Applications: mixtures and populations

MATH 2250 Lecture 06
Book sections 1.5 & 2.1

August 28, 2019

Mixture examples

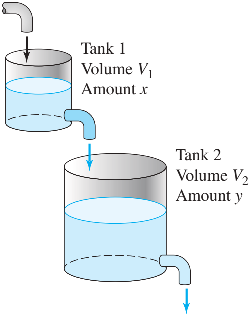
Example (Example 5, section 1.5)

A 120-gallon tank initially contains 90 lb of salt dissolved in 90 gallons of water. Brine containing 2 lb/gal of salt flows into the tank at the rate of 4 gal/min, and the well-stirred mixture flows out of the tank at the rate of 3 gal/min. How much salt does the tank contain when it is full?

Mixture examples

Example (Section 1.5, problem 39)

Suppose that in the cascade shown in the figure, tank 1 initially contains 100 gal of pure ethanol and tank 2 initially contains 100 gal of pure water. Pure water flows into tank 1 at 10 gal/min, and th eother two flow rates are also 10 gal/min. **(a)** Find the amounts $x(t)$ and $y(t)$ of ethanol in the two tanks at time $t \geqslant 0$. **(b)** Find the maximum amount of ethanol ever in tank 2.



Population models

If $P(t)$ is a population (of people, animal species, bacteria, etc.) as a function of time t, then a model for the evolution of this population is

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \beta P - \delta P,$$

where β and δ are the non-negative *birth* and *death* rates, respectively.

Both β and δ can be functions of time t and/or the population P.

Unbounded populations

If the death rate is zero, populations become unbounded.

Example (Section 2.1, problem 9)

The time rate of change of a rabbit population P is proportional to the square root of P. At time $t = 0$ (months) the population numbers 100 rabbits and is increasing at the rate of 20 rabbits per month. How many rabbits will there be one year later? What is the population as $t \to \infty$?

Bounded populations

With non-zero death rates, we can model populations with bounds. A (very) popular model for this is called *logistic* growth, where the birth and death rates are

$$\beta = \beta_0 - \beta_1 P, \qquad \delta = \delta_0,$$

where β_0, β_1, and δ_0 are all constants. This results in the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = aP - bP^2,$$

for some constants a and b that depend on β_0, β_1, and δ_0. This is called the **logistic equation**.

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Example (Section 2.1, problem 21)

Suppose that the population $P(t)$ of a contry satisfies the differential equation $\frac{\mathrm{d}P}{\mathrm{d}t} = kP(200 - P)$ with k constant. Its population in 1960 was 100 million and was then growing at the rate of 1 million per year. Predict this country's population for the year 2020. What is the population as $P \to \infty$?

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In logistic models, this finite **limiting population** is sometimes called the **carrying capacity**.