INCLUDING SPARSE NOISY EPICARDIAL POTENTIAL MEASUREMENTS INTO BAYESIAN INVERSE ELECTROCARDIOGRAPHY

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Abstract— **Imposition of *a priori* constraints is needed to combat the ill-posedness of the inverse problem of electrocardiography. Solutions to this problem have not yet achieved clinical utility. Extra measurements from catheters inserted into cardiac veins, even though quite sparse, may help increase accuracy and robustness. In this paper, we study various Bayesian methods to incorporate sparse epicardial measurements in solutions to the inverse problem.**

Keywords— **Inverse Electrocardiography, Bayesian MAP estimation**

I. Introduction

Due to attenuation and spatial smoothing that occurs in the thorax, inverse electrocardiography—estimating cardiac electrical activity based on measured torso surface potentials and a geometric model of the torso—is an ill-posed inverse problem [1], [2] and the forward model matrix is badly conditioned. Thus small disturbances in the measurements lead to amplified errors in inverse solutions. The most common approach to this problem is deterministic "regularization", where the solution is a trade-off between the estimate that best represents the data and fidelity to an *a priori* regularization constraint imposed on the solution.

Recent reports have described statistical inverse approaches. In [3], the authors applied a statistical model and *a priori* knowledge of the epicardial potential correlation matrix in a Bayesian Maximum-A-Posteriori (MAP) framework. Another approach [4] used estimated temporal correlations of the epicardial potentials to optimally estimate the epicardial potentials jointly at all time instants. These methods often produce better results than deterministic methods. However, there is still need for improvement, especially in determining appropriate parameters of the statistical models.

Recent progress in the fabrication of multielectrode venous catheters permits simultaneous measurement of epicardial potentials from several catheters located in cardiac veins [5]. The coverage of these catheters on the epicardium is very sparse, but these extra measurements near the heart surface may be useful to improve the accuracy and the robustness of the inverse solution. Since these measurements are themselves noisy, we incorporate them in a statistical setting. We assume we can use *a priori* knowledge of epicardial potential correlations along with actual noisy measurements of sparsely-located leads.

II. Methods

The electrocardiographic inverse problem at a particular time instant can be defined as:

$$\mathbf{y} = \mathbf{A}\,\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{y}$ is an $M \times 1$ vector of torso potentials, $\mathbf{x}$ is the $N \times 1$ vector of epicardial potentials, $\mathbf{A}$ is the $M \times N$ matrix representing the forward solution, and $\mathbf{n}$ is the noise in the torso measurements. We also assume that we have noisy epicardial measurements $\tilde{\mathbf{x}}_\mathbf{m}$ at some subset of N_m ($N_m < N$) epicardial sites. The goal is to estimate $\mathbf{x} = [\mathbf{x_m}^T\ \mathbf{x_u}^T]^T$ *i.e.,* the union of the potentials at the measured and unmeasured sites, using both the torso measurements $\mathbf{y}$ and the partial epicardial measurements $\tilde{\mathbf{x}}_\mathbf{m}$. We model $\tilde{\mathbf{x}}_\mathbf{m}$ as $\mathbf{x_m}$ plus an additive noise term $\mathbf{e_m}$, and partition $\mathbf{A} = [\mathbf{A_m}\mathbf{A_u}]$.

We consider the following methods: **1.** (**SUBT-M**) Let $\mathbf{x_m} = \tilde{\mathbf{x}}_\mathbf{m}$, subtract this part from the overall relationship, and solve the reduced inverse problem $(\mathbf{y} - \mathbf{A_m}\,\tilde{\mathbf{x}}_\mathbf{m}) = \mathbf{A_u}\,\mathbf{x_u} + \mathbf{n}$ for $\mathbf{x_u}$ using Tikhonov regularization. Note that this is essentially a *deterministic* approach. **2.** (**MAP-hybrid**) A hybrid approach that first uses statistical estimation based on *a priori* training data to estimate all epicardial potentials from the sparse measurements, then in a second step uses these estimates along with the torso measurements to solve the inverse problem. **Step 1:** (**LLS-epi**) We modify the method suggested in [5] to estimate missing epicardial potentials rather than activation times. The unmeasured potentials ($\tilde{\mathbf{x}}_\mathbf{u}$) are estimated from the measured ones $\tilde{\mathbf{x}}_\mathbf{m}$ and training datasets that include values for both measured and unmeasured sites: $\hat{\tilde{\mathbf{x}}}_\mathbf{u} = \bar{\mathbf{x}}_\mathbf{u} + \mathbf{C}_{um}\ \mathbf{C}_{mm}^{-1}\ (\tilde{\mathbf{x}}_\mathbf{m} - \bar{\mathbf{x}}_\mathbf{m})$ where $\bar{\mathbf{x}}_\mathbf{m}$ and $\bar{\mathbf{x}}_\mathbf{u}$ are the mean values of $\tilde{\mathbf{x}}_\mathbf{m}$ and $\tilde{\mathbf{x}}_\mathbf{u}$ respectively, $\mathbf{C}_{mm}$ is the covariance of $\tilde{\mathbf{x}}_\mathbf{m}$, and $\mathbf{C}_{um}$ is the cross-covariance between $\tilde{\mathbf{x}}_\mathbf{m}$ and $\tilde{\mathbf{x}}_\mathbf{u}$. **Step 2:** (**MAP-hybrid**) We use the results of Step 1 in a Bayesian MAP estimation setting with $\mathbf{n} \sim \mathrm{N}(\mathbf{0},\ \sigma_n^2\mathbf{I})$, $\mathbf{e_m} \sim \mathrm{N}(\mathbf{0},\ \sigma_{e_m}^2\mathbf{I})$, and $\mathbf{x} \sim \mathrm{N}(\bar{\mathbf{x}},\ \mathbf{C}_{xx})$ where $\bar{\mathbf{x}} = [\tilde{\mathbf{x}}_\mathbf{m}^T\ \hat{\tilde{\mathbf{x}}}_\mathbf{u}^T]^T$ and

$$\mathbf{C}_{xx} = \begin{bmatrix} \sigma_{e_m}^2\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \sigma_{e_m}^2\mathbf{I} + \mathbf{C}_{ww} \end{bmatrix}.$$

$\mathbf{C}_{ww} = \mathbf{C}_{uu} - \mathbf{C}_{um}\ \mathbf{C}_{mm}^{-1}\ \mathbf{C}_{um}^T$ is the covariance of estimation error in $\hat{\tilde{\mathbf{x}}}_\mathbf{u}$ (*i.e.,* $\tilde{\mathbf{x}}_\mathbf{u} - \hat{\tilde{\mathbf{x}}}_\mathbf{u}$) where $\mathbf{C}_{uu}$ is the covariance of $\tilde{\mathbf{x}}_\mathbf{u}$. Then the MAP estimate of $\mathbf{x}$ is $\hat{\mathbf{x}} = \left(\mathbf{A}^T\mathbf{A} + \sigma_n^2\ \mathbf{C}_{xx}^{-1}\right)^{-1}\left(\mathbf{A}^T\mathbf{y} + \sigma_n^2\ \mathbf{C}_{xx}^{-1}\bar{\mathbf{x}}\right)$. **3.** (**LLS-extraD**) Here we simply treat the sparse epicardial measurements as extra data and define a new measurement vector $\mathbf{v} = [\mathbf{y}^T\ \tilde{\mathbf{x}}_\mathbf{m}^T]^T$. Then we estimate $\mathbf{x}$ using linear least squares estimation: $\hat{\mathbf{x}} = \bar{\mathbf{x}} + \mathbf{C}_{xv}\ \mathbf{C}_{vv}^{-1}\ (\mathbf{v} - \bar{\mathbf{v}})$ where $\bar{\mathbf{v}}$ and $\bar{\mathbf{x}}$ are the mean values of $\mathbf{v}$ and $\mathbf{x}$ respectively, $\mathbf{C}_{vv}$ is the covariance of $\mathbf{v}$, and $\mathbf{C}_{xv}$ is the cross-covariance be-

TABLE I

AVERAGE OF RE OVER THE QRS INTERVAL (MEAN±STD)

SUBT-M	MAP-CZM	LLS-epi	MAP-hybrid	LLS-extraD
0.68±0.18	0.71±0.31	0.87±0.23	0.65±0.29	0.65±0.29

tween $\mathbf{x}$ and $\mathbf{v}$. **4.** (**MAP-CZM**) We use the Bayesian MAP estimation method suggested in [3] which assumes $\mathbf{x} \sim \mathrm{N}(\mathbf{0},\ \mathbf{R}_{xx})$ where $\mathbf{R}_{xx}$ is the correlation matrix of $\mathbf{x}$. Note that this method does not take advantage of the extra measurements and is included here for comparison.

III. RESULTS

We simulated venous catheter potentials by selecting sparse leads from a 490-electrode sock applied to an isolated canine heart and added normally distributed zero mean *i.i.d.* noise at 30dB SNR. We simulated torso potentials using a boundary element solution to Laplace's equation for a human shaped torso tank in which the heart was suspended. We then added noise at 25dB SNR to the simulated torso surface potentials and tried the various inverse solutions. In this work $N = 490$, $M = 771$ and we set $N_m = 42$.

We obtained the *a priori* information for the statistical methods from a database using a "leave-one-beat-out" protocol, where we excluded the test beat from the training dataset [5], [6]. The training set consisted of beats from the same experiment where every beat was paced at a different location on the ventricles. The same training database was used for all statistical methods.

Table I shows relative error (RE) averaged over the QRS interval. On average, MAP-hybrid and LLS-extraD had the lowest and LLS-epi had the highest RE. Fig. 1 shows a sample case of isopotential maps of the epicardial potentials. We compare the estimated epicardial potentials to the original map in the top left panel. The statistical methods (*i.e.,* MAP-CZM, LLS-epi, MAP-hybrid and LLS-extraD) performed better than SUBT-M in that these methods better preserved the wavefront. LLS-extraD and MAP-hybrid results showed improved fidelity of the wavefronts in the 5 o'clock and 11 o'clock positions over those from the LLS-epi and MAP-CZM methods.

IV. DISCUSSION

There are three types of measurements used here: *a priori* database measurements used to estimate parameters of a statistical model, $\mathbf{y}$, and $\tilde{\mathbf{x}}_\mathbf{m}$. In this study, using only the prior information (MAP-CZM) or prior information along with sparse epicardial measurements (LLS-epi) preserved wavefronts better than using everything but the prior measurements (SUBT-M) but achieved worse RE. Including all three types of information (MAP-hybrid and LLS-extraD) produced the most accurate results. The "leave-one-beat-out" protocol is simple and useful to test the performance of the statistical algorithms when we have a training dataset that represents the test data well. But it is unrealistic, as one rarely has access to beats from the same study in clinical settings. Future research will include testing of the methods with different datasets and making fuller use of the information in the measured subsets of epicardial potentials and the *a priori* database.

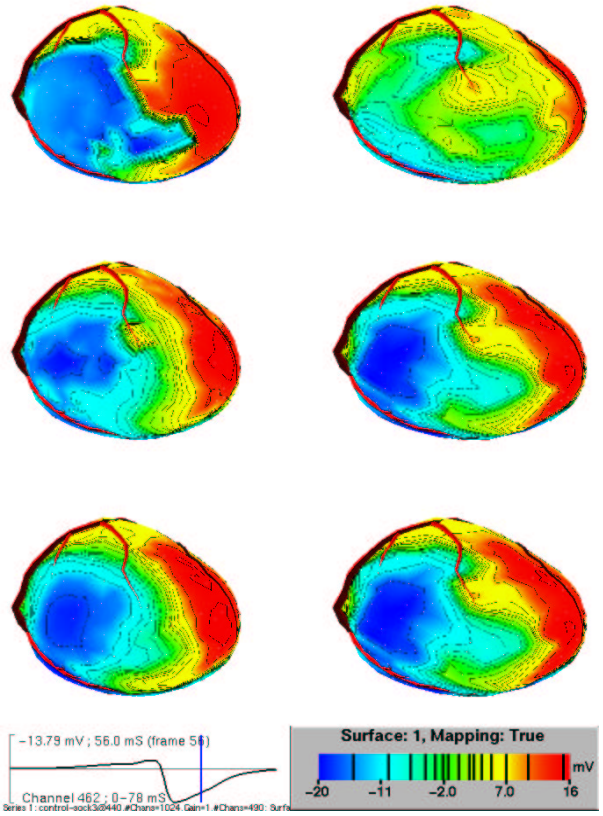

Fig. 1. Isopotential maps of epicardial potentials. Top-left: Original epicardial maps, Mid-left: SUBT-M, Bottom-Left: MAP-CZM, Top-Right: LLS-epi, Mid-Right: MAP-hybrid, Bottom-right: LLS-extraD. Display is an anterior view of the epicardial electrode array. The position of the anterior cardiac vein is drawn in red.

ACKNOWLEDGMENTS

This work was supported by the NIH National Center for Research Resources (NCRR) and the Whitaker Foundation. Y.S. thanks the Turkish Higher Education Council for their support of her graduate studies.

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