Midterm 2
Name: $\qquad$
MATH2250, Section 04

This test is:

- closed-book
- closed-notes
- no-calculator
- 50 minutes

Indicate your answers clearly, and show your work. Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

For questions with multiple parts, clearly indicate your solution to each portion of the question.

Fun fact of life: if your work is not legible, I will not be able to read it. The ramifications of this outcome should be clear.

There are 4 questions with multiple parts; each question is worth a total of 10 points.
All pages are one-sided. If on any problem you require more space, use the back of the page.

## Do Not Turn this page until directed to Begin

1. (10 pts total) Given the following matrix and vectors:

$$
\boldsymbol{A}=\left(\begin{array}{ccc}
1 & 1 & -2 \\
0 & 1 & -2 \\
0 & 1 & 1
\end{array}\right), \quad \boldsymbol{v}=\left(\begin{array}{c}
-1 \\
1 \\
2
\end{array}\right), \quad \boldsymbol{w}=\left(\begin{array}{c}
2 \\
0 \\
3
\end{array}\right)
$$

a) (3 pts) Compute $\boldsymbol{A} \boldsymbol{v}$.
b) (3 pts) Compute $\operatorname{det}(\boldsymbol{A})$.

$$
\boldsymbol{A}=\left(\begin{array}{ccc}
1 & 1 & -2 \\
0 & 1 & -2 \\
0 & 1 & 1
\end{array}\right), \quad \boldsymbol{v}=\left(\begin{array}{c}
-1 \\
1 \\
2
\end{array}\right), \quad \boldsymbol{w}=\left(\begin{array}{c}
2 \\
0 \\
3
\end{array}\right)
$$

c) (4 pts) Compute the solution $\boldsymbol{x}$ to $\boldsymbol{A} \boldsymbol{x}=\boldsymbol{w}$.
2. (10 pts total) Define $\boldsymbol{B}$ and $\boldsymbol{d}$ as

$$
\boldsymbol{B}=\left(\begin{array}{cccc}
1 & 1 & 2 & 1 \\
1 & -1 & 0 & 3 \\
3 & 1 & 4 & 6
\end{array}\right), \quad \boldsymbol{d}=\left(\begin{array}{c}
1 \\
0 \\
1
\end{array}\right)
$$

Compute the reduced Echelon form of the augmented coefficient matrix for this system and use it to compute all solution(s) $\boldsymbol{x}$ to $\boldsymbol{B} \boldsymbol{x}=\boldsymbol{d}$.
3. (10 pts total) This question concerns vector spaces and subspaces.
a) ( 4 pts ) Let $V$ be the set of points $\boldsymbol{x}=\left(x_{1}, x_{2}, x_{3}, x_{4}\right)$ in $\mathbb{R}^{4}$ satisfying

$$
\begin{aligned}
& x_{1}+x_{2}=x_{3} \\
& x_{1}-x_{3}=0
\end{aligned}
$$

Is $V$ a subspace? If so, prove it. If not, demonstrate why it is not a subspace.
b) ( 3 pts ) Let $W$ be the set of points $\boldsymbol{x}=\left(x_{1}, x_{2}, x_{3}, x_{4}\right)$ in $\mathbb{R}^{4}$ satisfying

$$
x_{1}+x_{4} \geq 0
$$

Is $W$ a subspace? If so, prove it. If not, demonstrate why it is not a subspace.
c) ( 3 pts ) Find a basis for the solution space in $\mathbb{R}^{3}$ for the following set of linear equations:

$$
\begin{array}{r}
x_{1}-2 x_{2}+3 x_{3}=0 \\
x_{1}-3 x_{2}+x_{3}=0
\end{array}
$$

4. ( 10 pts ) A motorboat is moving at $16 \mathrm{~m} / \mathrm{s}$ when its motor suddenly quits. 2 seconds later, the boat has slowed to $4 \mathrm{~m} / \mathrm{s}$. Assume that the resistance it encounters is proportional to $v^{3 / 2}$, where $v$ is the velocity in $\mathrm{m} / \mathrm{s}$. How far will the boat coast in all?
