Department of Mathematics, University of Utah

PDEs for Engineering Students

MATH 3150 – Section 004 – Spring 2018

Homework 9

Fourier Transform and PDEs

Due April, 23 2018

Text: *Linear Algebra and Differential Equations with Introductory Partial Differential Equations and Fourier Series*, custom edition for the University of Utah,

Note: Chapters 12-16 in the above textbook correspond to Chapters 1-4 and 10, respectively, in the Haberman textbook.

Section 16.4: # 10

Additional Problems: In the following problems, you may express solutions $u(x,t)$ to PDEs as a convolution if the convolution cannot easily be simplified.

P1. Compute (and reasonably simplify) the Fourier transform of the following functions:

a.) $f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$

b.) $f(x) = \begin{cases} \exp(-x), & x \geq 0 \\ 0, & x < 0 \end{cases}$

c.) $f(x) = \begin{cases} x\exp(-x), & x \geq 0 \\ 0, & x < 0 \end{cases}$

d.) $f(x) = \begin{cases} x(\cos x)\exp(-x), & x \geq 0 \\ 0, & x < 0 \end{cases}$

P2. Compute the solution $u(x,t)$ to the equation

$$\begin{aligned} u_t &= cu_x - au, & -\infty < x < \infty, \quad t > 0 \\ u(x,0) &= f(x). \end{aligned}$$

Here, c is an arbitrary real number and $a > 0$.

P3. Compute the solution $u(x,t)$ to the equation

$$\begin{aligned} u_{tt} &= c^2 u_{xx} - au, & -\infty < x < \infty, \quad t > 0 \\ u(x,0) &= f(x), \\ u_t(x,0) &= 0. \end{aligned}$$

Here, c is an arbitrary real number and $a > 0$.

P4. Compute the solution $u(x,t)$ to the equation

$$\begin{aligned} u_t &= k u_{xx}, & -\infty < x < \infty, \quad t > 0 \\ u(x,0) &= \delta(x), \end{aligned}$$

where $\delta(x)$ is the Dirac delta function centered at 0.
(This solution $u(x,t)$ is called the *Green's function* for this PDE.)

P5. Compute the solution $u(x,t)$ to the equation

$$\begin{aligned} u_{tt} &= u_{xxxx}, & -\infty < x < \infty, \quad t > 0 \\ u(x,0) &= f(x), \\ u_t(x,0) &= f''(x), \end{aligned}$$

where f is a given smooth function.