# Linear systems of DE's

MATH 2250 Lecture 33
Book section 7.1

November 20, 2019

# Systems of DE's

We have previously studied $n$th-order DE's for $y(x)$ of the form:

$$F(x, y, y', y'', \ldots y^{(n)}) = 0,$$

where $F$ may be a nonlinear function of $y$ and/or its derivatives.

We have constructive methods to solve this problem when $F$ is linear in $y$ and its derivatives.

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Our focus is now on systems of DE's, i.e., DE's for *multiple* variables. For two dependent variables $y(x)$ and $z(x)$, a system takes the form

$$F(x, y, y', y'', \ldots y^{(n)}, z, z', z'', \ldots, z^{(n)}) = 0,$$
$$G(x, y, y', y'', \ldots y^{(n)}, z, z', z'', \ldots, z^{(n)}) = 0,$$

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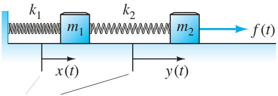
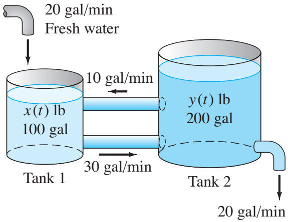
for two different functions $F$ and $G$.
In this class, we will focus exclusively on understanding *first-order* $(n = 1)$, *linear* systems of DE's.

# Motivation

Systems of DE's arise when unknowns are coupled with each other:



# Systems vs high-order scalar DE's

One key point to consider is that an $n$th order *scalar* DE can always be written as a size-$n$ *system* of first-order DE's.

## Example (Example 7.1.3)

Write the DE $x''' + 3x'' + 2x' - 5x = \sin 2t$ as a system of first-order DE's.

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## Example (Example 7.1.3)

Write the DE $x''' + 3x'' + 2x' - 5x = \sin 2t$ as a system of first-order DE's.

## Example (Example 7.1.4)

Write the following system as a system of first-order DE's:

$$\begin{aligned} 2x'' &= -6x + 2y \\ y'' &= 2x - 2y + 40 \sin 3t \end{aligned}$$

# Examples

We can also rewrite a first-order system as an $n$th order scalar DE.

## Example (Example 7.1.5)

Compute the general solution to the first-order system

$$x' = -2y$$
$$y' = \frac{1}{2}x$$

# Examples

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## Example (Example 7.1.5)

Compute the general solution to the first-order system

$$\begin{aligned} x' &= -2y \\ y' &= \frac{1}{2}x \end{aligned}$$

## Example (Example 7.1.6)

Compute the general solution to the first-order system

$$\begin{aligned} x' &= y \\ y' &= 2x + y \end{aligned}$$