Math 2250 Lab 9

14 November 2019

Name: ____________________

uNID: ____________________

Instructions and Due Date:

- **Due:** 21 November 2019 at the start of the lab
- Work together!
- My office hours for this week: Mon 9:30am-10:30am WEB 1705 Tues 9am-10am LCB basement Math Center, 10am-11am WEB 1705,
- Additional help for the lab: Lab TA's will be in WEB 1705 M, T, W, Th from 8:30am-1pm. If you go there outside of my office hours, you can ask another section's TA for help. Otherwise, you can ask for help from tutors in the Math Center in the LCB basement.

1. Recall the definition of the Laplace transform:

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt.$$

For each of the following functions, compute the Laplace transform, and state for which values of s the transform exists.

(a) $f(t) = 1$ (constant function)

(b) $f(t) = e^{at}$, a a constant

(c) $f(t) = \sin(at)$, a a constant

2. The convolution of f and g is defined as the integral of the product of the two functions as following:

$$(f * g)(t) = \int_0^t f(t-\tau)g(\tau)d\tau = \int_0^t f(\tau)g(t-\tau)d\tau.$$

If $F(s) = \mathcal{L}(f(t))$ and $G(s) = \mathcal{L}(g(t))$, then we have $\mathcal{L}^{-1}(F(s)G(s)) = (f * g)(t)$.

Compute the inverse Laplace transform of each function of s.

(a) $F(s) = \dfrac{4}{s^5}$

(b) $F(s) = \dfrac{s+8}{s^2+4}$

(c) $F(s) = \dfrac{s}{(s^2+a^2)^2}$ (*Hint: Use the convolution theorem and trig identities*)

3. **Review Problem: Undetermined Coefficients (...In Reverse)**
Find a second order linear equation

$$L(y) = f(t) \tag{1}$$

with constant coefficients whose general solution is:

$$y = C_1e^{2t} + C_2e^{6t} + te^{3t} \tag{2}$$

(a) The solution contains three parts, so it must come from a nonhomogeneous equation. Using the two terms with undefined constant coefficients, find the characteristic equation for the homogeneous equation.

(b) Using the characteristic equation find the homogeneous differential equation. This should be the $L(y)$ we're looking for.

(c) Since we have used two terms from the general solution, let's turn our focus to the non-homogeneous part, y_p. Find y_p' and y_p'' and plug them into the homogeneous equation to find the $f(t)$ we're looking for.

(d) Find the second order linear equation with the given general solution $L(y) = f(t)$.