Midterm 2 **Name:** ______________________

MATH3150, Section 04 March 14, 2018

This test is:

- closed-book
- closed-notes
- no-calculator
- 80 minutes

Indicate your answers clearly, and show your work. Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

Fun fact of life: if your work is not legible, I will not be able to read it. The ramifications of this outcome should be clear.

There are 3 questions with multiple parts; each question is worth 20 points. Thus the entire exam is scored out of 60 points.

All pages are one-sided. If on any problem you require more space, use the back of the page.

Do not turn this page until directed to begin

1. (20 pts total) This question involves sketching Fourier Series and computing coefficients on the interval $x \in [-L, L]$.

Your sketches **must** include (i) illustration of behavior *outside* the interval $[-L, L]$ (along with behavior inside this interval), (ii) explicit markings showing jump discontinuities and the value of the series at those discontinuous locations.

a.) (5 pts) Sketch the Fourier Series for the function

$$f(x) = x$$

b.) (5 pts) Sketch the Fourier Series for the function

$$f(x) = \begin{cases} -1, & x \geq 0 \\ 1, & x < 0 \end{cases}$$

c.) (10 pts) Compute the Fourier Series coefficients for the function

$$f(x) = \begin{cases} -1, & x \geq 0 \\ 1, & x < 0 \end{cases}$$

You must compute the required integrals, and simplify the result to a reasonable extent.

2. (20 pts) Consider the wave equation,

$$u_{tt} = c^2 u_{xx}, \qquad\qquad 0 < x < L, \quad t > 0$$

along with the boundary conditions,

$$\begin{aligned} \frac{\partial u}{\partial x}(0,t) &= 0 \\ u(L,t) &= 0. \end{aligned}$$

which models a vibrating string with one endpoint fixed and another free. Use separation of variables to compute the natural frequencies of this vibrating string. You must show all work. (Note that you are not being asked to compute a solution u to this equation.)

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3. (20 pts) Derive the partial differential equation for a vibrating string in the simplest possible manner. You may assume the string has constant mass density ρ_0, you may assume the tension T_0 is constant, and you may assume small displacements (with small slopes).

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