Non-Cartesian POCSENSE

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Introduction: Projections onto convex set (POCS) formalism presents a powerful mathematical apparatus for solving many reconstruction problems that entail incomplete and inconsistent data. Recently, POCS formalism has been adopted for reconstruction of sensitivity encoded MRI data in fast and efficient iterative POCSENSE procedure [1, 2]. POCSENSE allows simple and computationally efficient inclusion of many nonlinear constraints (i.e., phase constraint) [3] in image reconstruction to improve image quality or to achieve high degrees of data acquisition speedup. However, the application of the original POCSENSE was previously limited to the Cartesian k-space sampling. In this work, we propose extension of POCSENSE technique that is able to handle data sampled on arbitrary trajectories.

Theory: The iteration of POCSENSE technique includes several projections onto data and additional convex sets (see Algorithm). Here, $s_i$ -- coil sensitivities, $i=1...N_c$, and $*$ denotes complex conjugation. Projection onto data set is accomplished by resetting the k-space samples at the trajectory positions to the original values. The reconstruction is accomplished on a Cartesian grid to make use of the fast Fourier transform algorithm (FFT). Hence, the projection is straightforward, if the original trajectory coincides with the Cartesian grid points. However, in POCS algorithms, which deal with data sampled at arbitrary positions, this projection operation gets more complicated [4].

Algorithm. POCSENSE (non-accelerated version).

Step 1: $g_i^{(n+1)} = P_{data}(s_i f^{(n)})$, $i=1, K N_c$ (Project in parallel onto data convex sets)

Step 2: $t^i = (N_c^{-1} \sum_{j=1}^{N_c} g_j^{(n+1)} s_j) / (N_c^{-1} \sum_{j=1}^{N_c} |s_j|^2)$ (Combine the projection results)

Step 3: $f^{(n+1)} = P_A(t^{(n+1)})$ (Project onto additional convex sets)

$m_{i,k}(r) = $ data acquired by the $i$-th coil at $k \cdot K$, $m_{i,k} \cdot k$-space values for the $i$-th coil image on the current iteration, $i=1, ... , N_c$. Then, the data projection step for the $i$-th coil to be used with arbitrary trajectories is defined as follows:

$$g_i^{(n+1)} = f^{data}(s_i f^{(n)}) \rightarrow a) m_{i,k}^{(n+1)} = F(s_i f^{(n)}); b) m_{i,k}^{(n+1)} = m_{i,k}^{(n)} + G_{AC}(m_{i,k} - G_{CA}m_{i,k}^*); c) g_i^{(n+1)} = F^{-1}(m_{i,k}^{(n+1)})$$

Here, $G_{AC}$ and $G_{CA}$ are interpolation operations transferring the data from the original trajectory to a Cartesian grid and vice versa, and $F$ and $F^{-1}$ are forward and inverse FFTs respectively. In order to update the k-space values, the current Cartesian k-space estimate is first regridded to the original trajectory. Then, the difference between acquired and updated values is taken to produce a set of error measurements. Finally, the error is transferred back to Cartesian grid and used to correct the k-space values.

Methods: We utilized standard gridding with a Kernel-Bessel kernel [5] to implement interpolation operations, and Pipe’s method [6] to find sampling density compensation function. Our implementation of Eq. (1) is depicted in Fig. 1. The parameters of the kernel were chosen as $L=3, B=14.1372$, and the $\gamma$ denotes complex conjugation. Projection onto data set is accomplished by resetting the k-space samples at the trajectory positions to the original values. The reconstruction is accomplished on a Cartesian grid to make use of the fast Fourier transform algorithm (FFT). Hence, the projection is straightforward, if the original trajectory coincides with the Cartesian grid points. However, in POCS algorithms, which deal with data sampled at arbitrary positions, this projection operation gets more complicated [4].

Figure 1. Implementation of data projection operator using gridding interpolation. Here, $C(k)$ is k-space convolution kernel, $c(r)$ -- its image space counterpart. Sampling density compensation is accomplished before each convolution operation.

Figure 2. Reconstruction of spiral data using non-Cartesian POCSENSE. Undersampled image (a) was obtained from full spiral dataset (c) by taking every second interleave. Image after 9 iterations of non-Cartesian POCSENSE is shown in (b). The reconstructed image is characterized by minimized residual aliasing artifact and improved resolution. At the same time, the noise level is higher then in the full image (c) that is common for any method of reconstruction of reduced datasets.

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References: