Math 2250Lab 3

12 Sep 2019

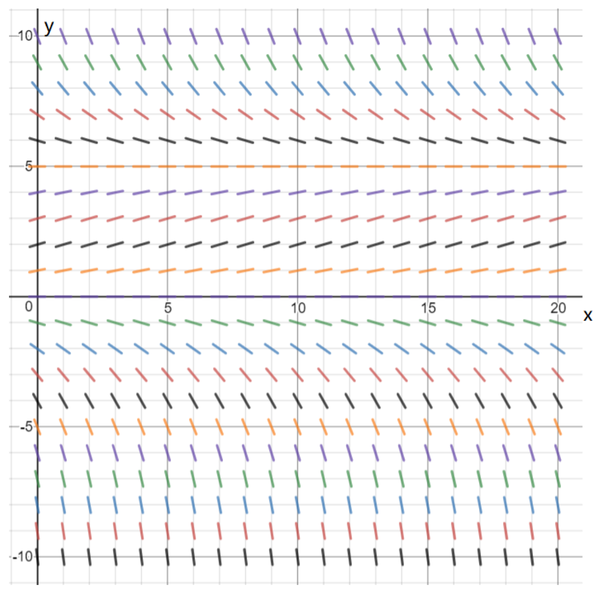
Name: ______________________

uNID: ______________________

<u>Instructions and due date:</u>

- **Due:** 19 Sep 2019 at the start of class.
- If extra paper is necessary, please staple it at the end of the packet.
- For full credit: Show all of your work, and simplify your final answers.
- Work together! However, your work should be your own (not copied from a group member).
- If you have any questions, please email me or visit me during office hours.
  Email address: jiao@math.utah.edu, office hour: Wednesday 10:00-12:00 am in WEB 1705.

1. (a) The following graph is the slope field of a logistic equation:



Using only information from the slope field, find all equilibrium solutions to the differential equation, draw the phase diagram and tell whether the equilibrium solutions are stable, unstable or semistable.

(b) Consider the differential equation

$$\frac{dx}{dt} = x^4 - x^3 - x^2 + x.$$

Find the critical points, equilibrium solutions, and draw the phase diagram. Clearly label each critical point/equilibrium as either stable "S", unstable "U", or semi-stable "SS". (**Hint:** in order to factor the right hand side of the equation, note that $x = 0$ and $x = 1$ are 2 roots of the polynomial, and you can use long division of polynomials.)

(c) Plot the slope field and 5 representative solution curves, including all equilibrium solutions. Set the $t-$axis range to go from -5 to 5 and the $x-$axis range to go from -5 to 5. (You can either draw the graph by hand or use softwares or online resources. For example, Geogebra and Wolfram Alpha are possible choices for plotting the slope field. If you choose to draw by hand, make sure your graph is reasonably rigorous.)

2. The growth of a certain bacteria in a reactor is assumed to be governed by the logistic equation:

$$\frac{dP}{dt} = k \cdot P(M - P)$$

where P is the population in millions and t is the time in days. Recall that M is the carrying capacity of the reactor and k is a constant that depends on the growth rate.

(a) Suppose that the carrying capacity of the reactor is 10 million bacteria, and that the peak growth rate is 3 million bacteria per day. Determine the constants k and M in the above equation.

(b) Supposing the bioreactor has 250,000 bacteria in it to begin with, find the number of bacteria in the tank. how long will it take for the population to reach 50% of the carrying capacity?

(c) Suppose that bacteria are being harvested from the tank continuously. Let h be the rate at which the bacteria are harvested in millions per day. Write down the new differential equation governing the bacteria population. What is the maximum rate of harvesting h that will not cause the population of bacteria to go extinct? (Below this rate there will always be a stable equilibrium point where P is positive).

3. **Comparing Growth Equations**
Growth models are frequently used to study biological topics. Three classic models are given below:

$$\text{Exponential growth:} \quad \frac{dx}{dt} = rx$$

$$\text{Von Bertalanffy growth:} \quad \frac{dx}{dt} = r(K - x)$$

$$\text{Logistic growth:} \quad \frac{dx}{dt} = (\frac{r}{K})x(K - x)$$

where r represents a growth rate and K represents a carrying capacity. In this exercise you will compare the behavior of these models with actual biological data and explore how these three models relate to one another.

(a) Find a general solution to each growth equation using arbitrary r and K.

(b) Now find an expression for the particular solution to each growth equation with $r = 0.5$, $K = 5 \times 10^5$, and $x(0) = 1 \times 10^4$.

(c) Using the math program of your choice, plot these particular solutions on the same set of axes. Staple your plot to the end of the lab.

(d) The values r and K in part (a) were chosen to fit the control data (denoted by diamonds) in the figure below. To obtain this data, the authors of the paper grew tumor cells in a controlled environment and measured how the number of cells grew over time. Which growth model do you think captures the growth behavior of the control group best and why?

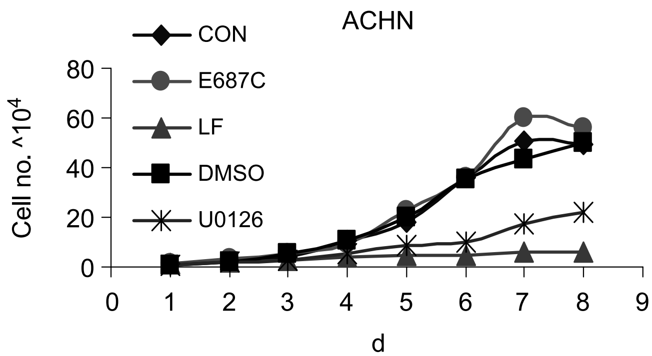


Figure 1: *
Figure 2c in 'Inhibition of MAPK Kinase Signaling Pathways Supressed Renal Cell Carcinoma Growth and Angiogenesis in vivo' by Huang et al.

(e) The logistic growth model has an inflection point. Find this inflection point by differentiating both sides of the differential equation with respect to t, then setting $\frac{d^2x}{dt^2} = 0$ and solving for x.

(f) Now look back to the diagram you made in part (c). Which growth model does the logistic curve look like before the inflection point? Which growth model does it look like after it reaches the inflection point?