VISUALIZING PATTERNS IN THE POINCARÉ PLOT OF A MAGNETIC FIELD

Allen R. Sanderson, University of Utah
Xavier Tricoche, University of Utah
Christoph Garth, University of Kaiserslautern
Scott Kruger, Tech-X Corporation
Carl Sovenic, University of Wisconsin
Eric Held, Utah State University
Joshua Breslau, PPPL

www.sci.utah.edu
THE UNIVERSITY OF UTAH

GOAL - develop a technique that allows rapid visualization and facilitation for the analysis of topological structures of magnetic fieldlines in a Poincaré plot.

DATA - magnetic fieldlines that are contained within the toroidal geometry of the DIII-D Tokamak fusion reactor as shown in Figure 1.

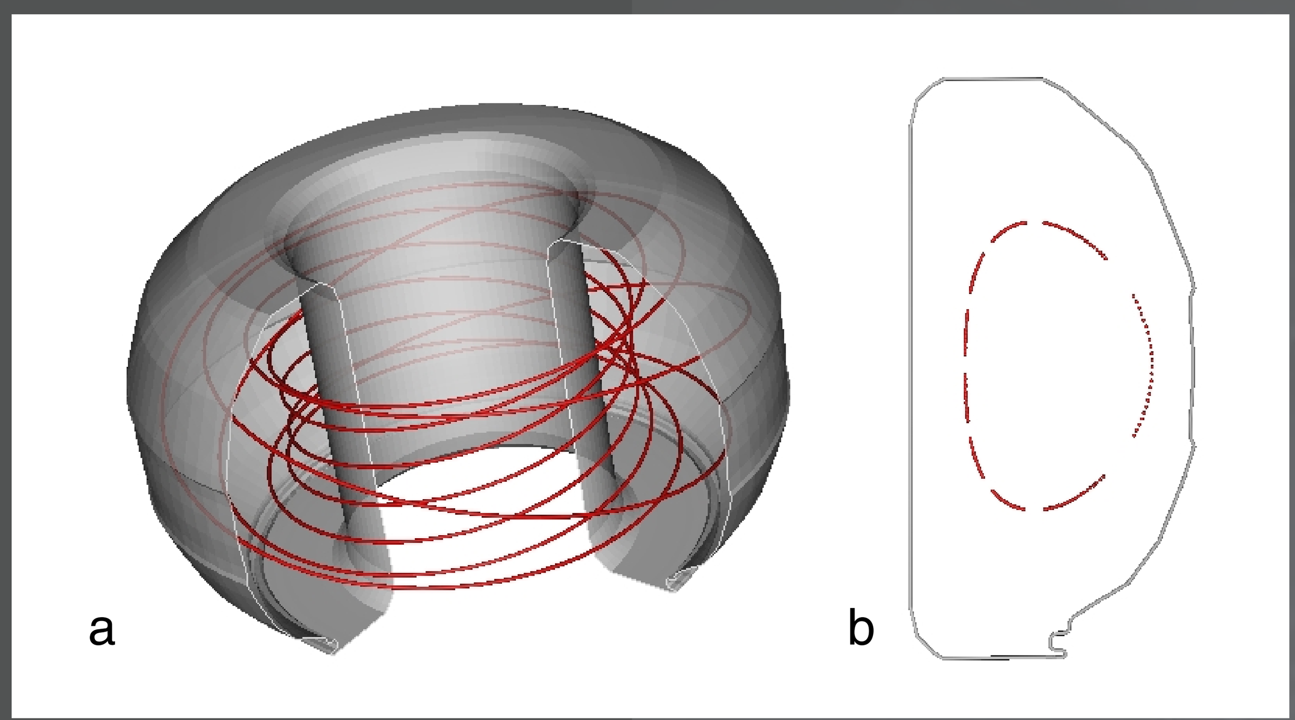

Figure 1. (a) Profile of the DIII-D Tokamak and a single quasi-periodic magnetic fieldline. (b) The corresponding Poincaré plot for the magnetic fieldline in (a) using 200 points.

PROBLEM - Computational expense and robustness, as well as obtaining many points, requires a large number of integration steps which is prone to numerical inaccuracies that may lead a fieldline to follow an erroneous path. In addition, a large number of points can lead to visual ambiguity when there is no correspondence between them.

PROPOSED SOLUTION - A geometric technique that is local in nature, and geared for parallel computations. Emphasis is on robustness and computational efficiency while ensuring reliable results for visual analysis.

GEOMETRIC ALGORITHM - A key property characterizing the behavior of a fieldline is its safety factor. It is defined as:

$$q = \lim_{n_T \to \infty} \frac{n_T}{\#\Theta(n_T)}$$

where $\#\Theta(n_T)$ denotes the number of poloidal rotations (measured along the minor circle of the torus) of a fieldline after n_T toroidal "windings" (around the major circle). Excluding the case of chaotic fieldlines, such a limit exists for a given fieldline. A rational q usually implies that the fieldline is periodic whereas irrational values of q correspond to fieldlines that are quasi-periodic. For both cases we determine a rational approximation of its safety factor as follows:

1) Number the successive intersections p_i of the fieldline with the Poincaré section.

2) For a given value of windings n_T, and for each $i \in \{0, .., n_{T-1}\}$ connect the point group $\{p_i, p_{i+nT}, p_{i+2nT}, ...\}$ using piecewise linear segments.

3) Connect points until a connection of one winding group intersects either itself or another group.

A valid value of n_T is found if the first connection in each group does not intersect any other connections. If the multiple winding's values are possible, arbitrarily select the winding that uses the greatest number of points.

This is demonstrated in Figure (2) for a fieldline that intersects the Poincaré section 30 times. It should be noted that the curves shown in Figure (1b) represent the cross section of an irrational surface, which is represented as five discrete curves instead of one continuous curve to underscore the winding. A second example is shown in Figure (3) for a fieldline that intersects the Poincaré section 75 times. When n_i is not correct intersection connections result as shown in Figure 4.

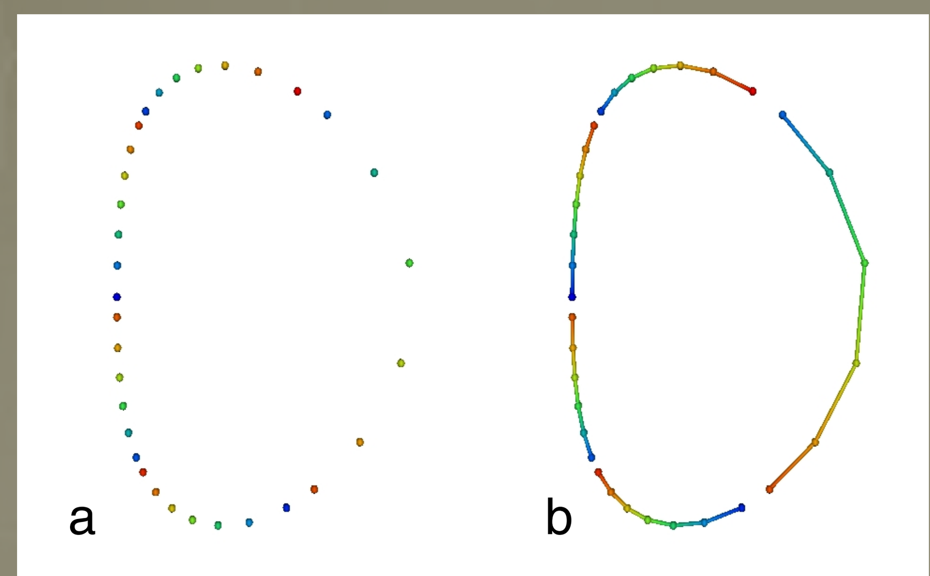

Figure 2 (a) the original 30 points (b) the points connected using a winding value of 5. The points are colored based on their intersection ordering, blue to red.

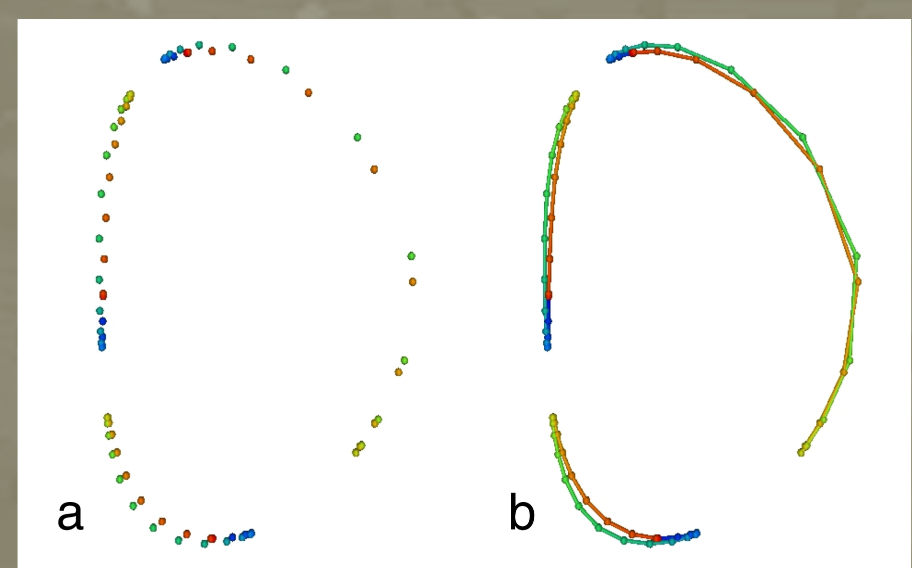

Figure 3 (a) the original 75 points (b) the points connected using a winding value of 3. The points are colored based on their intersection ordering, blue to red.

Above examples show Poincaré plots for individual fieldlines (which is rarely the case). Typically, 200 fieldines with 200 to 4000 intersecting points per fieldline are used. Besides the heavy computation required the resulting clutter makes the task of identifying the resulting structures very difficult. For example, Figure (5a) combines the Poincaré plots from Figures (2a) and (3a). It is not until the segments are connected that the structure becomes clear.

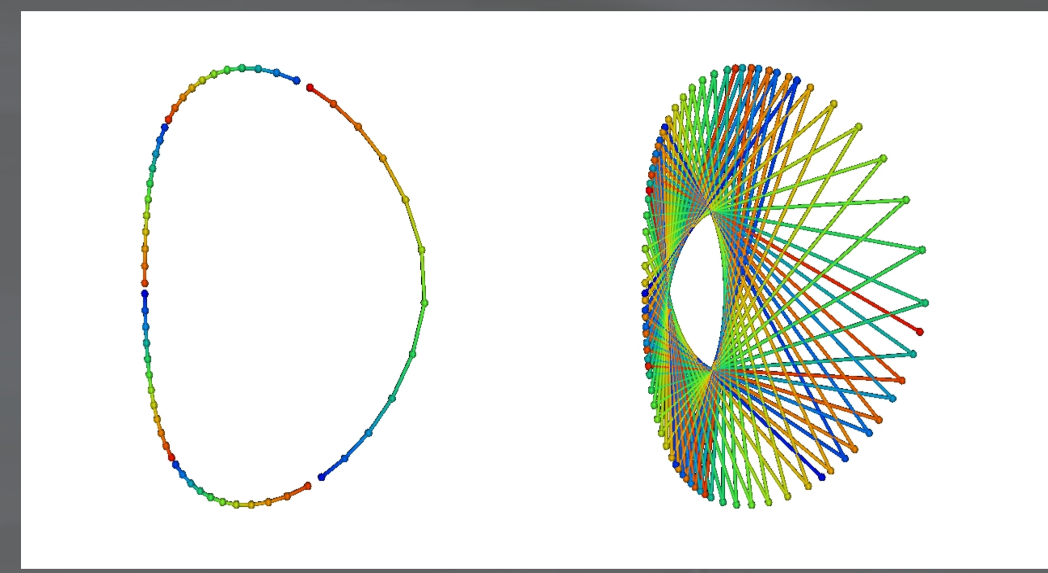

Figure 4 (a) a fieldline with the correct winding of 5 and (b) with the incorrect winding of 4 which results in intersecting connections.

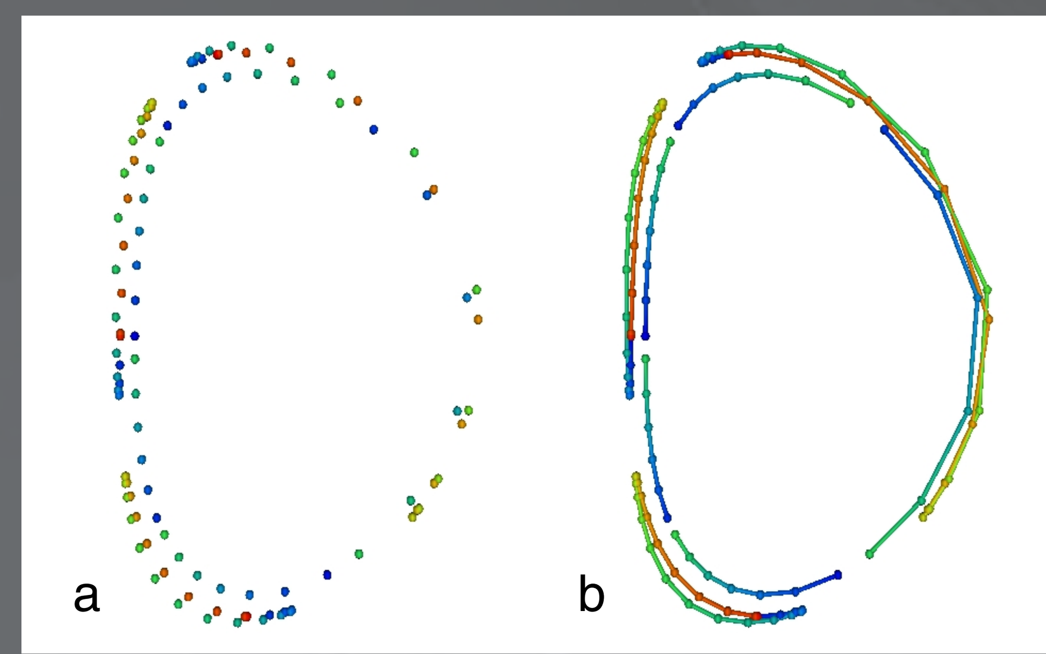

Figure 5 (a) the Poincaré plot for two fieldlines (b) the points connected using a winding value of 3 and 5 respectively. The points are colored based on their intersection ordering, blue to red.

IDENTIFYING PERIODIC FIELDLINES - When a fieldline is quasi-periodic, the intersecting points on the Poincaré plot (when fully connected) will form one of two topologies: a single closed curve or multiple closed curves, as shown in Figure (2b) and (3b) respectively. Within the latter, known as an island chain, there will be a single periodic fieldline that is near the geometric center of an island. The geometric center can be found by taking the median of the main sheet of the medial axis of the island. To achieve greater accuracy a new fieldline can be started at the geometric center and analyzed in an iterative fashion.

The technique has been applied to simulations of two different magnetic fusion reactors. The first example, Figure (6a), shows a series of island chains and two flux surfaces from a M3D simulation of the CDX-U Tokamak. The second example, Figure (6b), shows three island chains among a series of flux surfaces from a NIMROD simulation of the DIII-D Tokamak.

In the final example, we show a comparison between the typical Poincaré plot using 60 fieldlines with approximately 3700 intersecting points per fieldline (Figure (7d)) and our technique using a minimal number of points (approximately 140 intersecting points per fieldline, Figure (7a)) and connecting them based on their winding value n_i, Figure (7b). Because a minimal number of points are being used, the resulting surface contours are approximated by a series of linear segments, Figure (7b). These segments can be smoothed using spline interpolation thus giving a better over all approximation, Figure (7c).

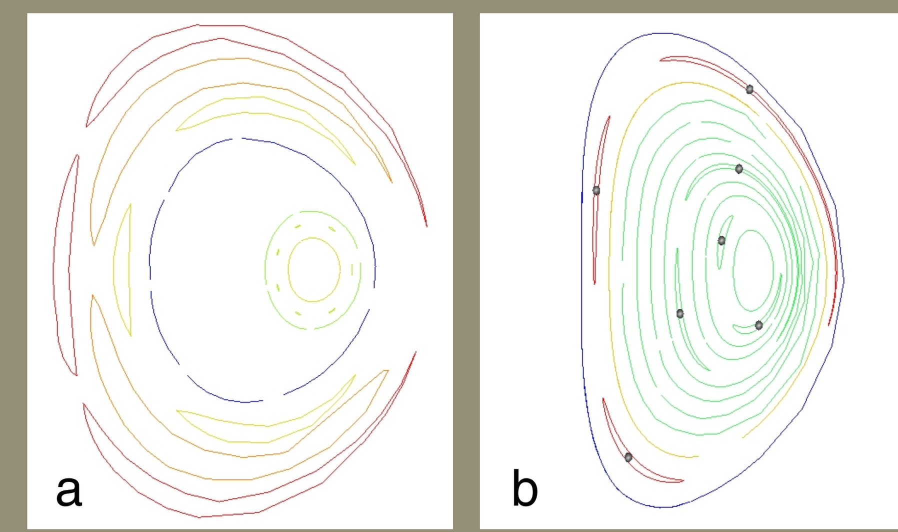

Figure 6 (a) A series of island chains and two flux surfaces from a M3D simulation of the CDX-U Tokamak. (b) Three island chains along with the location of their respective periodic fieldline among a series of flux surfaces from a NIMROD simulation of the DIII-D Tokamak. Each is colored based on its safety factor.

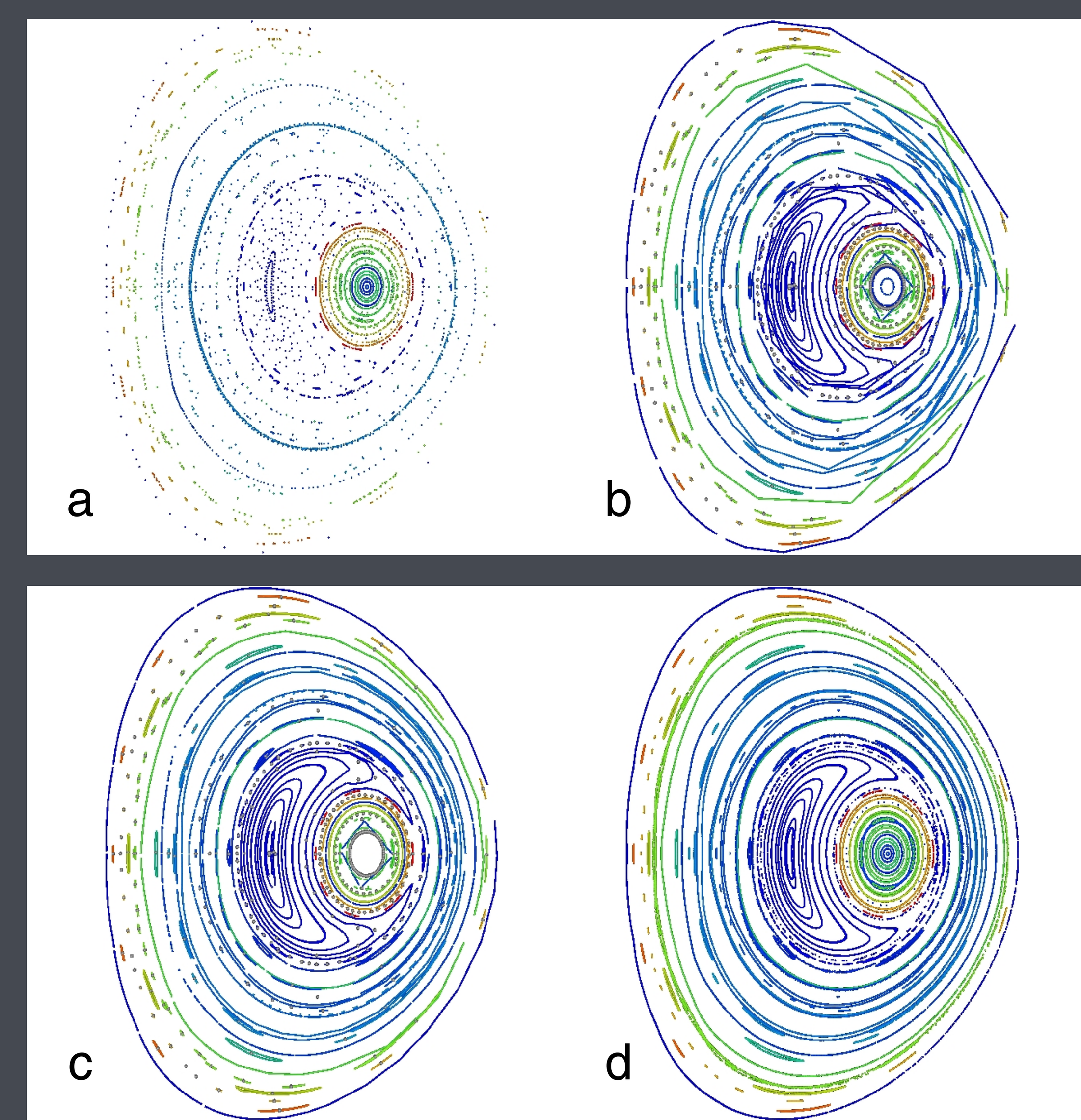

Figure 7 (a) minimal set of points for the Poincaré plot (b) the points connected as linear segments (c) the segments with spline smoothing (d) the original Poincaré plot. Each is colored based on its safety factor.

CONCLUSIONS - We have presented a geometric technique that allows for the creation of Poincaré plots using a minimal set of connected points, thus reducing the computational costs compared to traditional Poincaré plots. In addition, the technique aids in the analysis of the structure of the Poincaré plot and in particular, permits the identification of magnetic island chains and location of the periodic fieldline contained within them.

ACKNOWLEDGEMENTS - This work was supported, in part by the DOE SciDAC Center for Extended Magnetohydrodynamic Modeling using the SCIRun Problem Solving Environment.