Department of Mathematics, University of Utah

Orthogonal polynomials and spectral methods for PDE's

MATH 5750/6880 – Section 002 – Spring 2018

Course Information and Syllabus

Updated September 23, 2018

Instructor:	Akil Narayan
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Office:	WEB 4666
Office hours:	Monday 3-4pm, Tuesday 3-5pm
Office hours location:	WEB 4666

Class time and location: TTh, 10:45am-12:05pm, JWB 208

Section webpage: http://www.sci.utah.edu/~akil/math6880

Course Information: This is a 3-credit course.

Learning objectives: Upon successful completion of this course, a student should be able to:

- understand theoretical foundations of approximation with polynomials and Fourier Series
- implement algorithms to accomplish approximation tasks using orthogonal polynomials
- analytically devise and implement algorithms for computing numerical solutions to PDE's using spectral methods

Prerequisites: There are no explicit prerequisites for this course, but students should be comfortable with foundations of linear algebra, calculus, partial differential equations, and computer programming. Knowledge of basic numerical analysis is helpful.

Course description: Mathematical and computational foundations of approximations using Fourier Series and orthogonal polynomials; applications to solutions of differential equations. This course will emphasize both theory and computation. Some programming is required.

Text: There are no required textbooks for this course. Instead, course notes will be provided by the instructor. However, there are a number of reference textbooks that are excellent resources in general for this course. These optional reference texts are listed below. The book by Trefethen is perhaps the most accessible introduction, but emphasizes implementation in MATLAB. The code provided by the instructor in this class is written in Python.

Orthogonal Polynomials: Computation and Approximation, Walter Gautschi, Oxford University Press, ISBN 0-19-850672-4

Spectral Methods for Time-Dependent Problems, Jan S. Hesthaven & Sigal Gottlieb & David Gottlieb, Cambridge University Press, ISBN 0-521-79211-8

Spectral Methods: Algorithms, Analysis and Applications, Jie Shen, Tao Tang, & Li-Lian Wang, Springer Science & Business Media, ISBN 978-3-540-71041-7

Spectral Methods in MATLAB, Lloyd N. Trefethen, SIAM, ISBN 978-0-89871-465-4

Orthogonal polynomials, Gábor Szegö, American Mathematical Society, 4th edition, ISBN 978-0-8218-1023-1

Homework: Problem sets will be announced in-class and subsequently posted on the course website. Homework will be divided into bi-weekly assignments and collected in-class. Each homework assignment is worth equal weight. Only paper (hard) copies of assignments will be

accepted; electronic copies will *not* be accepted. Late assignments of any form will *not* be accepted without either prior approval from the instructor, or if a student provides documentation showing extenuating circumstances.

Projects: This course will have 1 midterm project, and 1 final project/presentation that serves as the final exam. The midterm project will be collected on Thursday, October 4. Details and expectations for projects will be discussed in class during the semester.

Grading: Your course grade will be computed as follows.

- Homework 50%
- Midterm project 20%
- Final project 30%

Final letter grades will be assigned based on the following scheme:

- 92% - 100% — A
- 90% - 91% — A−
- 88% - 89% — B+
- 82% - 87% — B
- 80% - 81% — B−
- 78% - 79% — C+
- 72% - 77% — C
- 70% - 71% — C−
- 68% - 69% — D+
- 62% - 67% — D
- 60% - 61% — D−
- 0% - 59% — E

Important dates:

Aug 31	Last day to add, drop (delete), elect CR/NC, or audit classes
Oct 5	Midterm project due
Oct 19	Last day to withdraw from classes
Nov 30	Last day to reverse CR/NC option
Dec 7	Reading Day
TBD	Final project due

Class communication: An email list is set up with which I shall send out information not communicated during class. This email list will also be used to communicate class information in the case of unusual circumstances affecting the the logistics of the class. If you are not officially registered for the class but wish to be on the roster, please discuss it with me.

If you are registered for the course, but do not receive the course email announcements to your University of Utah email address, please notify me immediately. It is not possible for me to arrange delivery of these emails to a non-Utah account, but you can forward your Utah emails to

other email addresses. (Navigate to `http://www.cis.utah.edu`, login, and change your UMail settings.)

The section website will be used to communicate more technical matter of the class (e.g. problem sets, lecture summaries, etc.).

In the event of a major campus emergency, course requirements, deadlines and grading percentages are subject to change that may be necessitated by a revised semester calendar or other circumstances. The above two methods, in addition to the coursewide website, are reliable means of getting information about changes to the course.

ADA Statement: The University of Utah seeks to provide equal access to its programs, services and activities for people with disabilities. If you will need accommodations in the class, reasonable prior notice needs to be given to the Center for Disability Services, 162 Olpin Union Building, 581-5020 (V/TDD). CDS will work with you and the instructor to make arrangements for accommodations. All written information in this course can be made available in alternative format with prior notification to the Center for Disability Services.

Student responsibilities and integrity: All students are expected to maintain professional behavior in the classroom setting, according to the Student Code, spelled out in the Student Handbook. Students have specific rights in the classroom as detailed in Article III of the Code. The Code also specifies proscribed conduct (Article XI) that involves cheating on tests, plagiarism, and/or collusion, as well as fraud, theft, etc. Students should read the Code carefully and know they are responsible for the content. According to Faculty Rules and Regulations, it is the faculty responsibility to enforce responsible classroom behaviors, and I will do so, beginning with verbal warnings and progressing to dismissal from and class and a failing grade. Students have the right to appeal such action to the Student Behavior Committee.
`http://regulations.utah.edu/academics/6-400.php`

Semester calendar

(Subject to change!)

Day	Date	Topic
Tuesday	August 21, 2018	Hello
Thursday	August 23, 2018	Orthogonal polynomials: the three-term recurrence
Tuesday	August 28, 2018	Examples: orthogonal polynomial families, connection coefficients
Thursday	August 30, 2018	Christoffel-Darboux relation, miscellaneous properties
Tuesday	September 4, 2018	Gaussian quadrature, computing approximations
Thursday	Septebmer 6, 2018	**(HW 1 due)** Python and algorithms
Tuesday	September 11, 2018	Elementary functional analysis
Thursday	September 13, 2018	Sturm-Liouville theory
Tuesday	September 18, 2018	Sturm-Liouville theory: recurrence coefficients
Thursday	September 20, 2018	**(HW 2 due)** Error estimates for best L^2 approximation
Tuesday	September 25, 2018	Computing Galerkin approximations
Thursday	September 27, 2018	Interpolative approximations
Tuesday	October 2, 2018	Theory for interpolative approximations
Thursday	October 4, 2018	**(Midterm project due)** Review/catch-up
Tuesday	October 9, 2018	No Class: Fall break
Thursday	October 11, 2018	No Class: Fall break
Tuesday	October 16, 2018	Fourier Series: fundamentals, and the FFT
Thursday	October 18, 2018	Fourier Series: best approximation
Tuesday	October 23, 2018	Fourier Series and Chebyshev polynomials
Thursday	October 25, 2018	ODE solutions: The Galerkin method with periodicity
Tuesday	October 30, 2018	Orthogonal polynomials: Non-periodic Galerkin solutions
Thursday	November 1, 2018	**(HW 3 due)** Elementary numerical analysis for linear stationary PDE's: Lax-Milgram
Tuesday	November 6, 2018	Multidimensional formulations
Thursday	November 8, 2018	Review/catch-up
Tuesday	November 13, 2018	Numerical solution of linear initial value problems
Thursday	November 15, 2018	The Galerkin method for periodic time-dependent problems
Tuesday	November 20, 2018	**(HW 4 due)** Stability of Fourier Galerkin and collocation methods
Thursday	November 22, 2018	No Class: Thanksgiving
Tuesday	November 27, 2018	Galerkin and collocation methods for non-periodic hyperbolic PDE's
Thursday	November 29, 2018	Stability and convergence
Tuesday	December 4, 2018	TBD
Thursday	December 6, 2018	TBD