Midterm 3 **Name:** ______________________________

MATH2250, Section 04 November 8, 2019

This test is:

- closed-book
- closed-notes
- no-calculator
- 50 minutes

Indicate your answers clearly, and show your work. Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

For questions with multiple parts, **clearly indicate** your solution to each portion of the question.

Fun fact of life: if your work is not legible, I will not be able to read it. The ramifications of this outcome should be clear.

There are 4 questions with multiple parts; each question is worth a total of 10 points.

All pages are one-sided. If on any problem you require more space, use the back of the page.

DO NOT TURN THIS PAGE UNTIL DIRECTED TO BEGIN

1. (10 pts total) Compute the following forward/inverse Laplace transforms using any method you wish. The Laplace transform of $f(t)$ is defined as

$$F(s) = \int_0^\infty f(t)e^{-st}\,\mathrm{d}t,$$

and

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0), \qquad \mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

a) (3 pts) Compute the Laplace transform of $f(t) = 3t^2 + 4$.

b) (3 pts) Compute the Laplace transform of $f(t) = \sin 3t + e^{2t}$.

c) (4 pts) Compute the inverse Laplace transform of

$$F(s) = \frac{s+6}{s^2-9}.$$

2. (10 pts) Compute the solution to the following initial value problem using any method you choose.

$$y'' - 3y' + 2y = 4, \quad y(0) = 1, \;\; y'(0) = 1$$

3. (10 pts total)

a) (5 pts) Find a linear homogeneous constant-coefficient equation with the general solution:

$$y(x) = (Ax + B)\cos x + (Cx + D)\sin x$$

b) (5 pts) Consider the following model for simple harmonic motion in a mass-spring system:

$$4x''(t) + 12x(t) = 0, \quad x(0) = 1, \; x'(0) = 1.$$

Compute the solution $x(t)$, and compute the amplitude and period of motion resulting from this model.

4. (10 pts total) This question concerns particular solutions.

a) (5 pts) Compute a particular solution to the DE

$$y^{(3)} - 3y'' + 2y' = 1 + e^x$$

b) (5 pts) Compute the solution to the initial value problem,

$$x'' - 6x' + 8x = 2, \quad x(0) = 0, \;\; x'(0) = 0$$