Algebraic MultiGrid with Multiple Right-Hand Side Treatment for an Efficient Computation of EEG and MEG Lead Field Bases

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ABSTRACT

Iterative solver techniques are used for the computation of EEG and MEG lead field bases for finite element method based volume conductor mode ling. Within this paper, we will discuss a new efficient strategy, the algebraic multigrid preconditioned conjugate gradient method with simultaneous treatment of multiple right-hand sides. We will show that this solver leads to a much higher cache hit rate, which speeds the computation by more than a factor of 2. Together with the concept of the EEG and MEG lead field bases, the complexity of realistic high resolution anisotropic finite element forward modeling within the EEG/MEG inverse problem is significantly reduced and can now be performed in approximately the same time as boundary element head modeling.

KEY WORDS

EEG/MEG Source Reconstruction, Finite Element Method, Lead Field Bases, Algebraic MultiGrid, Preconditioned Conjugate Gradient Methods, Treatment of Multiple Right-Hand Sides, Cache Algorithms

INTRODUCTION

When choosing the Finite Element (FE) method for volume conductor modeling within the EEG/MEG inverse problem, the construction of the lead field bases requires "number of EEG/MEG sensors" many solutions of large sparse systems of linear equations [Wolters, 2004]. Therefore, preconditioning techniques for the iterative solution process are important to speed the computation. It was previously shown that the Algebraic MultiGrid preconditioned Conjugate Gradient (AMG-CG) method is a very efficient solver for inhomogeneous anisotropic high resolution FE forward modeling [Wolters 2001, Wolters, 2002, Mohr, 2003]. Within this paper, we will discuss a new strategy for a further speedup, the simultaneous treatment of multiple right-hand sides.

METHODS

In a first step, compared to the AMG-CG presented, e.g., in [Wolters, 2001], general algorithmical improvements were implemented for the new Multiple Right-Hand Side AMG-CG (MultiRHS-AMG-CG) [Haase, 2003].

The old memory management for the stiffness and interpolation matrices was replaced by the classical Compact Row Storage (CRS) format in order to decrease the number of cache misses. Within the AMG algorithm, defect calculation follows forward Gauss-Seidel (GS) smoothing and both operations require matrix-vector operations. For symmetric stiffness matrices, parts of the matrix-vector operation from the last GS smoothing can be efficiently stored and reused by the defect calculation, a merging which leads to a reduction of the operation count. The AMG-procedure on the next coarser level is called with a zero- initial correction vector. This can be used, too, so that the first forward GS smoothing sweep on the coarser levels is reduced to half of the arithmetic and memory operations. If a V-cycle is chosen, i.e., only one pre-smoothing sweep is performed, the special structure of this smoother on the coarser levels furthermore leads to a reduction of the subsequent defect computation.

Since the RHSs in the lead field bases approach are computed beforehand and the stiffness matrix remains the same, we can simultaneously solve for a whole block of RHSs. The most computationally expensive operations in the AMG-CG method are the matrix-vector operations within the CG and within the AMG components smoothing, defect calculation, interpolation and prolongation. If the vector for one RHS is exchanged against a whole block of vectors for multiple RHSs and if this block is not stored as a matrix, but as a long vector (first the first entries of the RHSs, then the second entries etc., resulting in a long vector), then each matrix entry only has to be accessed once and can be multiplied to all corresponding values in the block-vector. This procedure results in much higher cache hit rates, which speeds the computations. For the simultaneous treatment of 3 RHS, the inner loops were manually unrolled, leading to a further reduction of the solver time.

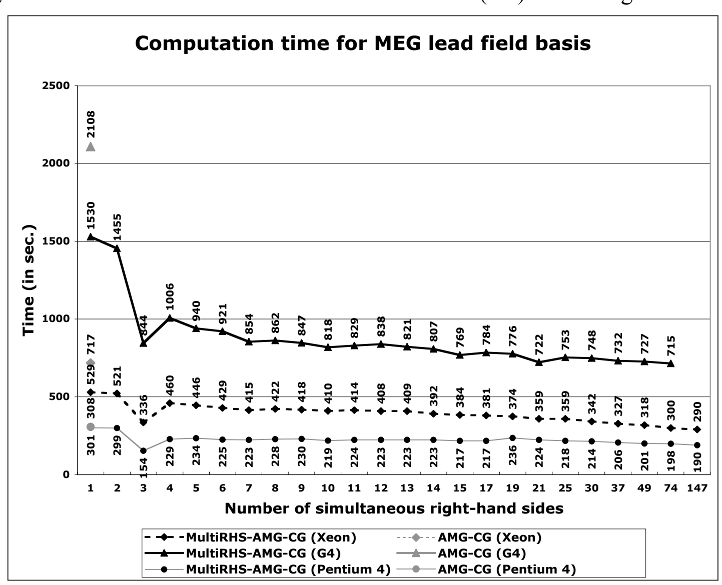

Figure 1 Time for the computation of the MEG lead field basis on a Mac-OSX PowerBook G4 and on Red-Hat Linux PC's with either Xeon or Pentium 4 architecture using the conventional AMG-CG and the new AMG-CG with simultaneous treatment of multiple right-hand sides.

RESULTS

As a basis for our computations, we chose a realistic anisotropic tetrahedral FE model with 147287 nodes and 892115 elements, a 71 electrode EEG and a 147 channel MEG configuration. We compared the computation time for the construction of the EEG and MEG lead field bases for the Jakobi-preconditioned CG (J-CG), the symmetric Incomplete Cholesky preconditioned CG without fill-in (symIC(0)-CG) and the AMG-CG (see [Wolters, 2001] for these approaches) with the new MultiRHS-AMG-CG while varying the number of simultaneously treated RHSs. Speedup tests were performed on three different platforms, a Mac-OSX with PowerBook G4 proc (1Ghz, 512 KB cache), a Red-Hat Linux PC with Xeon proc (2.4Ghz, 512 KB cache) and a Red-Hat Linux PC with Pentium 4 proc (3.2 Ghz, 1024 KB cache). The computation time for the EEG/MEG lead field bases with J-CG (symIC(0)-CG) on those three platforms were 4978/13361 sec. (2620/7261 sec.), 1543/4061 sec. (729/2089 sec.) and 929/2512 sec. (498/1322 sec.). The results for AMG-CG and MultiRHS-AMG-CG for MEG and EEG are shown in Figs. 1 and 2. The computation time for a specific number of simultaneous RHSs is indicated above the curves.

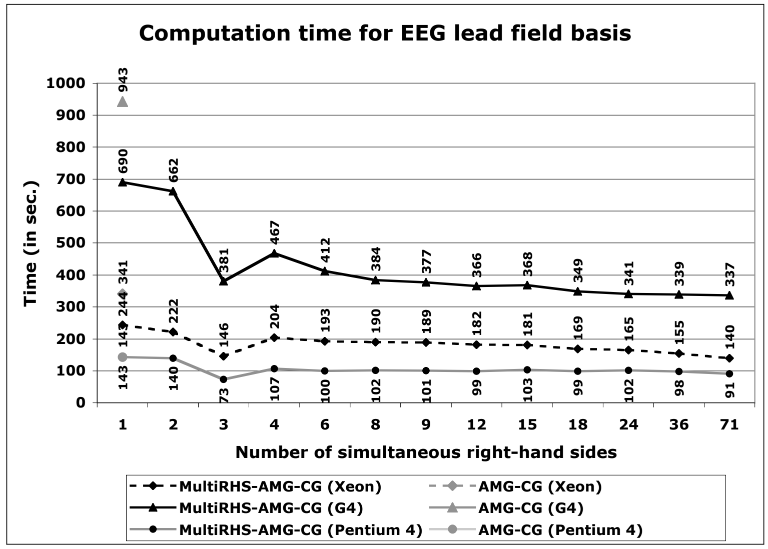

Figure 2 Time for the computation of the EEG lead field basis on a Mac-OSX PowerBook G4 and on Red-Hat Linux PC's with either Xeon or Pentium 4 architecture using the conventional AMG-CG and the new AMG-CG with simultaneous treatment of multiple right-hand sides.

DISCUSSION

On all platforms, the treatment of multiple RHSs within the new MultiRHS-AMG-CG reduced the computation time for the EEG and MEG lead field bases by at least a factor of 2. With the manual unrollment of inner loops for the simultaneous treatment of 3 RHS, this approach belongs to the fastest on all platforms so that we would recommend that choice for the RHS parameter. On platforms with a smaller cache and a slower access to the main memory, the improvement of the data-structures by means of the CSR storage for stiffness and interpolation matrices led to a further speedup factor of up to 1.38.

The combination of the lead field bases concept [Wolters, 2004] with the presented MultiRHS-AMG-CG solver for the setup phase reduces significantly the complexity of anisotropic high resolution finite element head modeling. The computation for the presented head model with nearly a million tetrahedral elements can be performed on a single processor platform in roughly the same time as boundary element head modeling. If the resolution still has to be increased (e.g., to the resolution of the MRI), the use of the parallel version of our software NeuroFEM-Pebbles [Anwander, 2002] [Wolters, 2002] gets necessary in order to distribute the memory.

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