POCSENSE: POCS-based Reconstruction Method for Sensitivity Encoded Data

Eugene G. Kholmovski[1,2], Alexei A. Samsonov[1,3], Dennis L. Parker[1,2,4]

[1]Departments of Physics, University of Utah, Salt Lake City, Utah, USA; [2]MIRL, University of Utah, Salt Lake City, Utah, USA; [3]SCI Institute, University of Utah, Salt Lake City, Utah, USA; [4]Department of Radiology, University of Utah, Salt Lake City, Utah, USA

A novel method for reconstruction of undersampled data from multiple receivers has been developed. The method uses coil sensitivity profiles and alternating Projection Onto Convex Sets (POCS) formalism to recover missing k-space data. POCS formulation of the problem permits natural incorporation of valid constraints (coil sensitivity profiles, acquired k-space data, and image support) in the reconstruction process as convex sets. The method does not require any type of computationally expensive matrix inversion operation and is highly time efficient. The feasibility and efficiency of the proposed method were demonstrated using phantom and human MRI data.

Introduction

Recently proposed methods of reconstruction of undersampled data from multiple receivers such as SENSE [1] and SMASH [2] show the great potential of parallel acquisition methods for scan time reduction. SENSE represents a general approach to algebraic reconstruction from aliased data in image space. SMASH generates missing k-space lines by linear combination of acquired data from multiple coils. Both methods have serious limitations: SMASH requires special coil configurations, while SENSE needs computationally expensive matrix inversion operations for image reconstruction.

A novel method for reconstruction of undersampled data from multiple coils using their sensitivity profiles and POCS formalism [3] has been developed. Resulting image is an intersection of convex sets defined by coil sensitivity profiles, acquired k-space data, and image support.

Method

Let K be a set of k-space positions corresponding to the sampling on the Cartesian grid that gives the desired field of view and resolution, $\tilde{K}_i(\boldsymbol{k}), \boldsymbol{k} \in \tilde{K}$ - k-space data acquired by i-th coil and preprocessed to have support on K, and $S_i = S_i(\boldsymbol{r})$ - i-th coil sensitivity profile. Denoting $g^{(n)} = g^{(n)}(\boldsymbol{r})$ and $g_i^{(n)} = g_i^{(n)}(\boldsymbol{r})$ to be the resulting image and i-th coil image after the n-th iteration, we define the following projection operators:

$$P_1^i : P_1^i g^{(n)} = S_i g^{(n)},$$

$$P_2 : P_2 g_i^{(n)} = F^{-1}\{T \cdot F\{g_i^{(n)}\}\} = F^{-1}\{T \cdot G_i^{(n)}\} = F^{-1}\{\tilde{G}_i^{(n)}\},$$

where F and F^{-1} are forward and backward Fourier transforms.

$$\tilde{G}_i^{(n)}(\boldsymbol{k}) = T \cdot G_i^{(n)} = K_i(\boldsymbol{k}) \cdot W(\boldsymbol{k}) + G_i^{(n)}(\boldsymbol{k}) \cdot (I - W(\boldsymbol{k})),$$

$$P_3 : P_3 g^{(n)}(\boldsymbol{r}) = \begin{cases} g^{(n)}(\boldsymbol{r}), & \boldsymbol{r} \in M \\ 0, & \boldsymbol{r} \notin M \end{cases},$$

where P_1^i is a projection onto the set of images acquired by the coil with sensitivity S_i, P_2 is a projection onto the set of images with k-space equivalent to the acquired k-space data, P_3 is a projection on the set of images with support M in image space, I is the identity matrix. $W = W(\boldsymbol{k})$ is a weighting matrix that in the case of Cartesian acquisition has binary entries corresponding to the sampling pattern.

Reconstruction Algorithm

Step 1: $g_i^{(n)} = P_3 P_2 P_1^i g^{(n-1)}, \qquad i = 1 \ldots N$,

Step 2: $g^{(n)} = \left(\sum_{i=1}^{N} w_i g_i^{(n)} S_i^*\right) \Big/ \left(\sum_{j=1}^{N} w_j S_j S_j^*\right), \; w_i = 1/\sigma_i^2$

where N is the number of coils used to acquire undersampled data, σ_i is noise standard deviation in i-th channel; S_i^* is complex conjugate of S_i. Step 2 is the weighted least mean square reconstruction of image from coil images and sensitivity profiles. The algorithm iterates between step 1 and 2 after some non-zero initial guess, for example, the image support mask. In the case of real time acquisitions, the result of reconstruction from the previous time frame could be used as an initial guess for the current frame.

Results

The proposed reconstruction technique was tested using computer simulation and real MR data. Reference and folded images of a phantom and a healthy volunteer were acquired on a 1.5 GE SIGNA with LX NV/CVi, gradients (GE Medical Systems, Milwaukee, WI) using custom built bilateral temporal lobe phased array coils. Sensitivity profiles and image mask (support) were obtained by methods similar to ones described in [1]. Figure 1. shows the result of POCSENSE reconstruction of a phantom image acquired with maximum reduction factor R. Aliasing was completely removed. However, the resulting image had increased noise, especially in areas where both coil sensitivity profiles has low values. The image reconstructed from complete data has a similar noise distribution. Figure 2. demonstrates the results of POCSENSE reconstruction of real brain data. The resulting image has no visible artifacts.

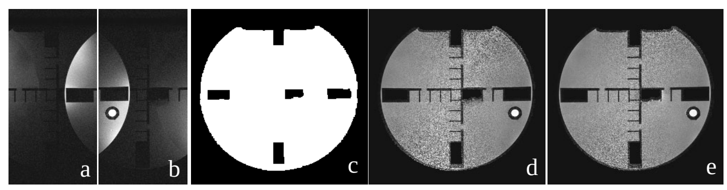

Figure 1. Reconstruction of phantom image by POCSENSE with maximum reduction factor (R=2, N=2). (**a**) and (**b**) folded images; (**c**) initial guess; (**d**) reconstructed image after 15 iterations of POCSENSE; (**e**) POCSENSE reconstructed image from completely sampled data (R=1).

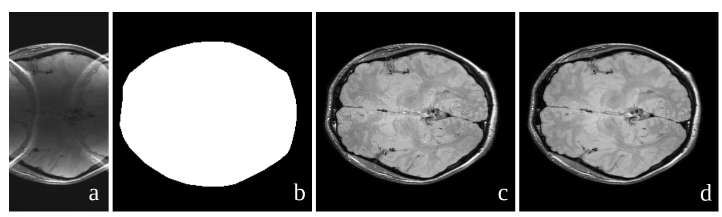

Figure 2. Reconstruction of brain image by POCSENSE from four receiver data (N=4). (**a**) standard “sum of squares” image (R=2); (**b**) initial guess; (**c**) reconstructed image after 10 iterations of POCSENSE. (**d**) POCSENSE reconstructed image from completely sampled data (R=1).

Discussion

A novel reconstruction technique for sensitivity encoded data based on POCS formalism has been developed. The resulting image is found as intersection of several convex sets by POCS iterations. The method does not require any type of matrix inversion operation and is based on the computationally efficient fast Fourier transform. The typical reconstruction time for a 256x256 image with reduction factor R=2 and N=2 is close to 1 second on a mid range PC. Rate of convergence of the proposed method depends on choice of initial guess, data acquisition reduction factor, and coil geometry. Algorithm may be easily modified to include additional constraints to achieve faster convergence or improve resulting image quality.

References

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