a textbook. Please note that it is very normal for producing good scientific writing to require a **lot** of time.
- Extreme verbosity is unnecessary (and should be avoided), but enough detail should be provided so that the general idea of your solution is reasonably clear to someone educated in the course material. Any plots of tables included should be referenced in the explanatory text.
- Do not include any source code text in your report.
- Your solution report is largely an explanation of the mathematics required for your solution, possibly along with a compilation of numerical results.
- Number all figures and tables, e.g., `Figure 2`, `Table 1`, etc.

Code expectations

- All computer code should be located in the `code` subfolder.
- Include a README text file that explains the purpose of the main files in the subfolder. You should write your README file with the intention of giving a clear guide to a recipient about what they are supposed to do once they receive your code.
- Each figure/table in your report should be paired with a file in the `code` subfolder that reproduces that figure/table. E.g., if you are coding in Matlab and your report contains `Figure 2` and `Table 1`, files `figure_2.m` and `table_1.m` should exist in the `code` subdirectory, and running those files should reproduce the figures. (If using a compiled language, binaries named, e.g., `figure_2.o` and `table_1.o` should be generated by your makefile.)

Example submission

I have provided an example submission for this assignment online:
`https://github.com/akilnarayan/math6610-homework-0/`
Please check this for a demonstration of the above procedures.