Midterm 1 **Name:** ______________________________

MATH3150, Section 04 February 14, 2018 ♥

This test is:

- closed-book
- closed-notes
- no-calculator
- 80 minutes

Indicate your answers clearly, and show your work. Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

Fun fact of life: if your work is not legible, I will not be able to read it. The ramifications of this outcome should be clear.

There are 3 questions with multiple parts; each question is worth a total of 20 points.

All pages are one-sided. If on any problem you require more space, use the back of the page.

Do not turn this page until directed to begin

1. (20 pts total) This question concerns linearity and equilibrium solutions.

a.) (5 pts) Compute the equilibrium solution for the following PDE:

$$\begin{aligned} u_t &= u_{xx}, \qquad 0 < x < 1, \;\; t > 0 \\ u(x,0) &= u_0(x) \\ u(0,t) &= 2 \\ \frac{\partial u}{\partial x}(1,t) &= 1 \end{aligned}$$

b.) (7 pts) Determine the value of β such that the PDE

$$\begin{aligned} u_t &= u_{xx}, \qquad 0 < x < 1, \;\; t > 0 \\ u(x,0) &= u_0(x) \\ \frac{\partial u}{\partial x}(0,t) &= 1 \\ \frac{\partial u}{\partial x}(1,t) &= \beta, \end{aligned}$$

has an equilibrium solution. (You need not compute the equilibrium solution.)

c.) (8 pts) Suppose L is an operator acting on a function u. What does it mean for L to be linear? (You must give a definition.)

2. (20 pts) Solve the following eigenvalue problem: find all eigenvalues λ and eigenfunctions $\phi(x)$. You must show all work, including exhausting all possible values of λ.

$$\begin{aligned} \phi''(x) + \lambda\phi(x) &= 0, \qquad 0 < x < L \\ \phi'(0) &= 0, \\ \phi'(L) &= 0 \end{aligned}$$

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3. (20 pts) Compute the solution $u(x,t)$ to the following one-dimensional heat equation:

$$u_t = u_{xx}, \qquad 0 < x < 1, \quad t > 0$$

subject to the initial and boundary conditions

$$\begin{aligned} u(x,0) &= x^2(1-x)^2, \\ \frac{\partial u}{\partial x}(0,t) &= 0, \\ \frac{\partial u}{\partial x}(1,t) &= 0 \end{aligned}$$

Show all work. Your solution must be written down in terms of explicit, computable expressions or integrals, but you need not compute the values of these integrals. You may use any results derived from previous problem(s).

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