Visualizing Patterns in the Poincaré Plot of a Magnetic Field

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ABSTRACT

In the development of magnetic fusion reactors which will be the source for future low cost power physicists must be able to analyze the magnetic fields that confine the burning plasma. The corresponding magnetic fieldlines have a periodic or quasi-periodic behavior. In this paper we describe a simple and efficient geometric technique that allows for effective visualization and topological analysis of these fieldlines through their signature in the Poincaré map.

Keywords:

1 INTRODUCTION

Traditionally the behavior of periodic, quasi-periodic, and chaotic 3D orbits can be visualized using a Poincaré plot, which is obtained by intersecting an integral curve with a transverse 2D plane (Poincaré section). By collecting a large number of intersections a pattern may form, which is used to visualize and analyze the dynamics of the 3D system. For our application we are interested in the patterns exhibited by magnetic fieldlines that are contained within the toroidal geometry of a magnetic fusion reactor, Figure (1).

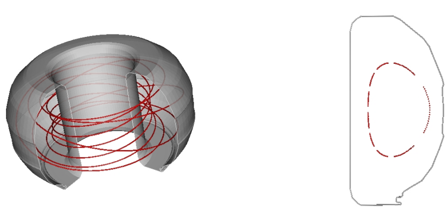

Figure 1: *(a) Profile of the DIII-D Tokamak and a single quasi-periodic magnetic fieldline. (b) The corresponding Poincaré plot for the magnetic fieldline in (a) using 200 points.*

Locating magnetic fieldlines that are periodic is an important component in understanding plasma transport in magnetic fusion research. As such, our motivation is to develop a technique that allows for the rapid visualization and facilitates the analysis of topological structures of magnetic fieldlines. The methods presented in this work are purely geometric, local in nature, and geared for parallel computations. Emphasis is on robustness and computational efficiency while ensuring reliable results for visual analysis.

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2 BACKGROUND AND PREVIOUS WORK

Previous researchers have located periodic fieldlines using numerical approaches such as those by [2]. However, these methods are computationally expensive and lack robustness, especially in the complicated geometries of modern experiments. Others have used a graph-based approach and machine learning techniques to classify the fieldlines [1]. However, in order to obtain an reasonable accuracy a large number, 2000-2500 of intersection points per fieldline were necessary. Obtaining this many points requires a large number of integration steps which is prone to numerical inaccuracies that may lead a fieldline to follow an erroneous path.

A related approach is that of Wischgoll and Scheuermann [4] that examines how a fieldline reenters a cell and reconnects. However, their method requires that closed orbits attract or repel neighboring streamlines which is not the case in solenoidal magnetic fields. It is also worth noting the work of Löffelmann et al. [3] who integrated 2D Poincaré plots with the original 3D flow for visualization purposes. However, their data contained synthetic periodic vector fields where the period was known.

3 GEOMETRIC ALGORITHM

A key property characterizing the behavior of a fieldline is its safety factor. It is defined as $q = \lim_{n_T \to \infty} \frac{n_T}{\#_\theta(n_T)}$, where $\#_\theta(n_T)$ denotes the number of poloidal rotations (measured along the minor circle of the torus) of a fieldline after n_T toroidal "windings" (around the major circle). Excluding the case of chaotic fieldlines, such a limit exists for a given fieldline. A rational q usually implies that the fieldline is periodic whereas irrational values of q correspond to fieldlines that are quasi-periodic. For both cases we determine a rational approximation of its safety factor as follows:

1) Number the successive intersections p_i of the fieldline with the Poincaré section.

2) For a given value of windings n_T, and for each $i \in \{0,..,n_T - 1\}$ connect the point group $\{p_i, p_{i+n_T}, p_{i+2n_T}, ...\}$ using piecewise linear segments.

3) Connect points until a connection of one winding group intersects either itself or another group.

A valid value of n_T is found if the first connection in each group does not intersect any other connections. If the multiple windings values are possible arbitrarily select the smallest winding value.

This is demonstrated in Figure (2) for a fieldline that intersects the Poincaré section 30 times. It should be noted that the curves shown in Figure (2b) represent the cross section of an irrational surface, which is represented as five discrete curves instead of one continuous curve to underscore the winding. A second example is shown in Figure (3) for a fieldline that intersects the Poincaré section 75 times. In this case, it is difficult to discern that the points form three distinct structures until they are connected.

Above examples show Poincaré plots for individual fieldlines which is rarely the case. Typically, 200 to 1000 fieldlines with 200 intersecting points per fieldline are used. Besides the heavy computation required the resulting clutter makes the task of identify-

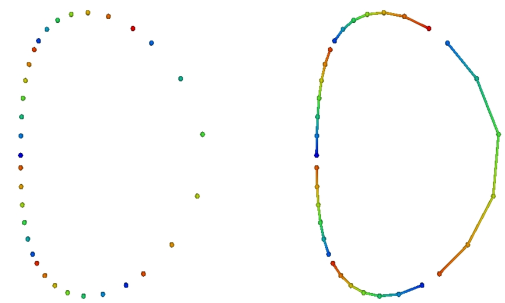

Figure 2: *(a) the original 30 points (b) the points connected using a winding value of 5. The points are colored based on their intersection ordering, blue to red*

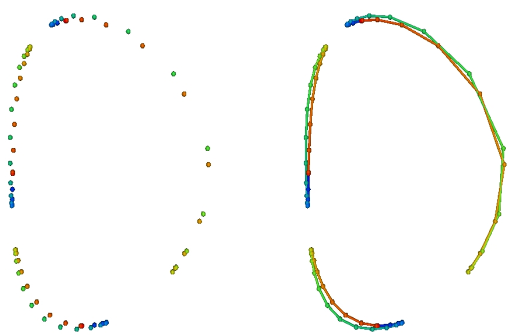

Figure 3: *(a) the original 75 points (b) the points connected using a winding value of 3. The points are colored based on their intersection ordering, blue to red*

ing the resulting structures very difficult. For example, Figure (4a) combines the Poincaré plots from Figures (2a) and (3a). It is not until the segments are connected that the structure becomes clear.

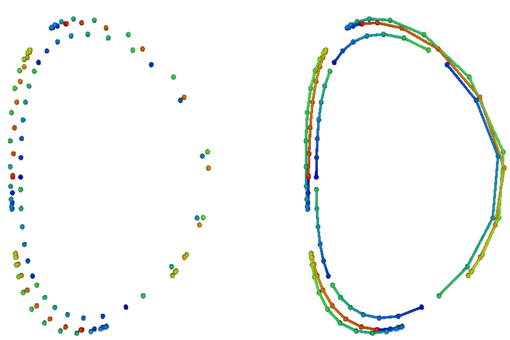

Figure 4: *(a) the Poincaré plot for two fieldlines (b) the points connected using a winding value of 3 and 5 respectively. The points are colored based on their intersection ordering, blue to red*

4 Periodic Fieldline Detection

When a fieldline is quasi-periodic the intersecting points on the Poincaré plot, when fully connected will form one of two topologies: a single closed curve or multiple closed curves, as shown in Figure (2b) and (3b) respectively. Within the latter, known as an island chain, there will be a single periodic fieldline that is near the geometric center of an island. The geometric center can be found by taking the median of the main sheet of the medial axis of the island. To achieve greater accuracy a new fieldline can be started at the geometric center and analyzed in an iterative fashion.

5 Results and Discussion

The technique has been applied to simulations of two different magnetic fusion reactors. The first example, Figure (5a), shows a series of island chains and two flux surfaces from a M3D simulation of the CDX-U Tokamak. The second example, Figure (5b), shows three island chains among a series of flux surfaces from a NIMROD simulation of the DIII-D Tokamak.

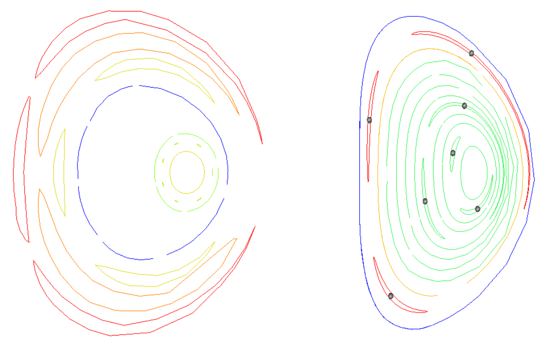

Figure 5: *(a) A series of island chains and two flux surfaces from a M3D simulation of the CDX-U Tokamak. (b) Three island chains along with the location of their respective periodic fieldline among a series of flux surfaces from a NIMROD simulation of the DIII-D Tokamak. Each is colored based on its safety factor.*

6 Conclusions

We have presented the application of a geometric technique that allows for the creation of Poincaré plots using a minimal set of connected points. This greatly reduces the computational costs when compared to traditional Poincaré plots. The technique permits the identification of magnetic island chains and location of the periodic fieldline contained within them.

Our on going work includes validating the technique in datasets containing chaotic fieldlines and the detection of the separatices that surround the island chains.

7 Acknowledgments

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