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Combining Numerical and Physiological Constraints in Inverse Electrocardiography

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Abstract. The inverse problem of ECG is ill-posed and the application of suitable constraints to the calculation of cardiac sources is a persistent challenge in solving the problem. In this paper we describe Bayesian approaches to extract useful constraints from sparsely sampled epicardial signals as well as a training set of epicardial maps, and use them to improve the quality of computed inverse solutions. We also use Bayesian metrics to evaluate the results

1. Introduction

The goal in the inverse electrocardiography (ECG) problem is to reconstruct the cardiac sources from torso potential measurements and an appropriate forward model. Attenuation and spatial smoothing in the thorax make the problem ill-posed and the forward model matrix is badly conditioned [1]. Thus, even small disturbances in the measurements lead to amplification of error in the inverse solution. The most common approach to combat this problem is deterministic "regularization", where the solution trades off between the estimate that best represents the data and fidelity to an *a priori* regularization constraint imposed on the solution. The basic limitation on performance of all methods for the ill-posed inverse ECG problem is the availability of good a priori information, and numerical methods to effectively use this information, and evaluate the resulting reconstructions.

Recent studies have described Bayesian approaches to the inverse ECG problem. These approaches provide a more general way to combine one or more types of physiological data, such as various physiological measurements, a forward model, and statistical prior information representing the cardiac sources [2, 3, 4]. The Bayesian models also offer statistical performance evaluation tools that are not generally available to deterministic approaches.

One source of additional physiological measurements comes from the use of multielectrode venous catheters that permit simultaneous measurements of epicardial potentials below the superficial coronary veins [5]. We have combined these sparse epicardial measurements with appropriate statistical prior models to estimate activation time and/or epicardial maps [5, 4]. This approach is not an inverse solution in the traditional sense-it does not require torso surface measurements or a forward model. One source of prior information for spatial or spatiotemporal statistical constraints are assumed correlation matrices or assumed structures of those matrices [2, 3]. Our current approach is to combine all these sources of inverse solution information and constraints in a Bayesian framework to investigate the utility of such a multifaceted approach [4].

In this paper we describe studies that examined the effect of the number and location of epicardial measurements used in such a multifaceted Bayesian approach. We also illustrate the use of the Bayesian predicted error covariance to quantify this effect. The ultimate goal is to recommend practices for venous catheter mapping that together with body surface potential measurements would allow accurate reconstruction of epicardial potentials.

2. Methods

We start with the standard formulation of the inverse problem as $\mathbf{y} = \mathbf{A}\,\mathbf{x} + \mathbf{n}$, where $\mathbf{y}$ is an $M \times 1$ vector of torso potentials, $\mathbf{x}$ is an $N \times 1$ vector of epicardial potentials, $\mathbf{A}$ is an $M \times N$ matrix representing the forward solution, and $\mathbf{n}$ is the noise in the torso measurements. In addition, we assume we have access to sparse noisy epicardial measurements

$\tilde{\mathbf{x}}_m$ at some subset of $N_k (N_k < N)$ epicardial sites $\tilde{\mathbf{x}}_m = \mathbf{x}_m + \mathbf{e}_m$.

We organize **A** and **x** such that the first N_1 elements of **x** correspond to measured leads, and first N_1 columns of **A** are the coefficients that multiply the known values of x, *i.e.*, $x = \left[x_m^T \; x_u^T\right]^T$ and $A = \left[A_m \; A_u\right]$ where subscripts *m* and *u* represent "measured" and "unmeasured" leads respectively. The goal is to estimate **x** using either only torso measurements, or both the torso measurements and the sparse epicardial measurements. In the latter case, one solves an augmented inverse problem $v = Dx + \tilde{n}$ where **v** consists of $\tilde{\mathbf{x}}_m$ appended to **y**, $\tilde{\mathbf{n}}$ consists of $\mathbf{e}_m$ appended to **n**, and **D** has a block [**I 0**] appended below **A**. We solved the inverse problem using Bayesian MAP estimation with prior: $x \sim N(\bar{x}, C_x)$ where $\bar{\mathbf{x}}$ is the mean, and $\mathbf{C}_x$ is the correlation matrix of **x**. The solutions then become:

$$\hat{x} = \left(A^T C_n^{-1} A + C_x^{-1}\right)^{-1}\left(A^T C_n^{-1} y - C_x^{-1}\bar{x}\right) \; and \; \hat{x} = \left(D^T C_{\tilde{n}}^{-1} D + C_x^{-1}\right)^{-1}\left(D^T C_{\tilde{n}}^{-1} v - C_x^{-1}\bar{x}\right) \quad (1)$$

respectively, where $\mathbf{C}_n$ and $\mathbf{C}_{\tilde{n}}$ are the covariance matrices of the noise vectors, **n** and $\tilde{\mathbf{n}}$, respectively.

3. Results and Discussions

We simulated sparse venous catheter measurements by adding normally distributed zero mean *i.i.d.* noise to a subset of potentials measured with an epicardial sock electrode array selected to lie close to the cardiac veins. We simulated torso potentials using a boundary element solution to Laplace's equation for a human shaped torso tank in which the heart was suspended and then added noise. In this study, N = 490, M = 771.

We obtained the *a priori* information for the statistical methods from a database of epicardial sock measurements using a "leave-one-beat-out" protocol in which we excluded the test beat from the training dataset [5, 4]. In the training dataset, we included beats from the same experiment with the same pacing site as the test beat.

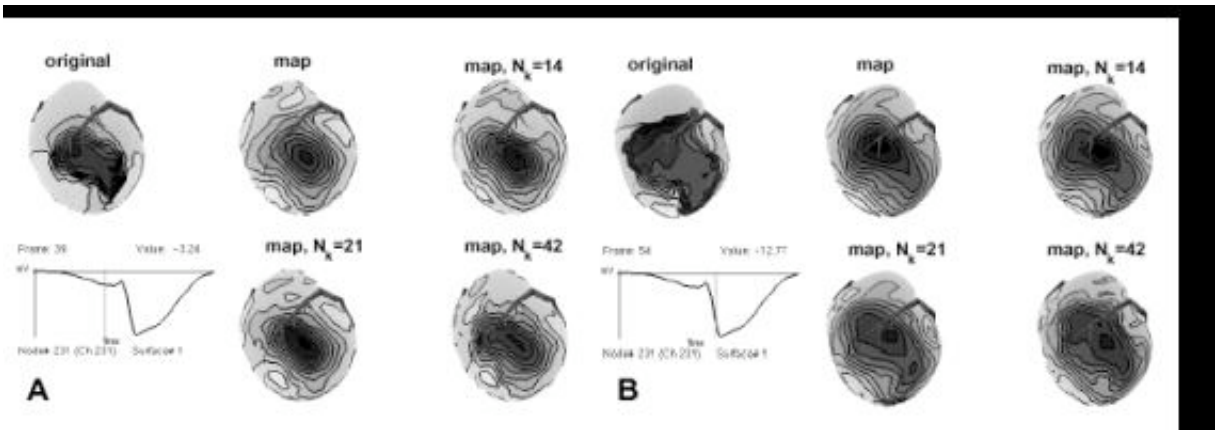

Figure 1. *Original and estimated epicardial potentials for a left ventricularly paced beat using an LV-paced training set. Panels A and B show the potentials at 39 and 54 ms after stimulus, respectively.*

Figure 1 contains the epicardial isopotential maps of the original beat and various inverse solutions at 39 and 54 ms after the application of the stimulus. The isopotential maps for $N_k = 21$ and $N_k = 14$ suggest that even with a small number of epicardial measurements there is substantial improvement in the fidelity to the original isopotential map compared to using only torso measurements. This fidelity becomes better as the number of epicardial measurements increases. Table 1 contains the theoretical and actual confidence intervals (CI) in millivolts, averaged over all epicardial leads. Theoretical values come from the Bayesian estimation error covariance while actual values were computed using the error between the inverse solution and the true values. The values in the table indicate that with 95% probability the true epicardial potentials, on average, fall within the $\left[(\hat{x}\text{-}CI)(\hat{x} + CI)\right]$ interval the accuracy increases as the CI value decreases. The actual and theoretical CI values were similar (e-xcept for N_1 = 42). The CI values decreased as the N_k increased.

Table 1. *Predicted and actual confidence intervals, mV, averaged over epicardial leads.*

N_k	*theoretical*	*actual*
0	5.41	5.01

14	4.65	4.90
21	4.36	4.85
42	3.84	5.08

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