Prior Model Selection for Bayesian Inverse Electrocardiography

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Introduction

The goal of the inverse problem of electrocardiography (ECG) is to reconstruct cardiac electrical sources using body surface potential measurements, and an appropriate mathematical model representing the thorax [1,2]. However, due to attenuation and smoothing that occurs in the thorax, the problem is ill-posed and reliable solutions require regularization. The application of suitable regularization constraints to the calculation of cardiac sources remains a persistent challenge.

Recent studies have described Bayesian approaches to the inverse ECG problem [3,4,5]. In addition to providing a more general way to formulate physiological constraints, these approaches also offer statistical performance evaluation tools that are not generally available with deterministic approaches. However, the Bayesian methods rely on the choice of prior probability density function (pdf); the better the prior model fits the epicardial potentials, the greater the reliability of the Bayesian estimates [3,4,5].

In our previous work, we estimated prior model parameters from a training dataset of previously recorded epicardial potentials [5,6]. In the earlier work [6] <why not put the earlier work first?>, we used a training dataset that pooled epicardial beats paced from various locations on the heart surface to capture the variability in the epicardial potentials. In a follow-up study [5], we compared Bayesian inverse solutions using two different training datasets to determine which performed better. These comparisons provided us with valuable insight into the nature of the training data selection problem, but did not provide a metric that would enable us to select the best training dataset among many candidates.

In this paper, we study a prior model selection criterion based on *the evidence*, a Bayesian metric defined as the marginal pdf of the measurements. The underlying hypothesis is that the prior model that best explains the available measurements (*i.e.* that yields maximum value of the *evidence*) is the best prior model among a set ofcandidates. This study extends the method reported in [7], in which there were only two, statistically independent, parameters, to include a more complicated model with many parameters and a full covariance matrix. The long-term goal is to automatically select the training data that would allow the most accurate reconstruction of epicardial potentials from a given set of training data and measurements.

Methods

Problem Definition: We can define the inverse electrocardiography problem at a particular time instant as: $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}$, where $\mathbf{y}$ is an Mx1 vector of torso measurements, $\mathbf{x}$ is an Nx1 vector of epicardial potentials, $\mathbf{A}$ is an MxN matrix representing the forward solution, and $\mathbf{n}$ is the noise in the torso measurements. The goal is to estimate $\mathbf{x}$ given $\mathbf{y}$ and $\mathbf{A}$, by applying appropriate statistical constraints (*i.e.,* by choosing an appropriate prior model).

Bayesian MAP Estimation: We used a Bayesian Maximum a Posteriori (MAP) approach to compute the inverse solution. This approach maximizes the posterior distribution of the epicardial potentials, which is based on the conditional probability distribution function (pdf) of the torso potentials conditioned upon the epicardial potentials along with the prior pdf of the epicardial potentials. This prior pdf represents our statistical knowledge about the epicardial potentials and can be fully represented by their mean vector, $\mathbf{x_o}$, and covariance matrix, $\mathbf{C_x}$, if we assume that the epicardial potentials have a multivariate Gaussian distribution, *i.e.* $\mathbf{x} \sim N(\mathbf{x_o}, \mathbf{C_x})$. The MAP solution is:

$$\mathbf{x_{inv}} = (\mathbf{A}^T \mathbf{C_n}^{-1} \mathbf{A} + \mathbf{C_x}^{-1})^{-1} (\mathbf{A}^T \mathbf{C_n}^{-1} \mathbf{y} - \mathbf{C_x}^{-1} \mathbf{x_o}) \quad (1)$$

where $\mathbf{C_n}$ is the covariance matrix of the noise vector, $\mathbf{n}$.

Bayesian Prior Selection based on Evidence: We assumed we have various candidate prior models corresponding to different training datasets of previously recorded epicardial potentials. From these training datasets, we first computed candidate model parameters, H^i <note the superscript here is very crowded up to the H> = $\{\mathbf{x_o}^i, \mathbf{C_x}^i\}$. Then, for each of these candidate models, we calculated the Bayesian evidence:

$$p(\mathbf{y} \backslash H^i) \sim N(\mathbf{A}\mathbf{x_o}^i, \mathbf{A}\,\mathbf{C_x}^i\,\mathbf{A}^T + \mathbf{C_n}) \quad (2)$$

Finally, we compared the evidence values and the resulting inverse solutions corresponding to each model in order to test the validity of our hypothesis.

Results

We recorded canine epicardial potentials recorded according to the protocol presented in [5]. We used a leave-one-experiment-out protocol to test the methods, *i.e.,* we selected a test beat from one experiment to simulate the measurements, and we pooled data from other experiments (using the other animals) to create the training dataset. We simulated the torso measurements from the test beat by multiplying the epicardial potentials by the forward matrix, and adding zero mean and *i.i.d.* noise at 30 dB SNR.

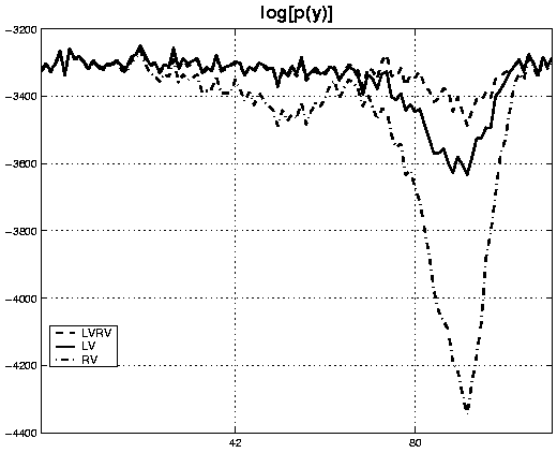

Fig. 1 Evidence values for LV-paced, RV-paced and mixed LVRV-paced priors

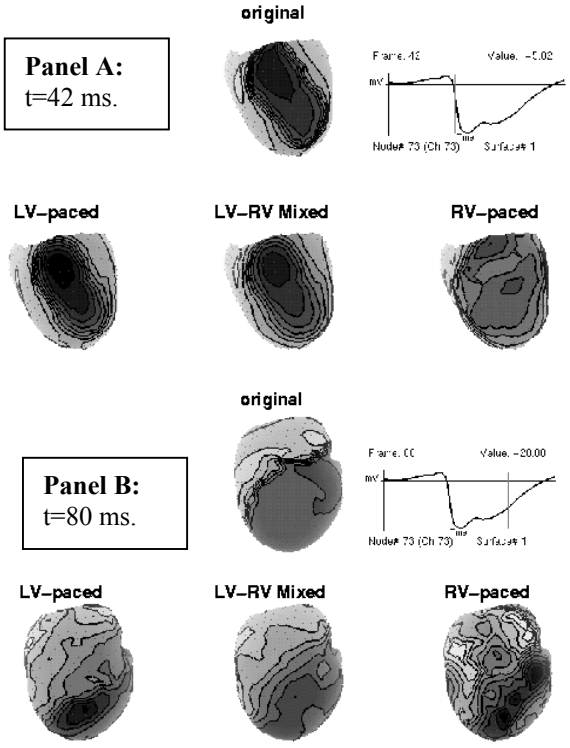

Fig. 2 Isopotential maps at two different time instances, using three different prior densities.

In this work, we study the feasibility of using the evidence to compare different models. Since this is only a feasibility study, we test our results in a very simple scenario, in which we have only three prior models, each using a different training set:

1. use a training set composed of only left ventricularly paced beats (LV-paced)
2. use a training set composed of only right ventricularly paced beats (RV-paced)
3. use a training set composed of both LV and RV paced beats (LVRV-paced)

The number of beats in each training set is the same <true??>>. The test beat is paced from the LV surface.

In Fig. 1, we plot evidence values found using Eq. (2) at all time instances in the QRS interval, for the three training sets. The evidence stays approximately the same for all three priors until around 25 ms. After that time, the evidence value corresponding to the RV-paced prior becomes smaller than the other two. This gap widens even more between time instances 70 and 100 ms. The evidence values corresponding to LV-paced prior are indistinguishable from that of LVRV-paced prior until around 75 ms; after that they are smaller.

In Fig. 2, we show reconstructed epicardial potential maps in the QRS interval for all three priors. The two panels in Fig. 2 show isopotential maps of the reconstructions as well as the original at 42 (top) and 80 (bottom) ms. In each panel, we plot the original isopotential map at the top, and the three reconstructions below. To the right of the original maps, we plot a single time series from one lead with that panel's time instant marked for reference. In each panel, the range is fixed for all isopotential maps. In all of these isopotential maps, darker regions represent negative potentials, lighter regions represent positive potentials, and the wavefront lies at the transition from darker to lighter regions. According to Fig. 1, at 42 ms, the evidence value for the RV-paced prior was smaller than the other two, so we would expect the reconstruction with the RV-paced prior to be worse than the others. At 80 ms, the evidence values from largest to smallest corresponded to the LVRV-paced prior, the LV-paced prior and the RV-paced prior. Again, according to our hypothesis, we expect to obtain the best reconstruction among the three

priors using the LVRV-paced prior, and the worst using the RV-paced prior.

Examining the maps, we see that at 42 ms, the reconstruction using the RV-paced prior is the worst of the three reconstructions. It is noisy, and the shape of the wavefront is not accurate. On the other hand, reconstructions using the other two priors have better fidelity to the original potential distribution. Among thesetwo, the LV-paced prior performs slightly better.

At 80 ms, the reconstruction using the RV-paced prior produces many noisy contours, and it is hard to differentiate one single dominant wavefront. The best reconstruction is with the LVRV-paced prior; the wavefront that lies along the 2 o'clock - 8 o'clock line in the original isopotential map is better reconstructed than the other two. This is also consistent with the evidence values: at this time instant, the LVRV-paced prior produced the highest evidence value, and the RV-paced prior produced the lowest evidence value.

Discussion

Previous studies on Bayesian MAP estimation applied to the inverse ECG problem have shown that it was important to use a "good" prior model in order to increase the reliability of the Bayesian reconstructions. In this paper, we studied Bayesian evidence, and examined a very simple prior selection scenario, with only three prior models. Even by using this simple simulation, we obtained results that support the hypothesis that the prior model that maximizes the evidence is a good choice of prior, at least among the proposed candidates. This idea can easily be extended to choose from a larger number of candidate priors. Alternatively, it is possible to use the evidence to design an adaptive algorithm in which one can determine whether to accept a training beat into a training dataset depending on the increase or the decrease in the evidence value due to including that training beat. The results of this study show that it is feasible to use the Bayesian evidence in both types of prior model selection scheme. However, more research is necessary to evaluate the evidence as a prior selection criterion for inverse bioelectric problems.

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